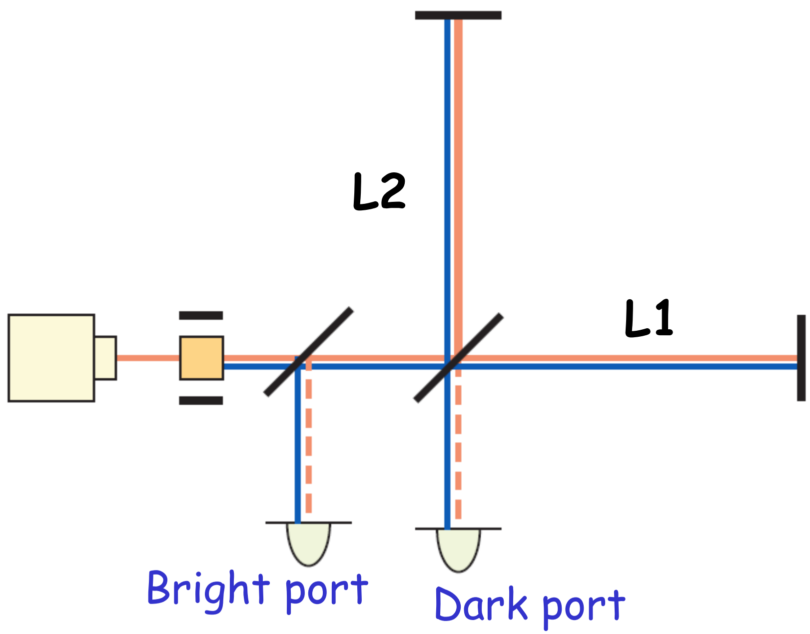
Michelson interferometer



$$L1 - L2 = \Delta\ell$$

$$\Delta\ell\,\omega / c = N\pi$$

$$\Delta\ell\,\omega_m / c = \alpha$$

Carrier at Bright port

$$E_0 e^{i\omega t}$$

Carrier at Dark port

$$i\psi_- E_0 e^{i\omega t}$$

SB at Bright port

$$\pm\frac{1}{2}E_1 e^{i(\omega\pm\omega_m)(t+\Delta\ell/c)} \pm \frac{1}{2}E_1 e^{i(\omega\pm\omega_m)(t-\Delta\ell/c)}$$

$$= E_1 e^{i(\omega\pm\omega_m)t}(\pm\cos\alpha - \psi_-\sin\alpha)$$

SB at Dark port

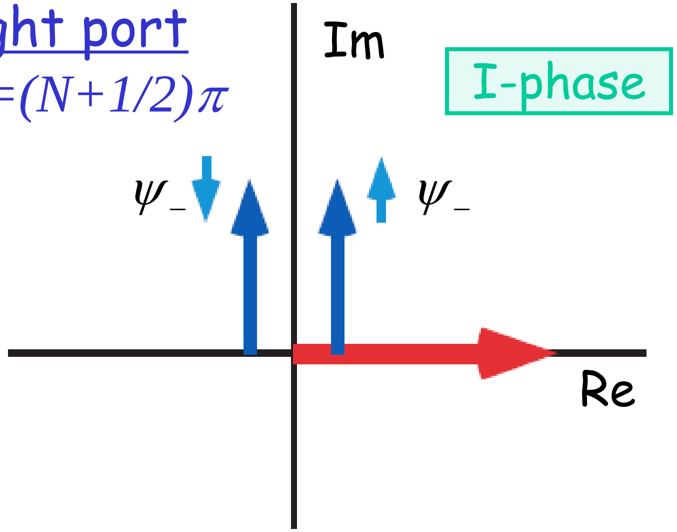
$$\pm\frac{1}{2}E_1 e^{i(\omega\pm\omega_m)(t+\Delta\ell/c)} \mp \frac{1}{2}E_1 e^{i(\omega\pm\omega_m)(t-\Delta\ell/c)}$$

