



Thermal Noise and Material Related Issues

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Overview

- thermal noise in GW detectors (revision)
- important material properties
- mechanical loss measurements
 - bulk
 - coatings
- (optics...)
- summary

Thermal noise in GW detectors

Thermal noise in GW detectors

- GW detectors are amongst the most sensitive tools today.
- operation at the technical and scientific limitations (noise, cross coupling, etc.)
- improving the instruments means fighting with physics
 - novel techniques (setups, cryogenics, etc.)
 - novel materials (change of material for optical components)
 - ...

Thermal noise in GW detectors

- The first generation of interferometric GW detectors (LIGO, Virgo, GEO600, TAMA300) reached their design sensitivities in a wide range of frequencies.

Thermal noise in GW detectors

- current detectors are based on:
 - fused silica optics (best optical material currently available)
 - fused silica or metal suspensions
- friction between the suspension and the optics can be avoided by using the low mechanical loss jointing technique of hydroxide catalysis bonding

Thermal noise in GW detectors

- Currently, the second generation of GW detectors is being built based on the experiences of the first generation of GW detectors and novel techniques that have been tested.

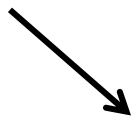
- further improvements:

- aLIGO, aVirgo, GEO-HF

fused silica, room temperatur,
monolithic suspensions
(demonstrated in GEO600 for
years)

- LCGT + CLIO

sapphire, cryogenics

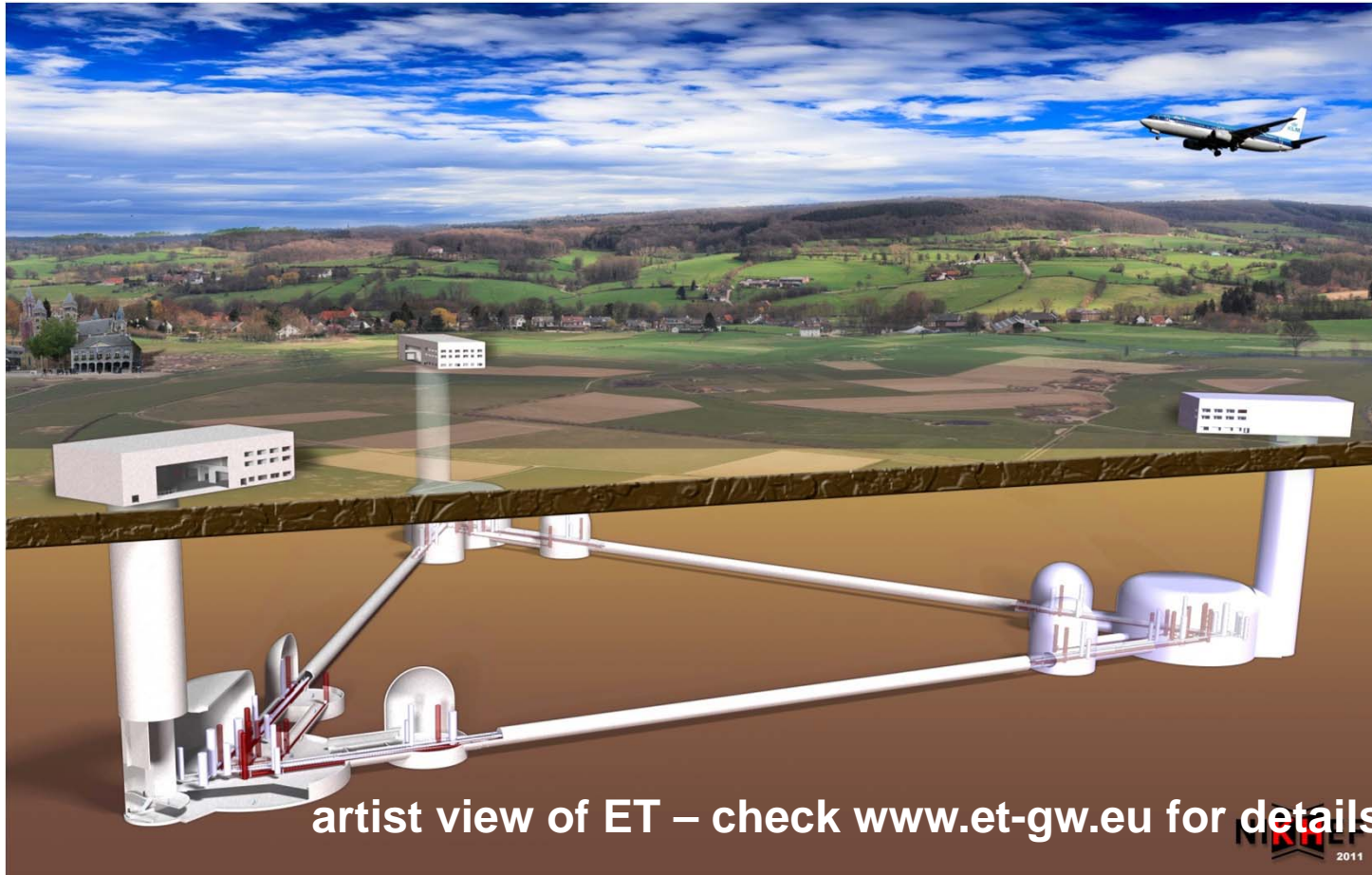


pioneering work in the field of cryogenic large scale interferometry

Thermal noise in GW detectors

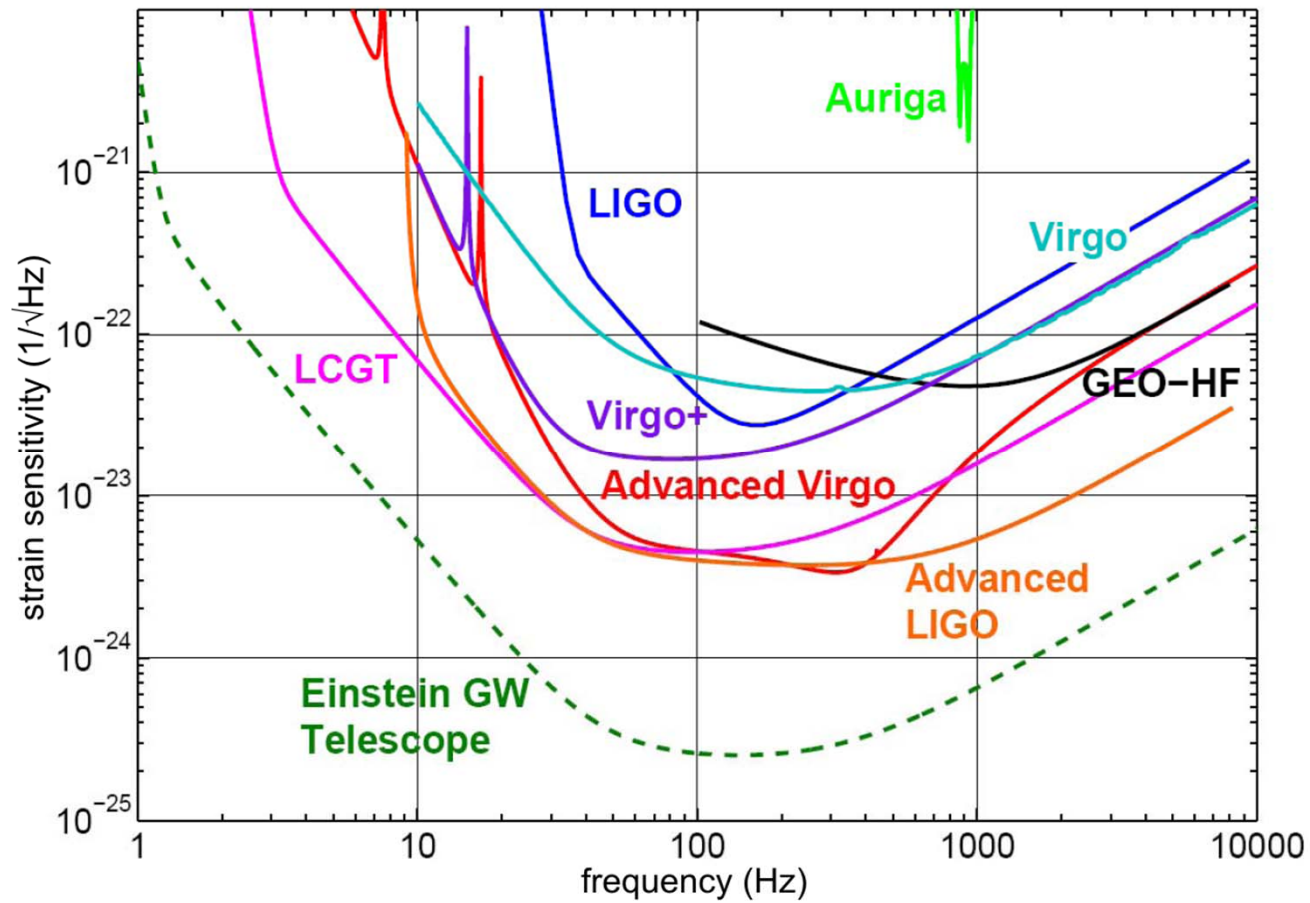
- As the second generation is designed and currently about to being built researchers started a conceptual design study for a 3rd generation detector – the Einstein Telescope design study.
- aims:
 - What technologies are needed to increase sensitivity by a factor of 10 compared to 2nd generation?
 - How might such a design look like?
 - Which materials should be used? Which design?
 - (How much does it cost?)
- homepage: www.et-gw.eu