$$= iE_1 e^{i(\omega\pm\omega_m)t}(\sin\alpha \pm \psi_-\cos\alpha)$$

Michelson interferometer

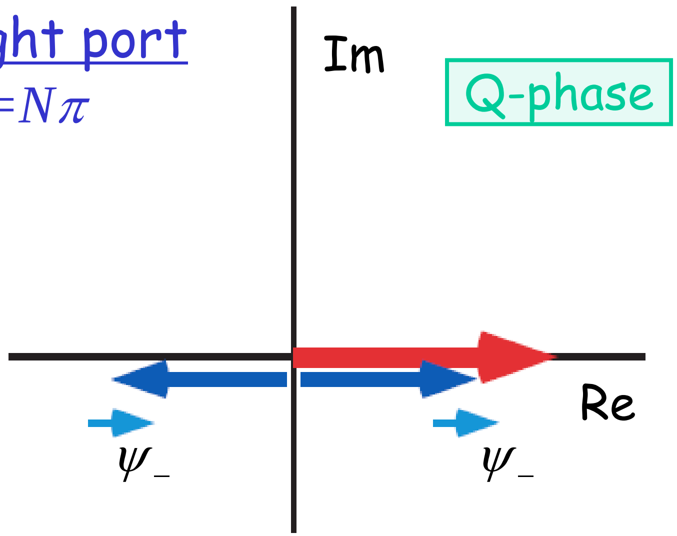
Bright port

$\omega_m t=(N+1/2)\pi$



Bright port

$\omega_m t=N\pi$



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Michelson interferometer

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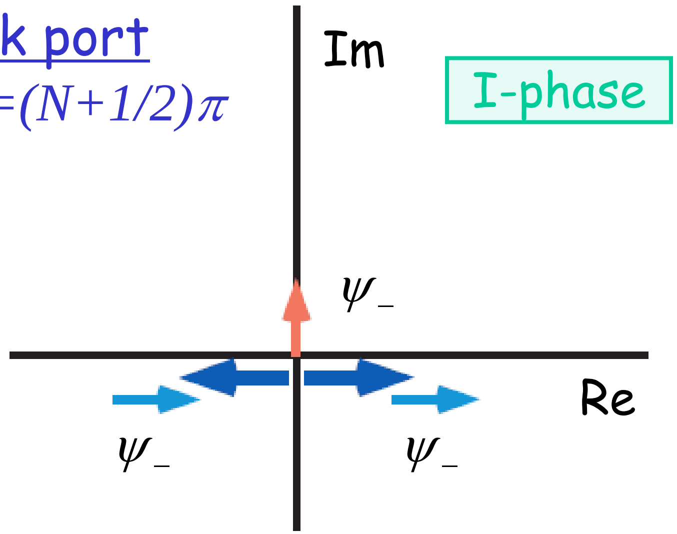
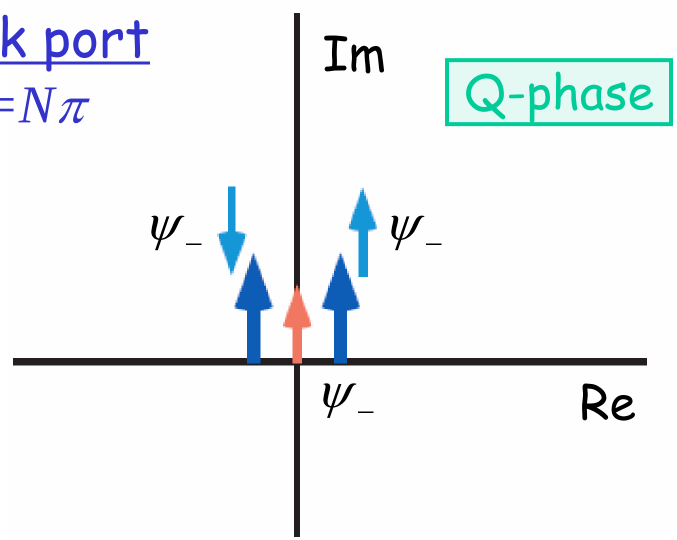


Dark port

$\omega_m t=N\pi$



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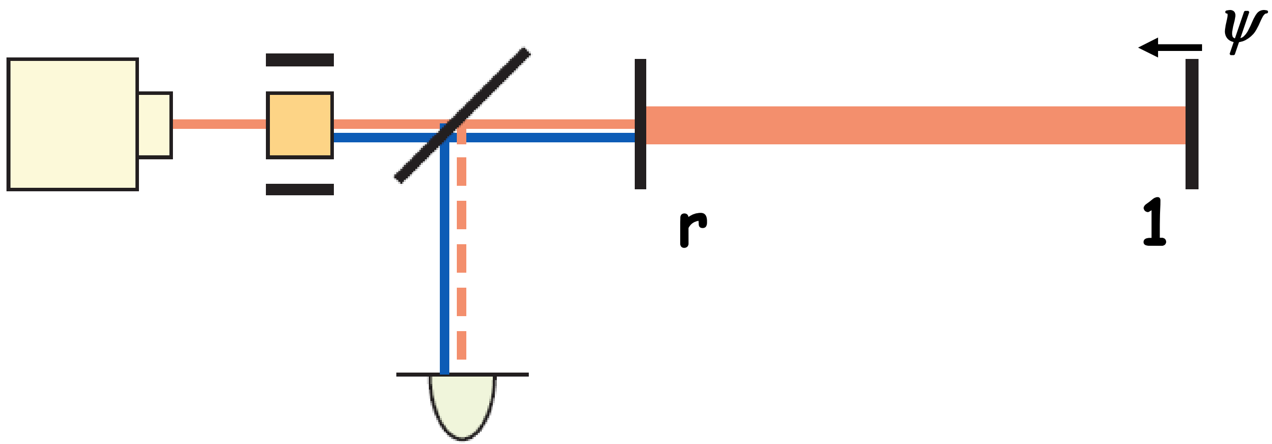
Pound-Drever-Hall technique

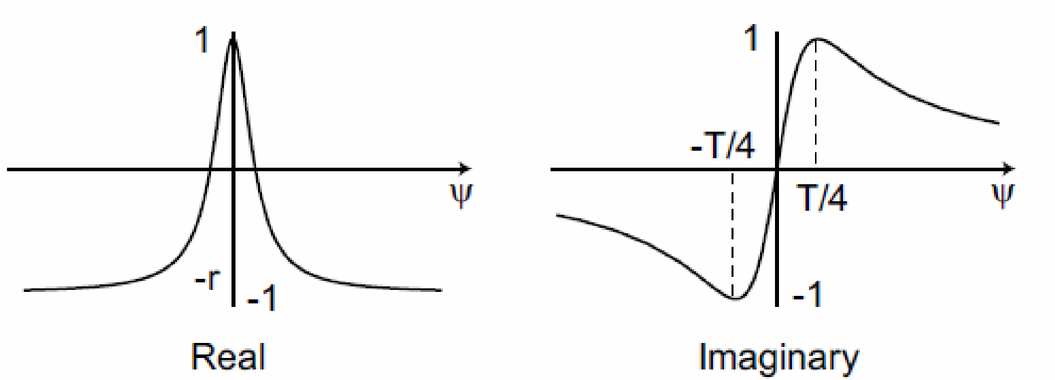


Complex reflectivity of a FP cavity is given as

T=t^2=1-r^2

$$-r+\frac{t^2e^{2i\Psi}}{1-re^{2i\Psi}}\cong\left[\frac{2T^2}{T^2+4\sin^2 2\Psi}-1\right]+i\left[\frac{4T\sin 2\Psi}{T^2+4\sin^2 2\Psi}\right]$$

The ψ dependence of the real and imaginary parts is…

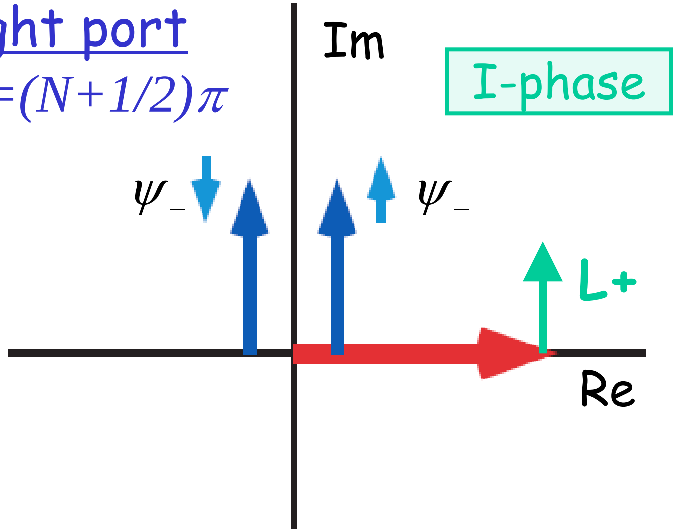


The imaginary part is linearly proportional to ψ.

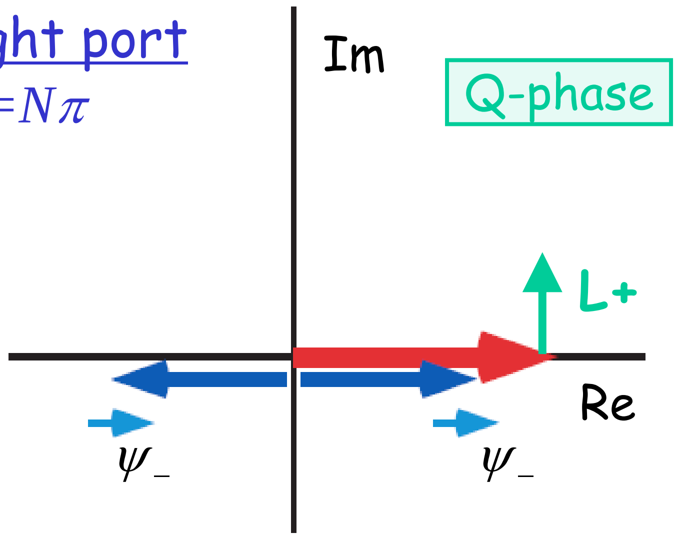
And it's amplified by 4/T.

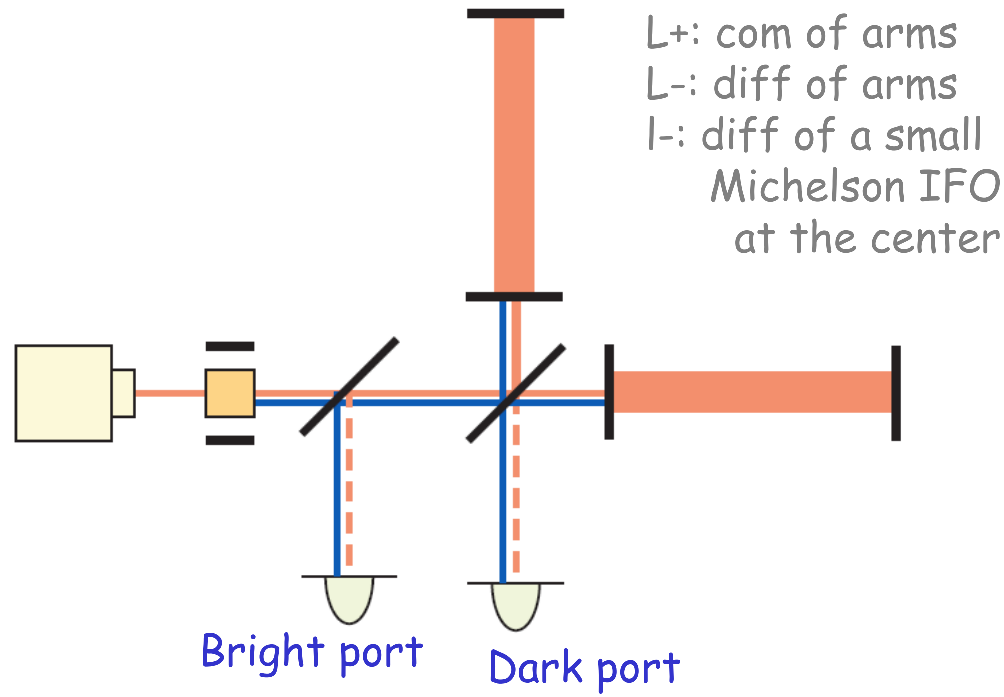
Fabry-Perot Michelson interferometer

Bright port
$\omega_m t = (N+1/2)\pi$



Bright port
$\omega_m t = N\pi$





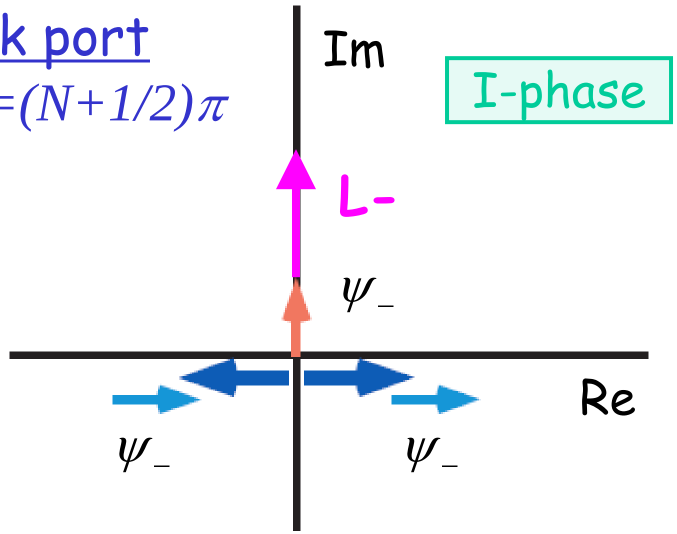
Common-mode FP signal (L+) appears as the imaginary part of carrier light.

The L+ signal can be taken in the I-phase at the bright port.