Thermal noise in GW detectors



Thermal Noise in GW detectors

improvement of the sensitivity between different generations of GW detectors:



Thermal noise in GW detectors

- Parts of the Einstein Telescope have to be operated at cryogenic temperatures to reduce thermal noise.
- natural links between ET and LCGT:
 - cryogenics
 - pulse tube vs. LHe cooling
 - contamination of the mirrors due to cryotrapping
 - general: pioneering technology in cryogenics

 - people exchange between Japanese and Einstein Telescope researchers now extended through personal exchange program

Thermal noise in GW detectors

- a reminder of thermal noise:
 - two different types
 - (1) fluctuating thermal energy → Brownian thermal noise
 - (2) fluctuating temperature → thermo-elastic, thermo-refractive, thermo-optic
temperature dependent parameter (e.g. CTE, dn/dT)
links temperature fluctuation and phase fluctuation of the detector

Thermal noise in GW detectors

- a reminder of bulk thermal noise

- ✓ Thermo-elastic noise:

$$S_{TE}^{ITM}(f, T) = \frac{4k_B T^2 \alpha^2 (1 + \sigma)^2 \kappa}{\pi^{5/2} \rho^2 C^2 f^2 w^3}$$

$\alpha=0$ possible for some materials, e.g. silicon (@ 18 and 125 K)

- ✓ Brownian thermal noise:

$$S_X^{ITM}(f, T) = \frac{2k_B T}{\pi^{3/2} f} \times \frac{1 - \sigma^2}{w Y} \times \phi_{\text{substrate}}(f, T)$$

beam diameter

temperature

- main message: material properties influence strongly the thermal noise – (nearly) all of them are temperature dependent

Thermal noise in GW detectors

- a reminder of bulk thermal noise

✓ Thermo-elastic noise:

$$S_{TE}(f, T) = \frac{8k_B T^2}{\pi^2 f} \frac{d}{w^2} \frac{\alpha_S C_F}{C_S} (1 + \sigma_s)^2 \tilde{\Delta}^2 g(\omega)$$

$$g(\omega) = \text{Im} \left[-\frac{1}{\sqrt{i\omega\tau_F}} \frac{\sinh \sqrt{i\omega\tau_F}}{\cosh \sqrt{i\omega\tau_F} + R \sinh \sqrt{i\omega\tau_F}} \right]$$

$$\tilde{\Delta}^2 = \left\{ \frac{C_S}{2\alpha_S C_F} \left(\frac{\alpha}{1-\sigma} \left[\frac{1+\sigma}{1+\sigma_s} + (1-2\sigma_s) \frac{E}{E_S} \right] \right)_{\text{AVG}} - 1 \right\}^2$$

$$\tau_F = \frac{d^2}{\kappa} \quad \text{and} \quad R = \sqrt{\frac{\kappa_F C_F^2}{\kappa_S C_S^2}}$$

✓ Brownian thermal noise:

$$S_x(f, T) \approx \frac{2k_B T}{\pi^2 f} \frac{d}{w^2} \frac{1}{Y} \left(\frac{Y'}{Y} \phi_{\parallel} + \frac{Y}{Y'} \phi_{\perp} \right)$$

beam diameter

temperature

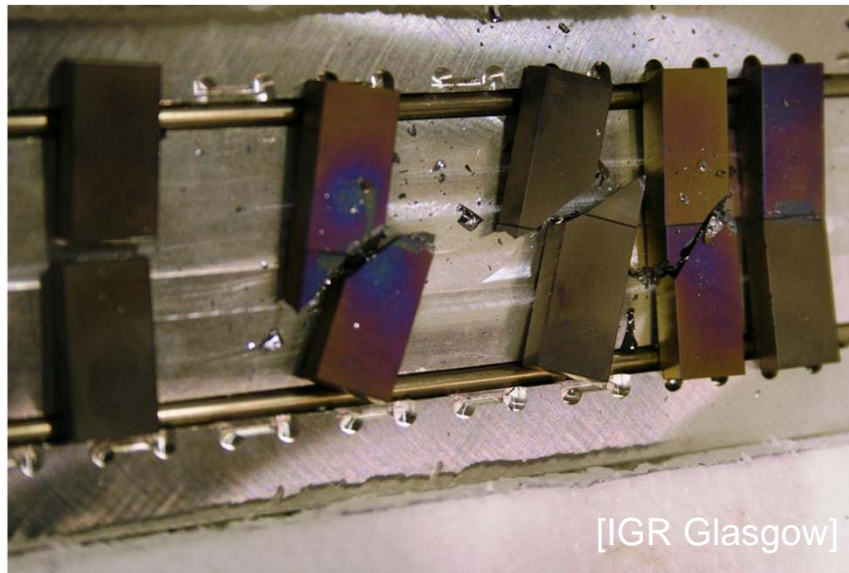
- similar with thermo-optical noise and finite corrections

Material properties

Overview of material properties

- mechanical properties

basic property: strength of the material (especially for suspension elements)



Overview of material properties

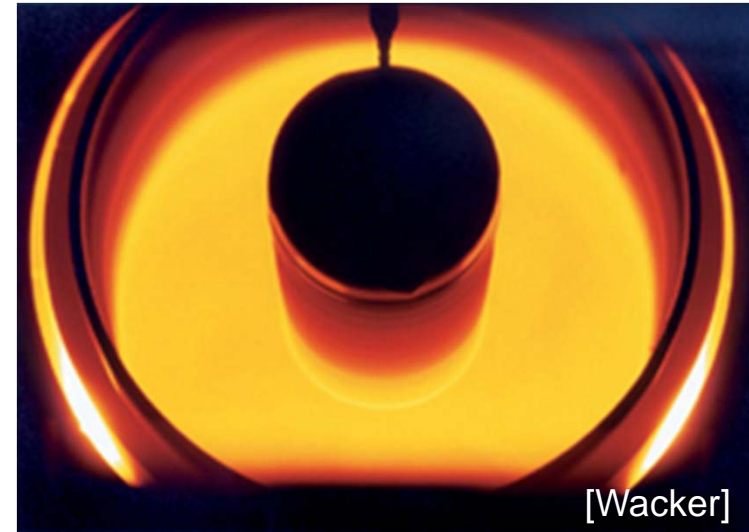
- mechanical properties

Young's modulus, Poisson ratio
mass density of the material

available size
(growing methods)

optimum crystal orientation

mechanical loss to determine Brownian thermal noise
(= main focus of this lecture)