Fabry-Perot Michelson interferometer

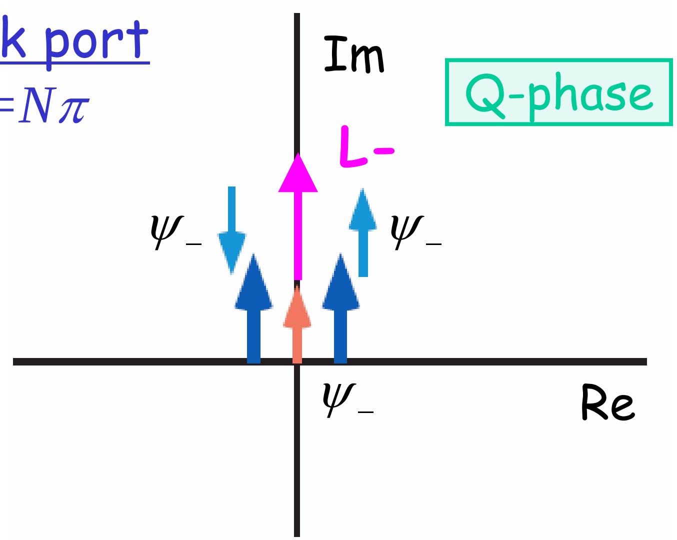
Dark port

$\omega_m t = (N+1/2)\pi$

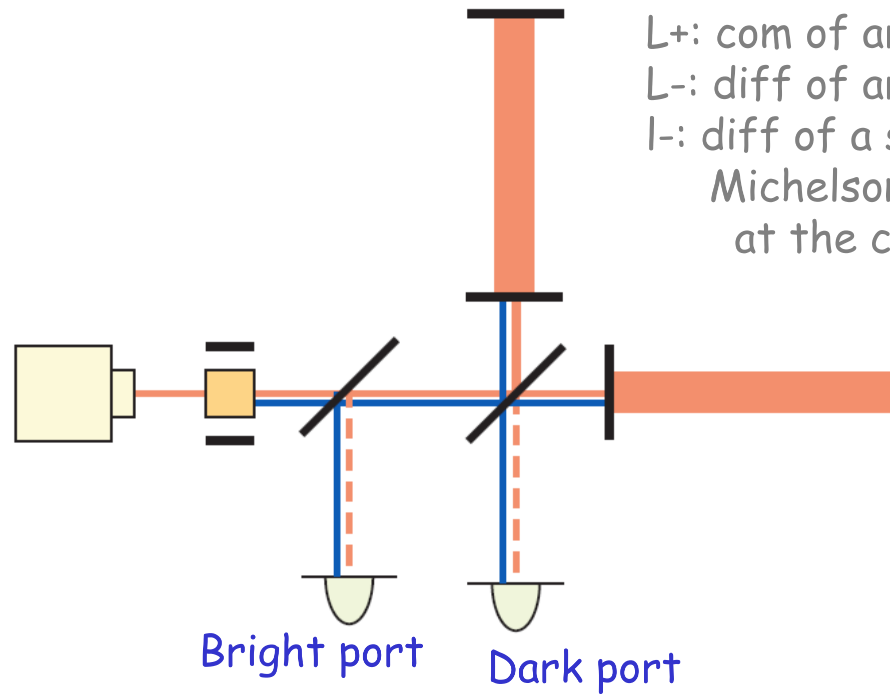


Dark port

$\omega_m t = N\pi$



L+: com of arms
L-: diff of arms
l-: diff of a small Michelson IFO at the center

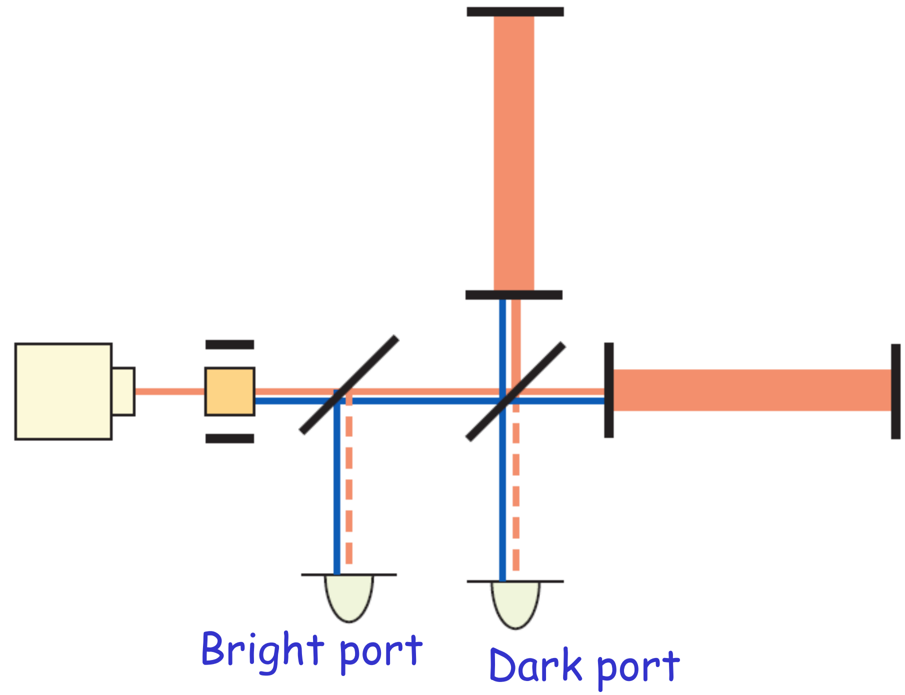


Differential-mode FP signal (L-) appears as the imaginary part of carrier light and it's larger than l- signal..

The L- signal can be taken in the Q-phase at the dark port.

Signal extraction scheme of FPMI

Port&phase	signals
BP I-phase	L+
BP Q-phase	l-
DP I-phase	(nothing)
DP Q-phase	L- and l-

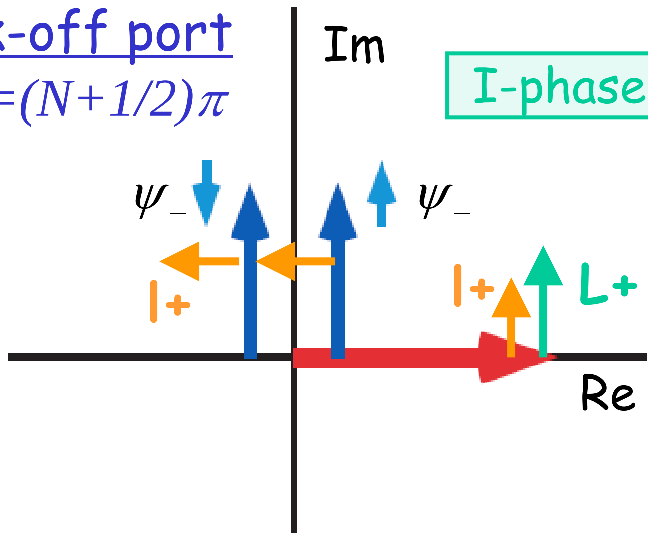


For simple Michelson IFO, the most efficient way is with $\alpha=\pi/2$.
For FPMI, L+ is proportional to $\cos\alpha$ and the others are to $\sin\alpha$,
α shall be chosen somewhere between 0 and $\pi/2$.

Power-recycled FPMI

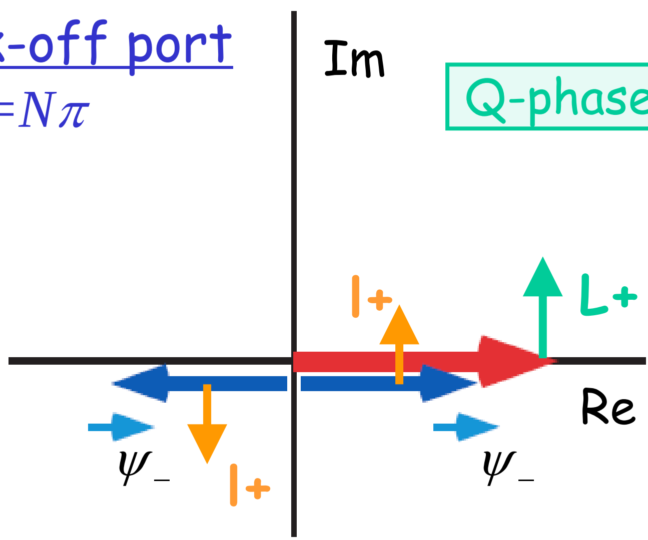
Pick-off port

$\omega_m t = (N + 1/2)\pi$

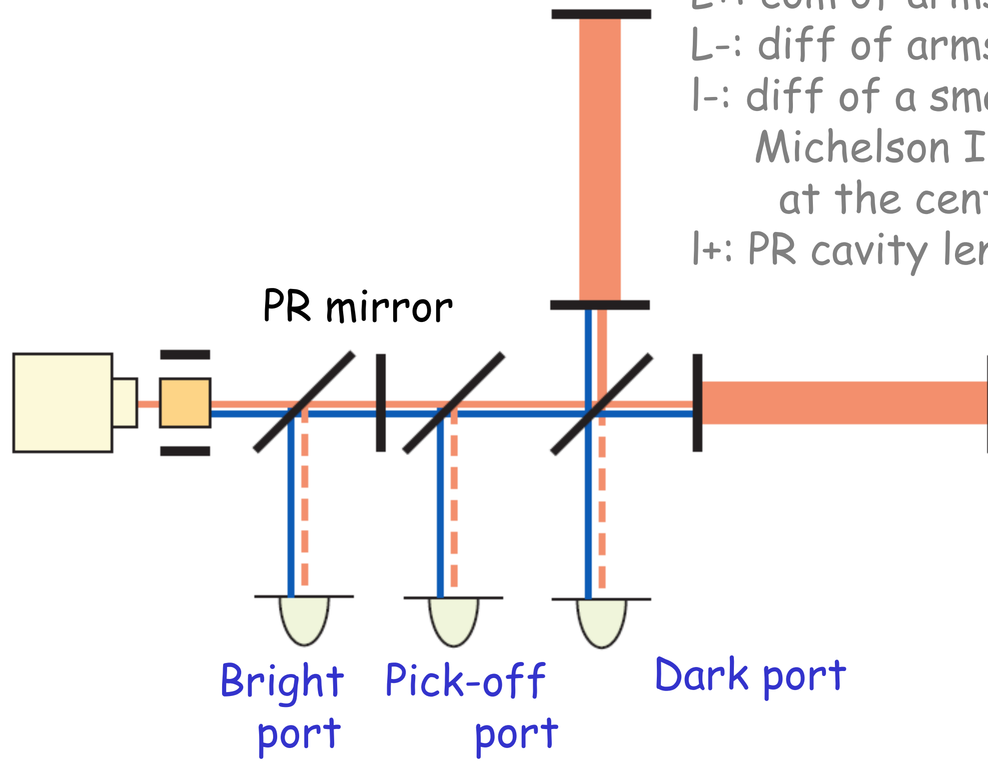


Pick-off port

$\omega_m t = N\pi$



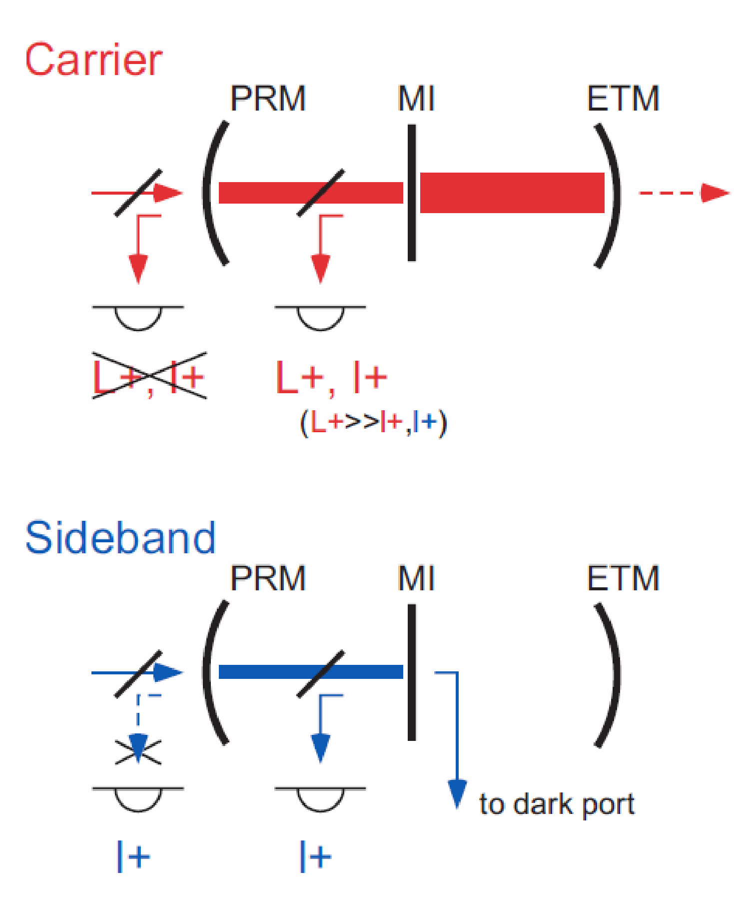
L+: com of arms
L-: diff of arms
l-: diff of a small Michelson IFO at the center
l+: PR cavity length



Q-phase signals (L- and l-) are same as FPMI and there is no problem.