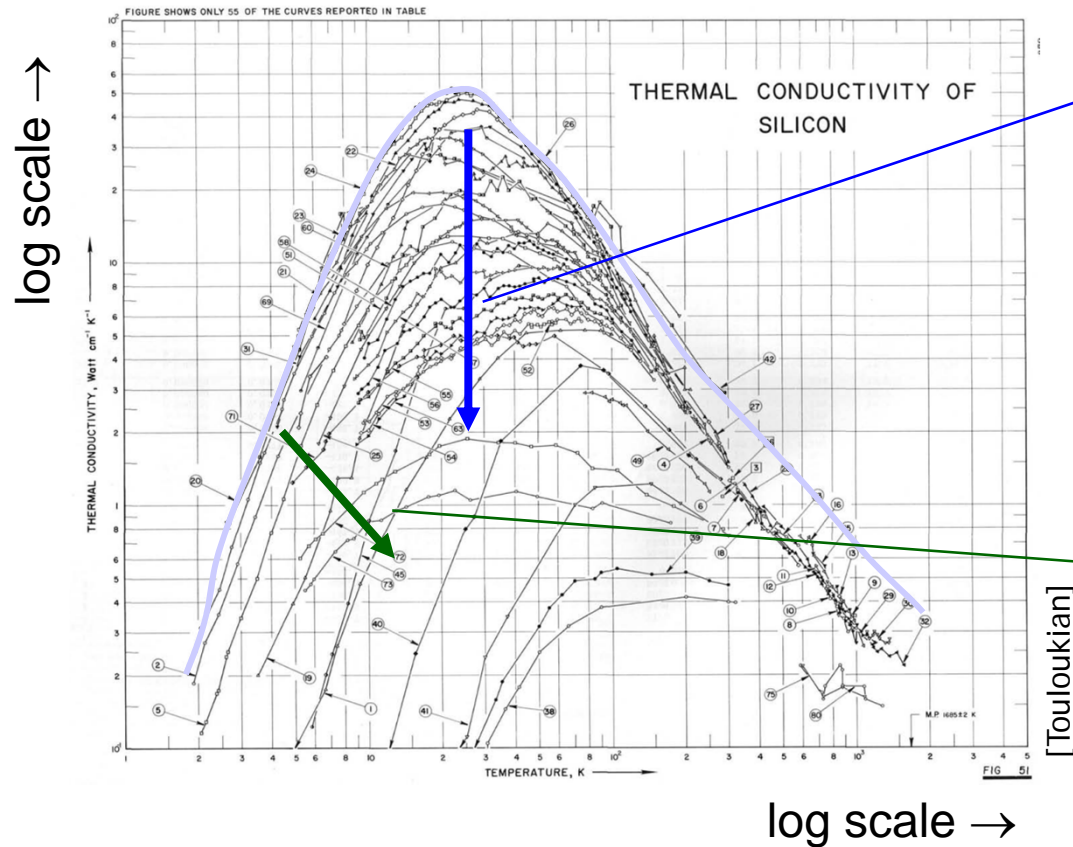


Overview of material properties

- thermal properties
 - strongly temperature dependent
 - crystalline materials follow „basic“ thermodynamics / statistics (Debye theory)
 - heat capacity ($\sim T^3$ at low temperatures for crystals)
 - thermal conductivity (removal of heat!)
 - coefficient of thermal expansion

Overview of material properties

- thermal properties
 - thermal conductivity



increasing impurity concentration (scattering of phonons on impurities) + size effects (boundary scattering)

not enough phonons left to carry heat

Overview of material properties

- thermal properties
 - coefficient of thermal expansion

larger CTE for cryst. materials compare to amorphous (reason: summation of tiny effects in the crystal)

usually CTE decreases with decreasing temperature

some materials show temperatures with $CTE = 0$
(e.g. silicon @ 18 and 125 K)

Overview of material properties

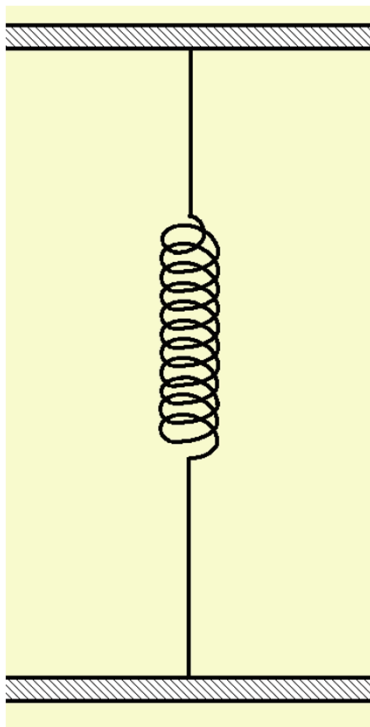
- optical properties
 - refractive index n
forming HR mirror coatings
 - thermo-refractive coefficient dn/dT
determines the thermo-refractive noise
 - optical absorption α
sets a fundamental limit to the minimum operational temperature

Mechanical loss

- Introduction -

Basics of mechanical loss

- elastic behaviour of a solid



Hook's Law

$$\sigma = E \cdot \varepsilon$$

σ ... stress

ε ... strain

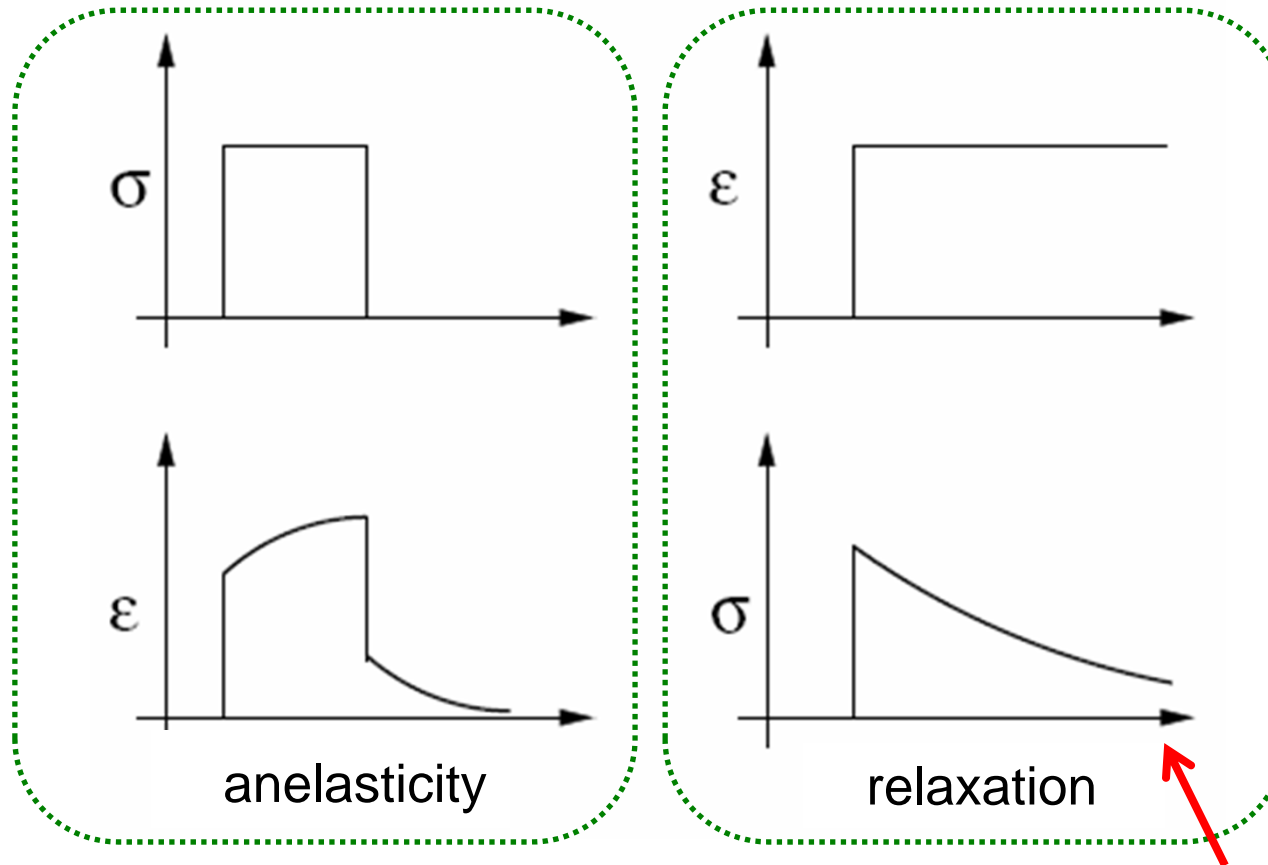
E ... Young's modulus

(German: modulus of elasticity)

instantaneous reaction, full recovery

Basics of mechanical loss

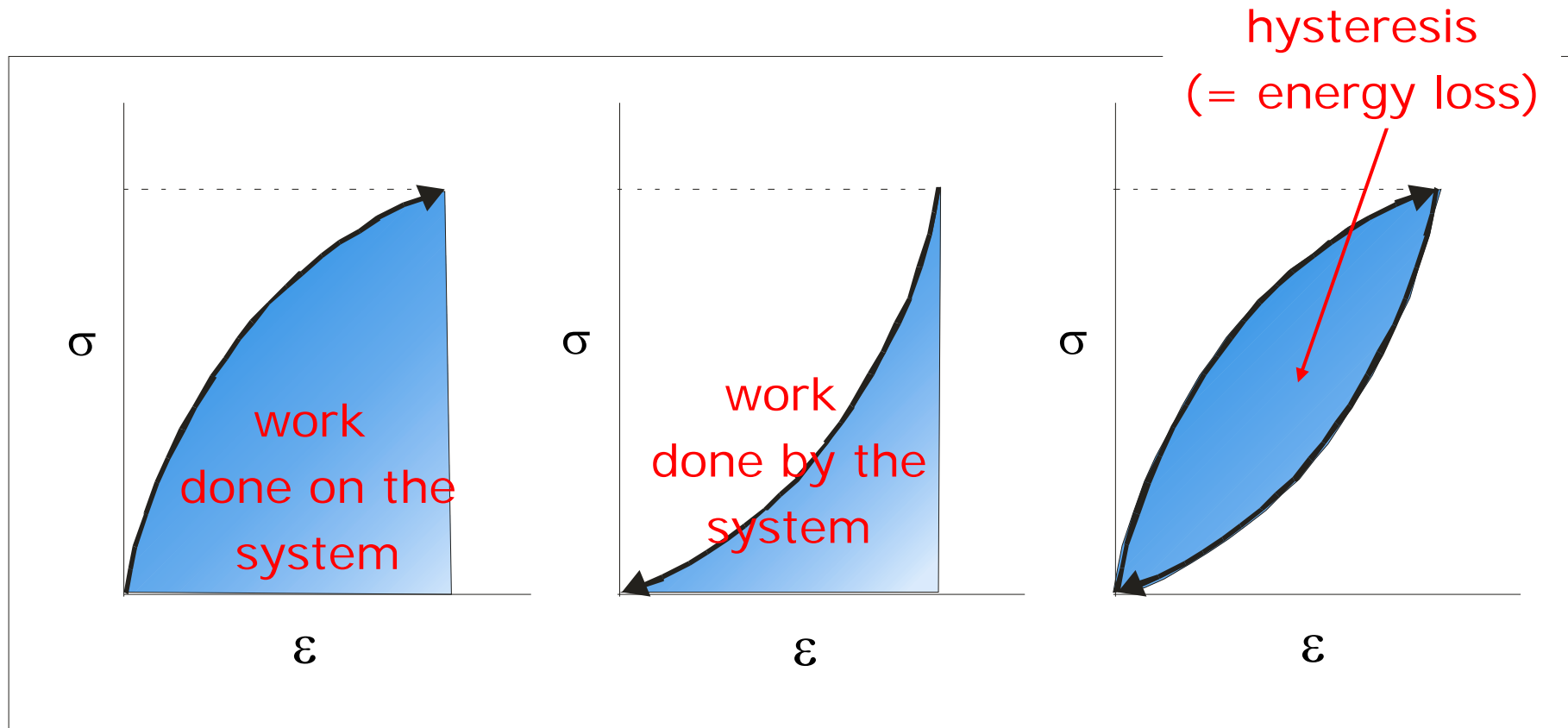
- anelastic behaviour of a solid



only partial instantaneous
reaction, full recovery after $t = \infty$

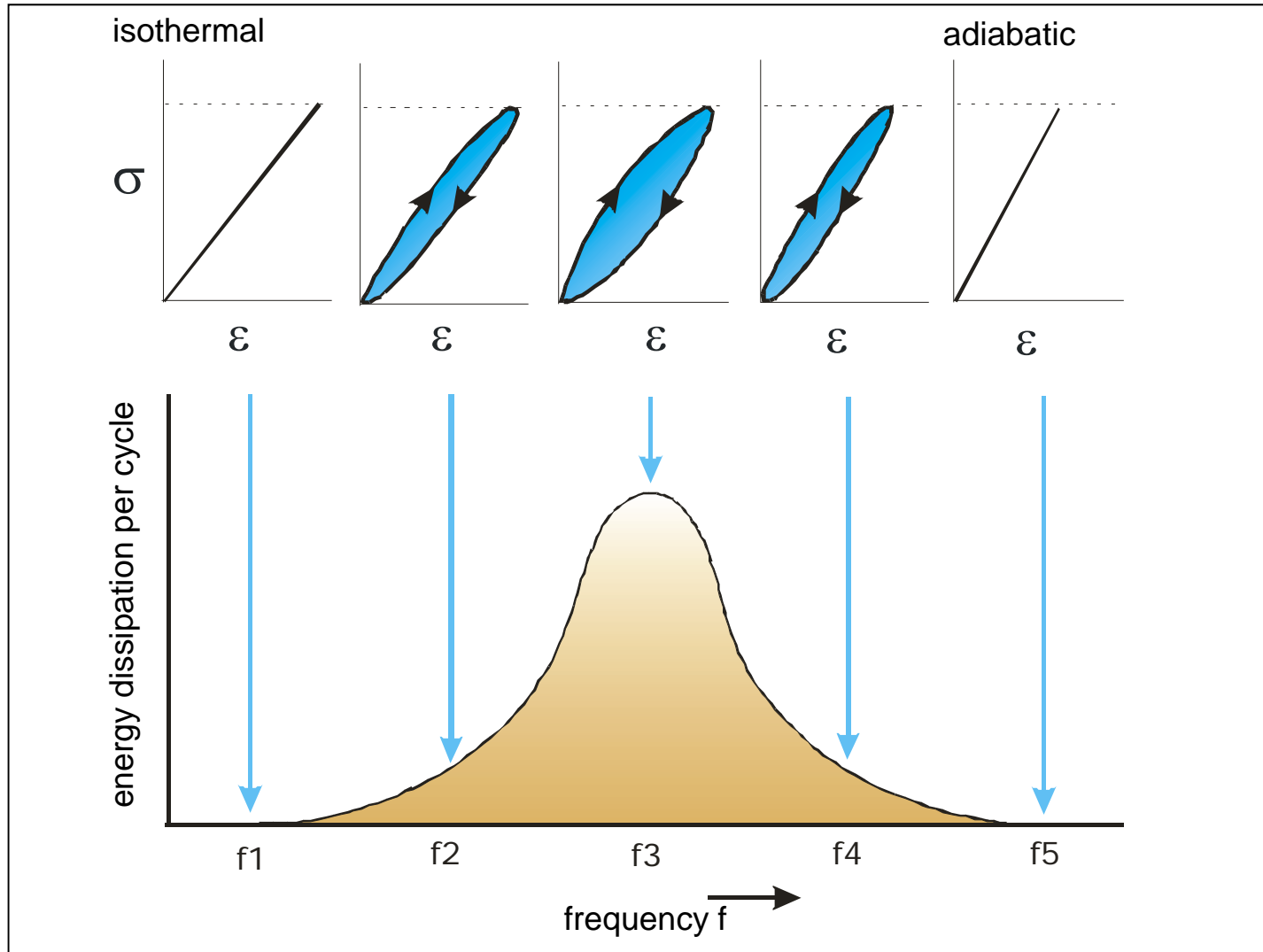
Basics of mechanical loss

- periodic process – anelasticity and mechanical loss



energy loss related to mechanical loss by means of: $\varphi = \frac{1}{2\pi} \frac{\Delta E}{E}$

Basics of mechanical loss



Basics of mechanical loss

- definition of the mechanical loss = phase lag between stress and strain
- measurement via the mechanical Q-factor at a resonance
- keep in mind:

The mechanical loss is a continuous function of frequency but we just probe it at certain frequencies (resonant frequencies) of a system → no full knowledge available.

Loss mechanisms

- There are many different origins of loss in solids.
- Focus on 3 dominant ones:
 - phonon-phonon interaction
 - thermo-elastic loss
 - impurity driven losses
- Phonon-phonon-damping (Akhiezer-/Landau-Rumer-Damping)

Phonons are forming a certain distribution when in equilibrium. At low frequency excitations the acoustic vibration (= phonon) modulates the lattice → new local equilibrium → redistribution consumes energy → loss.

Loss mechanisms

- Phonon-phonon-damping (Akhiezer-/Landau-Rumer-Damping)

Phonons are forming a certain distribution when in equilibrium. At low frequency excitations the acoustic vibration (= phonon) modulates the lattice → new local equilibrium → redistribution consumes energy → loss.