L+ and l+ appear both in the I-phase.
How can we separate them?

Power-recycled FPMI



We only see average length of two arms; let's regard them as one arm.

We can also think of the central Michelson IFO as a mirror of $r=\cos\alpha$ and $t=i\sin\alpha$.

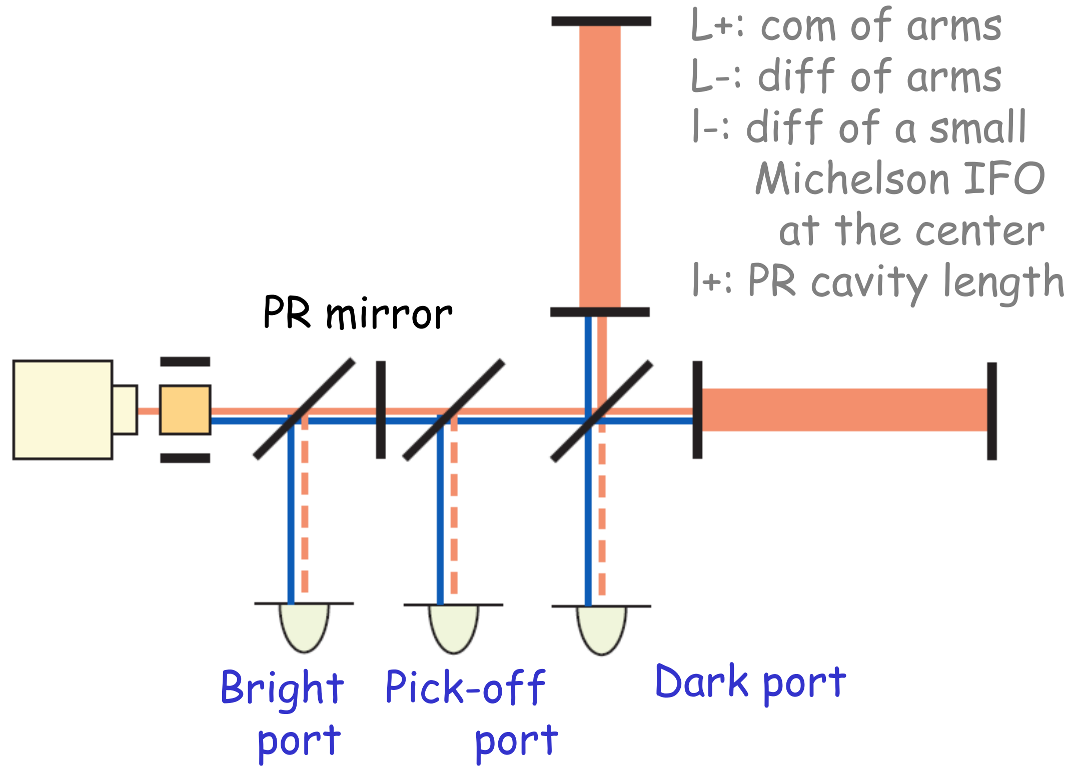
If $r_{PR}=\cos\alpha$, the sidebands will completely transmit through the dark port while the l+ signal will come back to the bright port.

$$t_{PR-MI} = \frac{t_{PR}\sin\alpha}{1-r_{PR}\cos\alpha} \rightarrow 1$$

Then L+ component on the carrier light has no light to beat with at BP, namely we can extract l+ separately from L+.

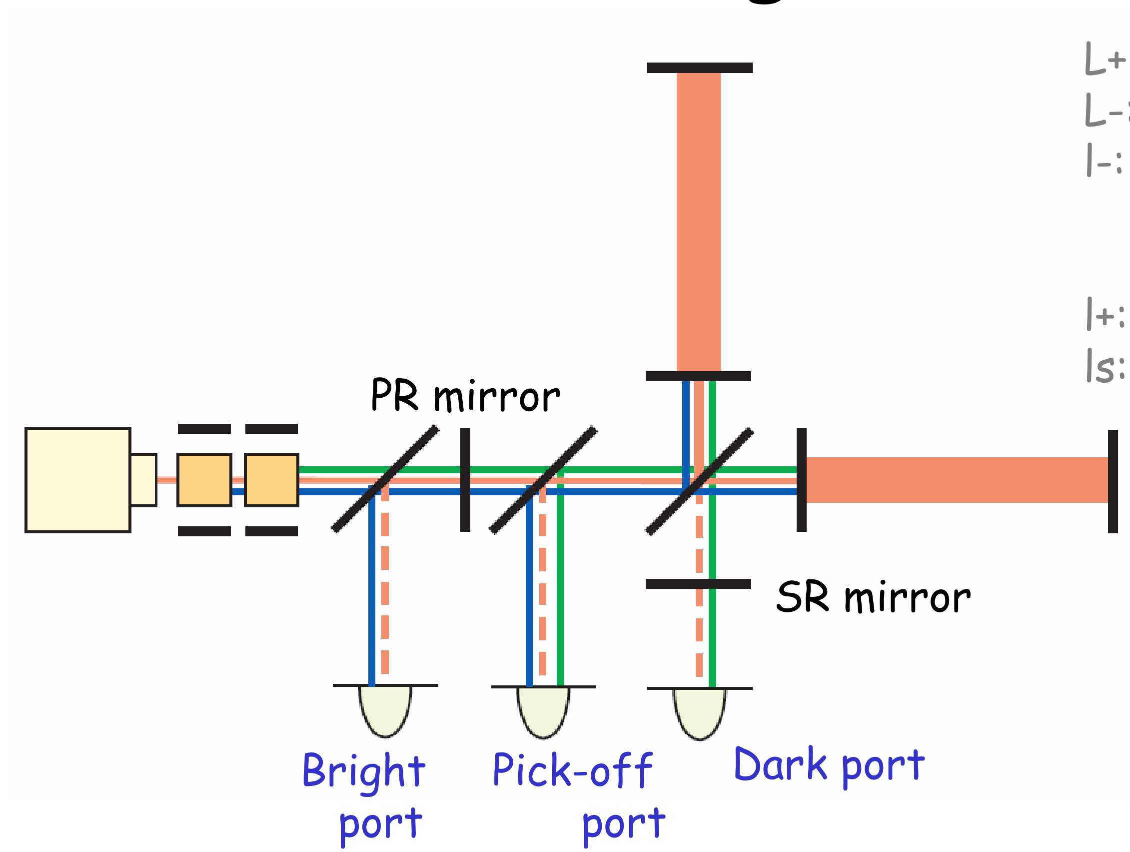
Signal extraction scheme of PRFPMI

Port&phase	signals
BP I-phase	l+
BP Q-phase	l-
PO I-phase	L+ and l+
PO Q-phase	l-
DP I-phase	(nothing)
DP Q-phase	L- and l-



For PRFPMI, the most efficient way is with $\alpha = \arccos(r_{PR})$.

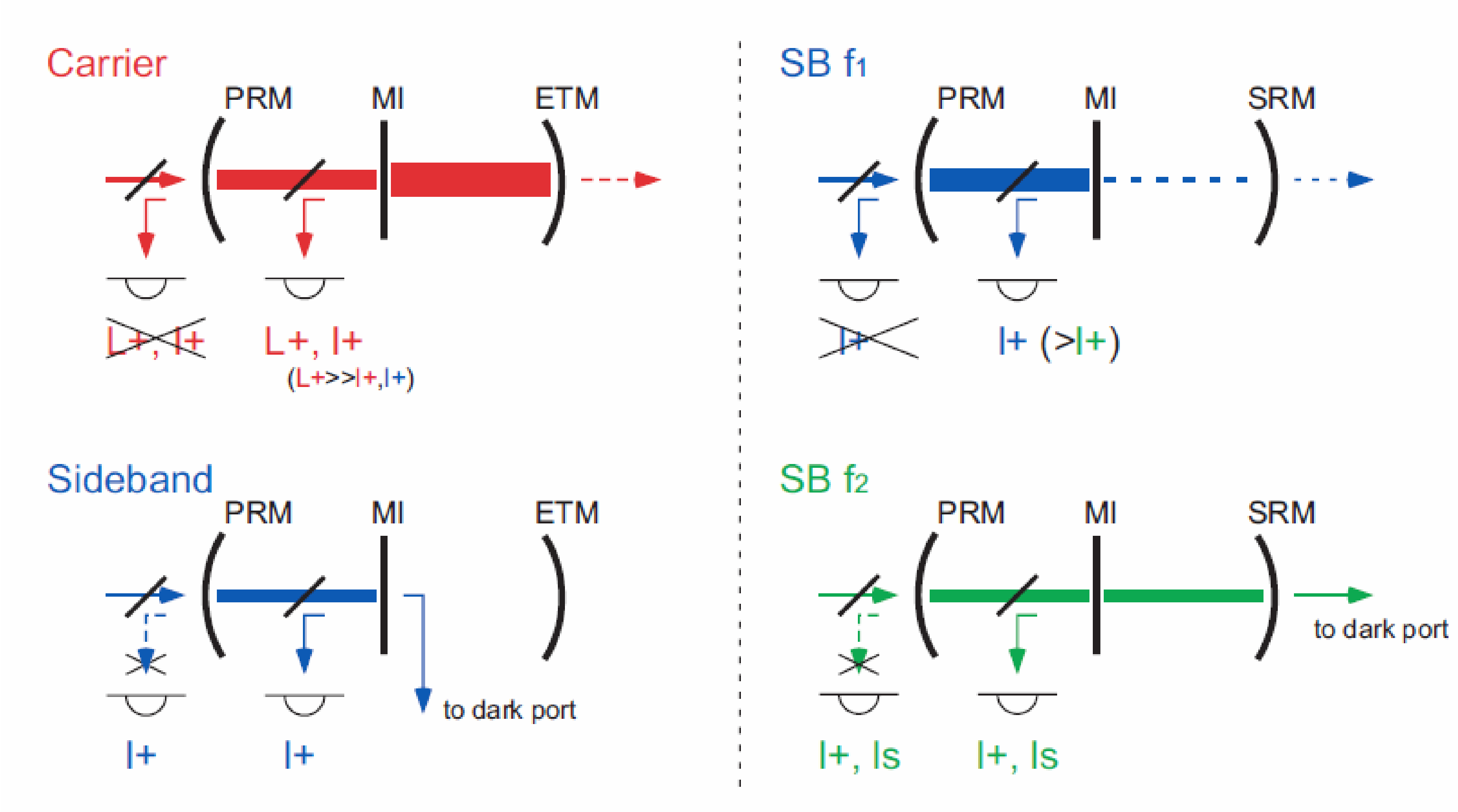
RSE (LCGT configuration)



L+: com of arms
L-: diff of arms
l-: diff of a small Michelson IFO at the center
l+: PR cavity length
ls: SR cavity length

ls signal appears also in I-phase (3 dof in I-phase).
We'll need 2 modulation sidebands.
What will be the most efficient way to control this?

RSE (LCGT configuration)



We can use the same trick to separate l+ and ls, beating the two sidebands.

Note that signals with l+ on f1 and f2 have opposite sign.