(Akhiezer loss)

If the phonon energy is high (high frequency vibration) the acoustic phonon directly interacts with the phonons of the given distribution → direct phonon scattering → redistribution consumes energy → loss.

(Landau-Rumer-Loss)

Basics of mechanical loss

- thermo-elastic damping

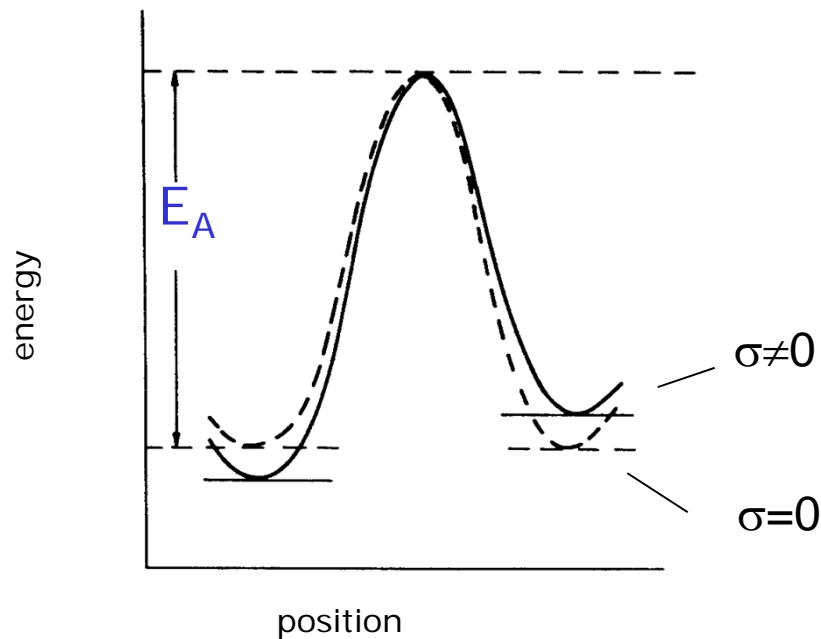
If a sample is deformed certain parts will be compressed or expanded → local heating or cooling (depending on CTE).
Sample is now in thermal non-equilibrium → heat flux → entropy is increased → loss.

- impurity driven damping

Impurities can occupy different positions in a lattice depending on the applied stress. If an external vibration is applied it might be energetically better to change positions → loss.

Basics of mechanical loss

- The transition between 2 (quasi-)stable positions can be modelled with a double-well potential:



$$\phi(\omega) = \Delta \frac{\omega\tau}{1 + (\omega\tau)^2}$$

„Debye peak“

Δ ... relaxation strength

τ ... relaxation time

thermally activated process:

$$\tau = \tau_0 e^{\frac{E_A}{k_B T}}$$

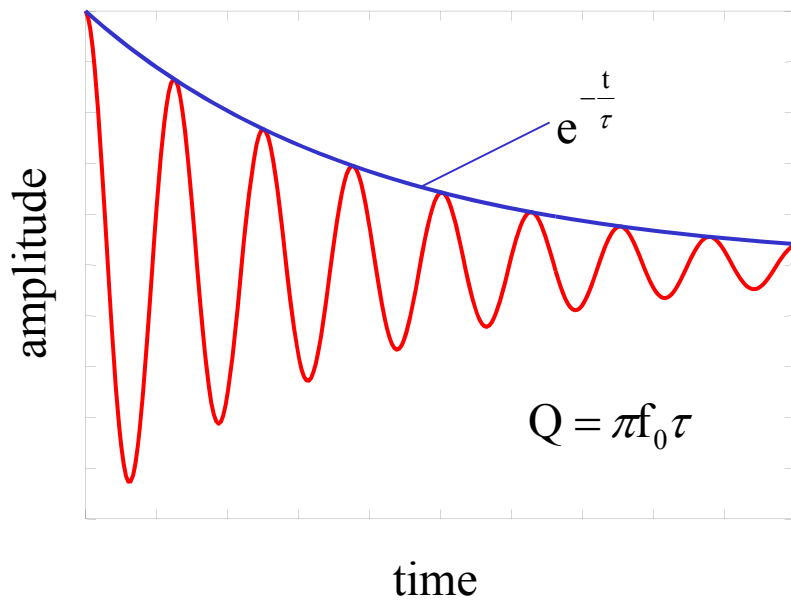
E_A ... activation energy

τ_0 ... relaxation constant

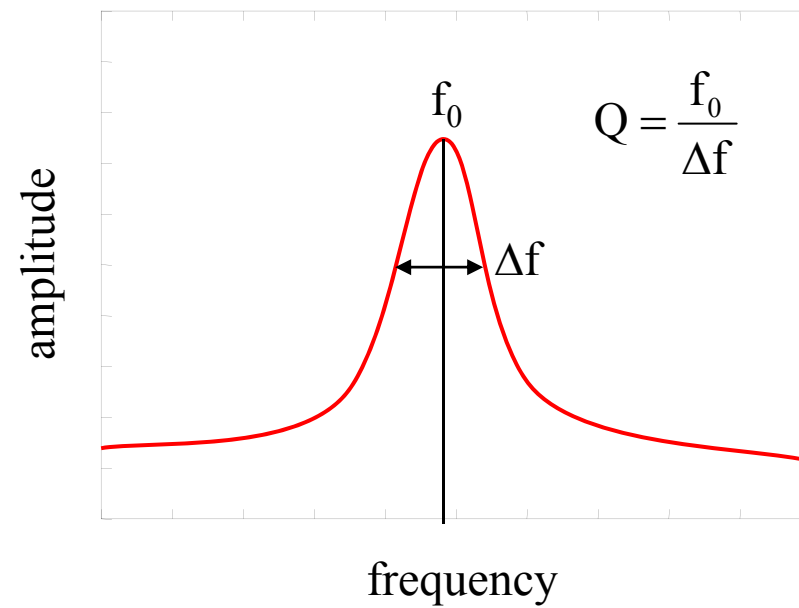
Basics of mechanical loss



- ring-down measurements



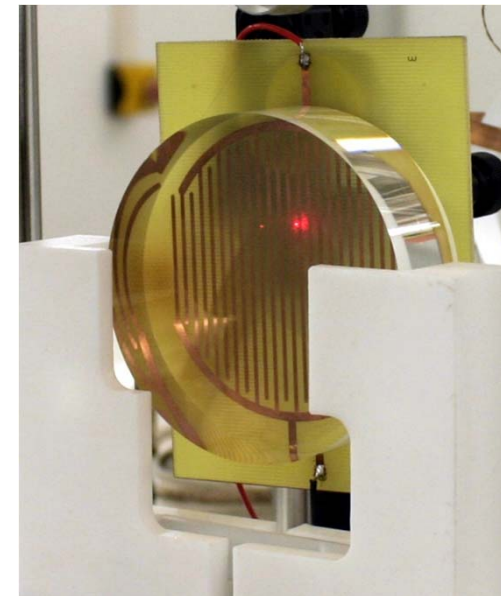
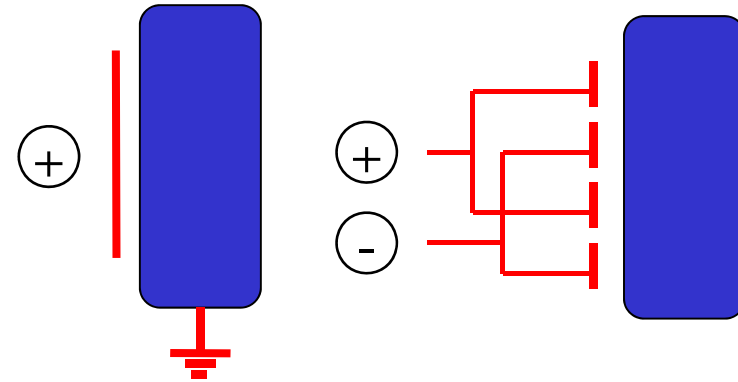
- bandwidth measurements

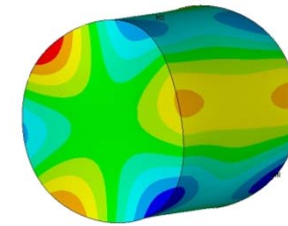
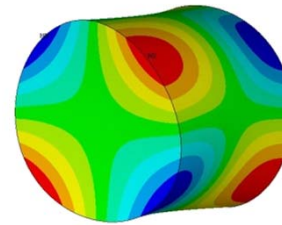
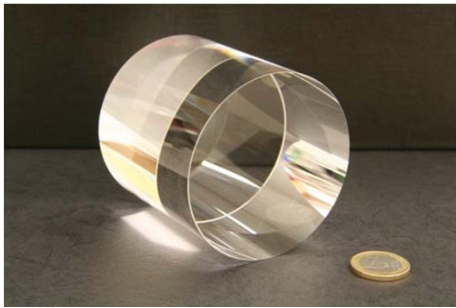


Basics of mechanical loss

- excitation of modes
 - mechanical (e.g. piezo)
 - electro-static

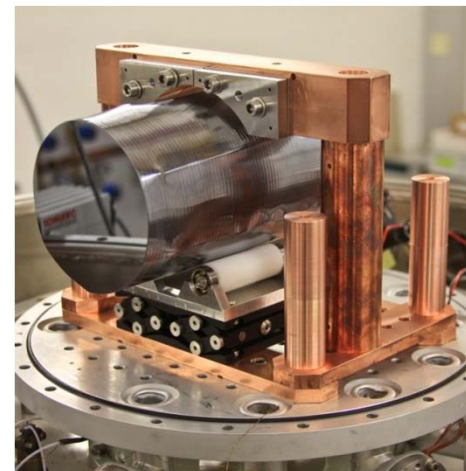
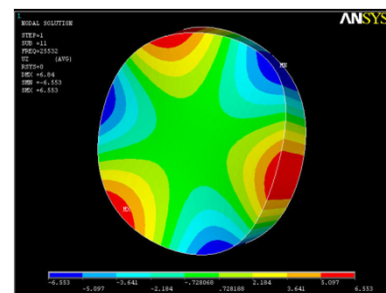
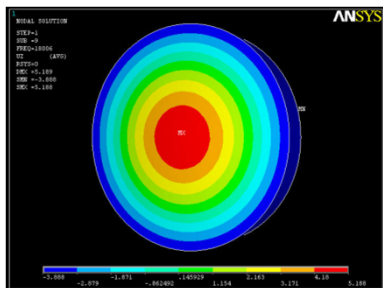
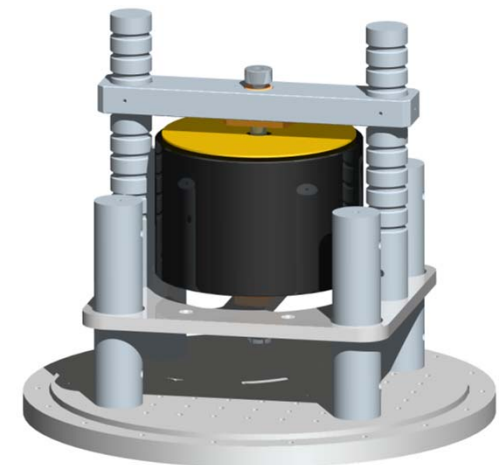
- vibration read-out
 - electrical read-out (capacitor)
 - optical read-out (e.g. optical lever, interferometric techniques)





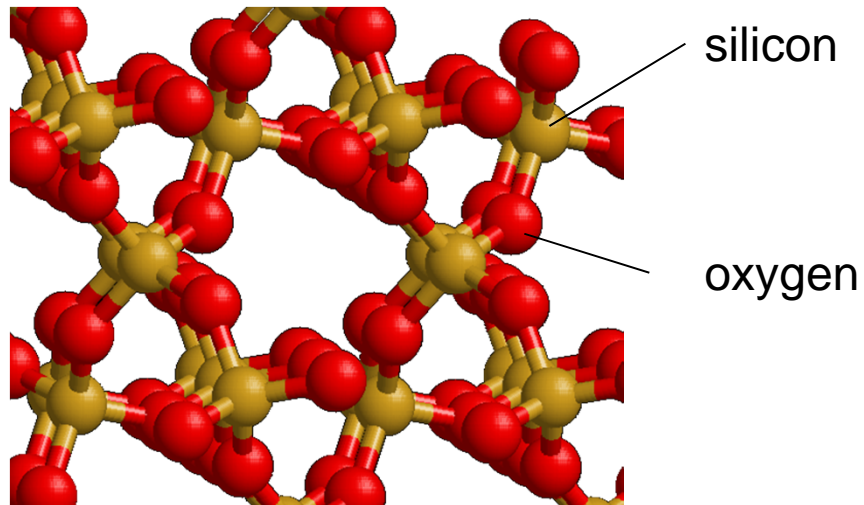
Mechanical loss

- Bulk Materials -



Selected examples - Quartz

- crystalline quartz is well known → toy material to investigate setups and data processing tools



cryst. quartz shows channels along its c-axis

hydrothermal growth of crystal

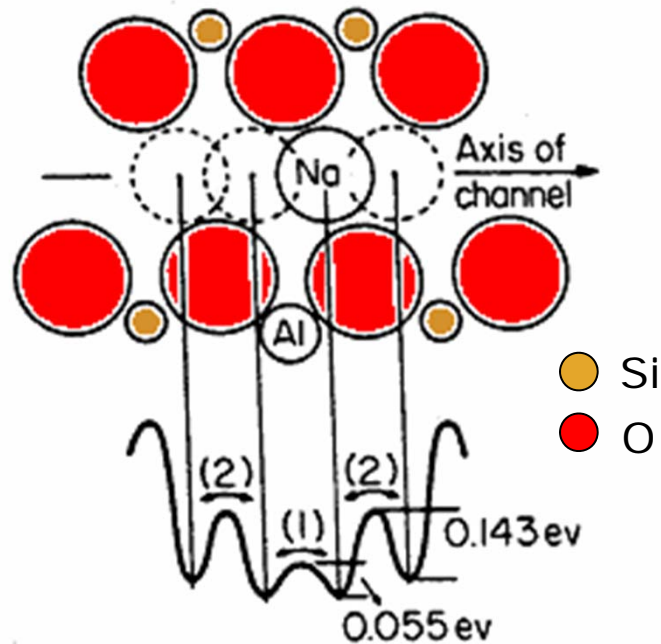


grown from solution under pressure (~ 500 bar) at elevated temperatures containing:

- water
- silicon dioxide
- sodium carbonate / hydroxide

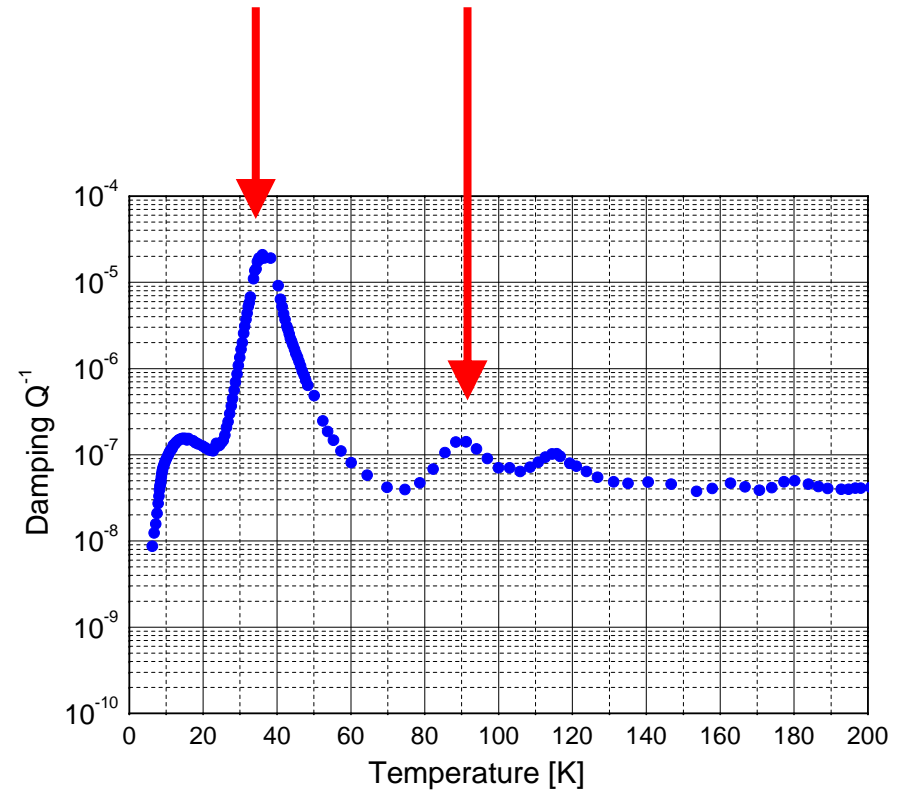
Selected examples - Quartz

impurities trapped in multiple well potential along the c-axis



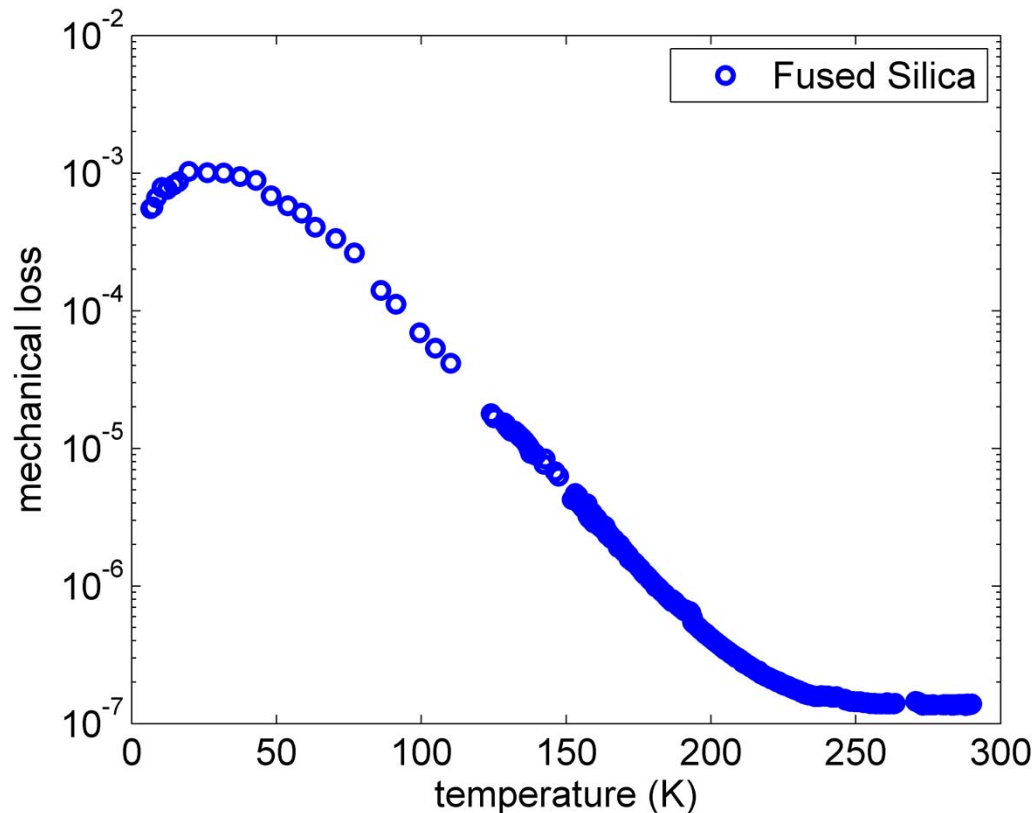
loss process is orientation dependent
→ detailed study needs different cryst. cuts from the same material

loss peaks associated with sodium



activation energy from experiment:
~ 55 meV

Selected examples – Fused Silica



fused silica:

- very low loss at room temperature reaching 4×10^{-10}
- large loss peak around 30K



not suited for cryogenic use in GW detectors

origin of the peak:

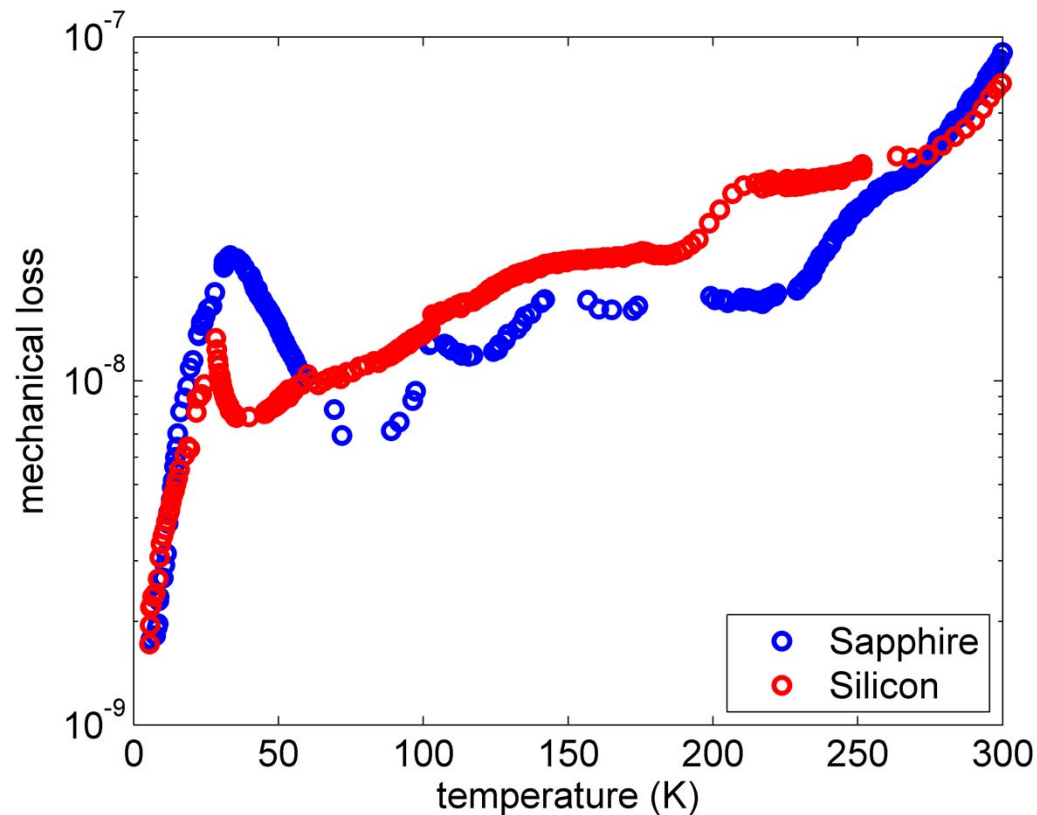
Amorphous silica has a near but **no** far **order**. Thus, loss processes get a **distribution** of loss **parameters**. The peak is the **superposition** of all of them.

Selected examples – Sapphire/Silicon

- crystalline materials needed for cryogenic operation
- different candidate materials have been discussed in the past
- possible candidate materials are sapphire (LCGT) and Si (ET)
- reasons:
 - both are optical materials (remember, FS is currently the best optical material)
 - both are available in rather large pieces
 - high thermal conductivity
 - coating techniques available
- while sapphire can be operated at 1064nm silicon demands a change of the laser wavelength due to its optical absorption

Selected examples – Sapphire/Silicon

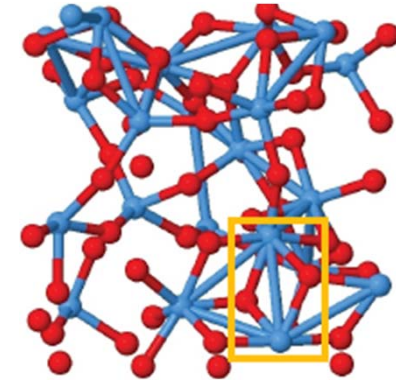
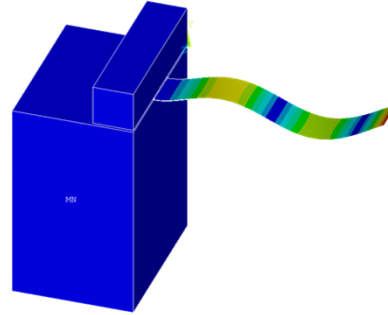
- mechanical loss of silicon and sapphire is comparable at cryogenic temperatures (Q's up to several 10^9 achieved)



test measurement of silicon
and sapphire samples
[U Jena]