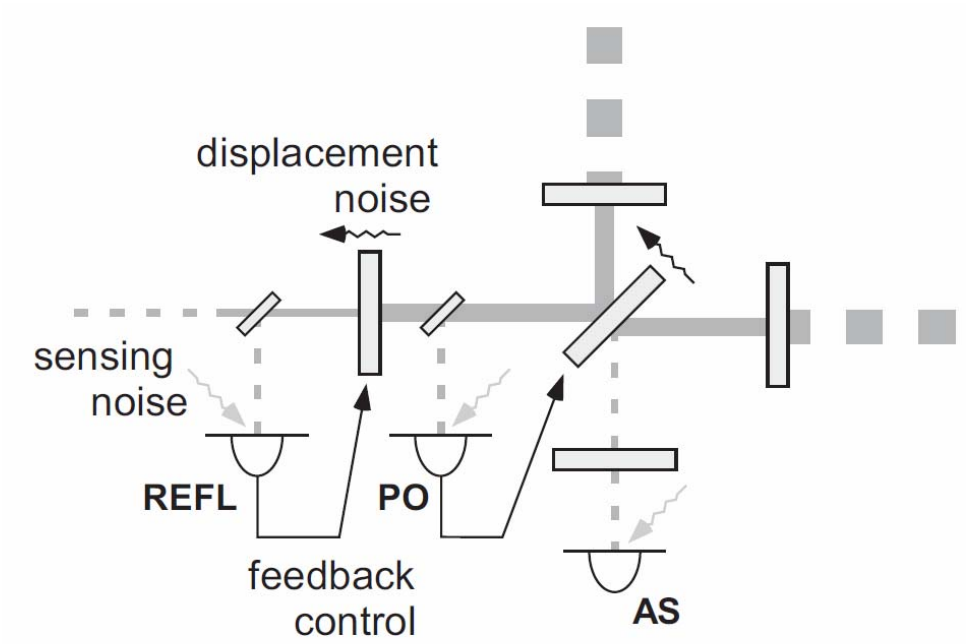
Good control scheme

There are actually many more to consider, like detuning, DC readout, etc. including practical issues like offsets, SB imbalance, etc.

How can we evaluate the control schemes?
What is the good control scheme?

It shall be all evaluated with the detector sensitivity.
Big signal with bad separation?
Good separation but small signal?
Control loop imposes sensing noise that may contaminate the detector sensitivity. We shall compare the control scheme in terms of this “loop noise”.

Loop noise

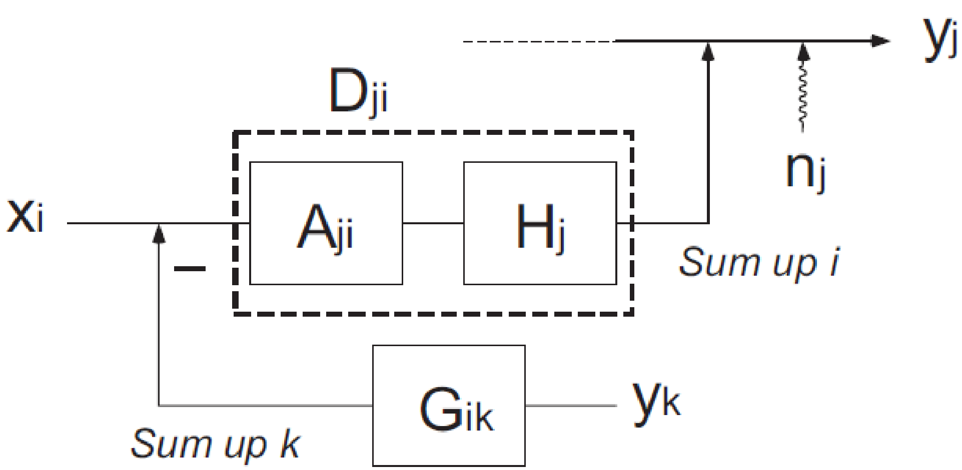


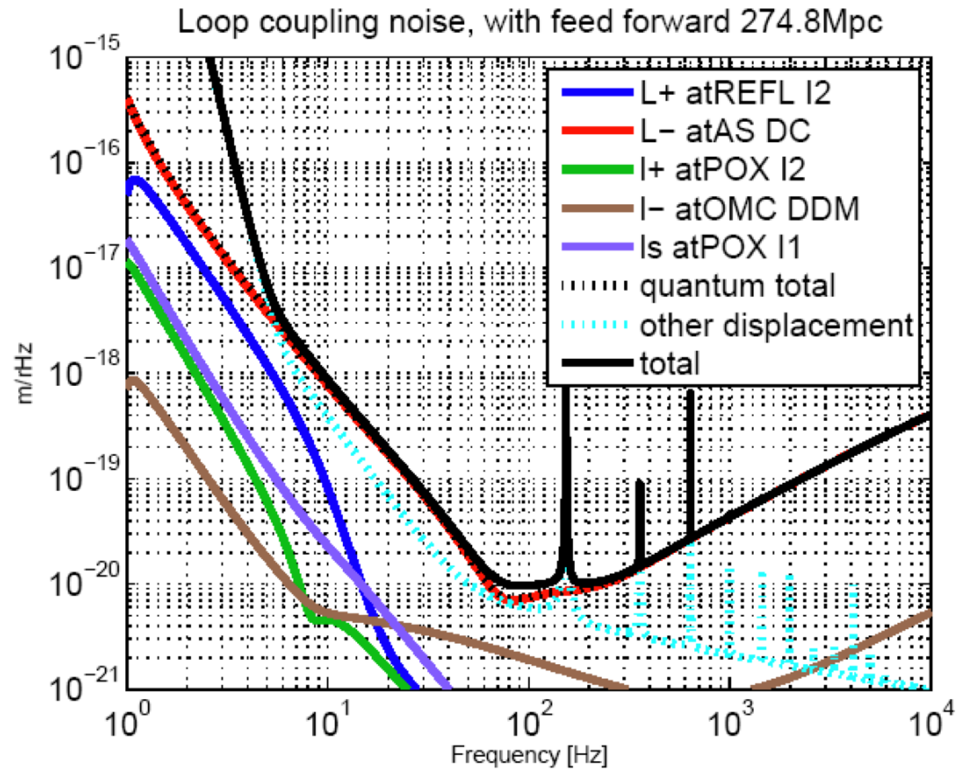
	$L+$	$L-$	lp	lm	ls
$L+$ port	1				
$L-$ port		1			
lp port			1		
lm port				1	
ls port					1

Shot noise turns to displacement noise via control loop.

The 1st order and 2nd order contributions are intuitively understandable though we shall consider all the contributions.

Loop noise





Control matrices to complete the loop-noise calculation.
Loop noise could actually limit the LCGT sensitivity.
There are still many things to do on the control scheme.