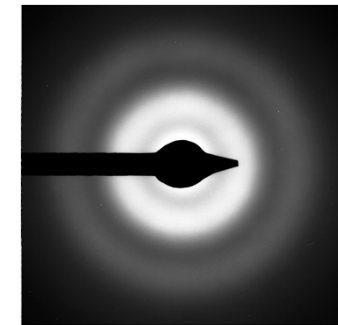
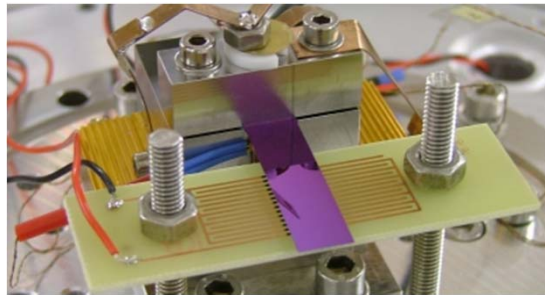
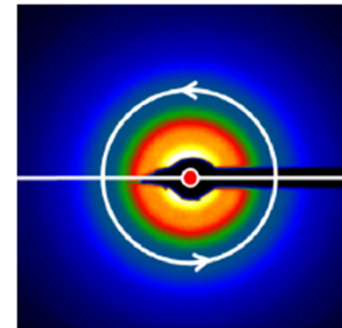
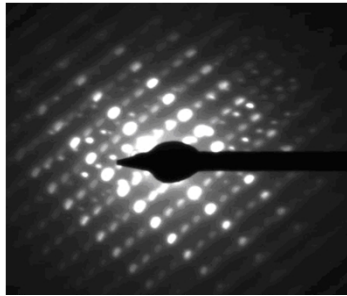
Selected examples – Sapphire/Silicon

- detailed investigation of intrinsic loss mechanisms in promising candidate bulk materials ongoing
- questions:
 - general loss processes?
 - impurities? tolerable level of impurities?
 - heat treatment to remove dislocations?
 - surface loss?
 - ...
- collaboration of several groups (Jena, Glasgow, Legnaro, starting collaboration with Japan) to investigate intrinsic damping of materials (coatings, bulk, surfaces, etc.)



Mechanical loss

- Coating Materials -

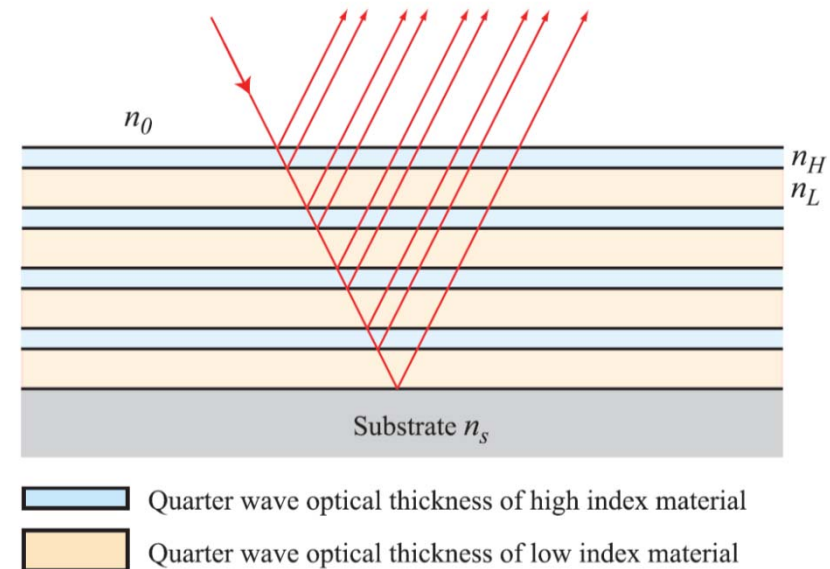


Optical coatings

- Requirements – high reflectivity, low optical absorption (< 1ppm)
- Multilayer coatings of dielectric materials, $\lambda/4$ thick
- Reflectivity from difference in refractive index, and number of layers, $2N$.

$$R_{2N} = \left(\frac{n_s f - n_0}{n_s f + n_0} \right)^2$$

$$f = (n_H/n_L)^{2N}.$$

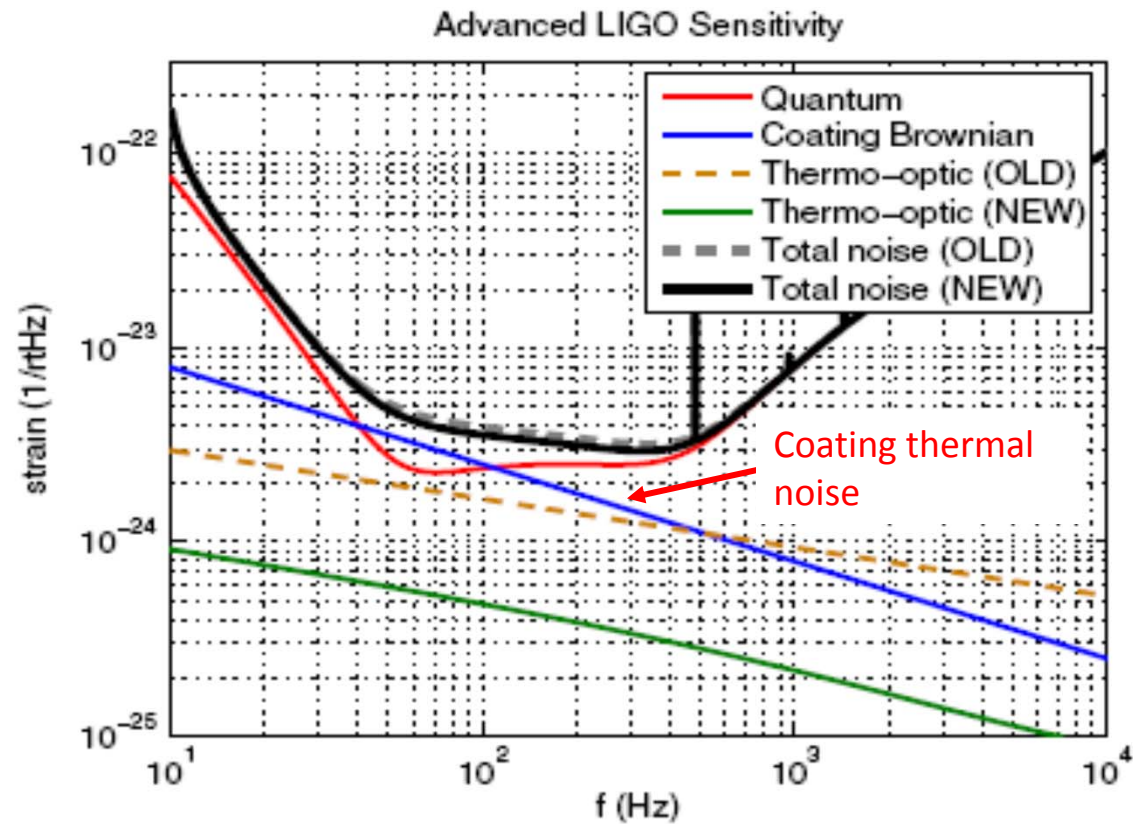


- Current detectors use silica ($n=1.45$) / tantala ($n=2.03$) coatings, ~ 15 bi-layers

Coating thermal noise

- Levin – interferometer most sensitive to mechanical loss close to the reflected laser beam
 - Thus mechanical loss of coatings is particularly important
- Coating loss dominated by the loss of the tantala layers
 - $\phi_{\text{tantala}} \sim 4 \times 10^{-4}$
 - $\phi_{\text{silica}} \sim 5 \times 10^{-5}$
- Measurements suggested no observable loss from coating layer interfaces (however recent results from LMA, Lyon, suggest some interface loss may be observable)
- Doping Ta_2O_5 with TiO_2 can reduce the loss by $\sim 40\%$.

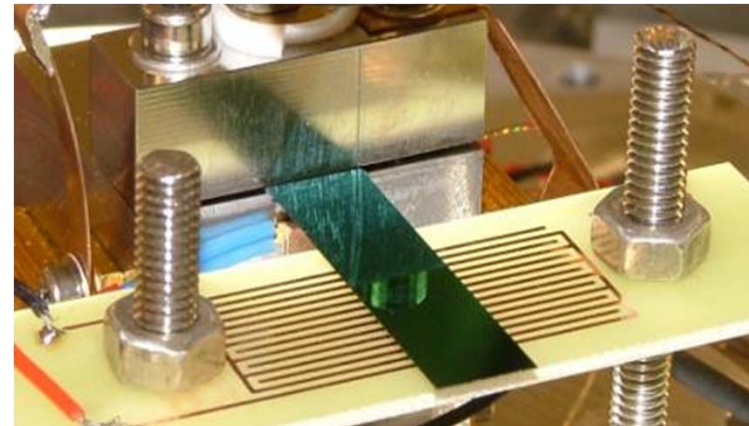
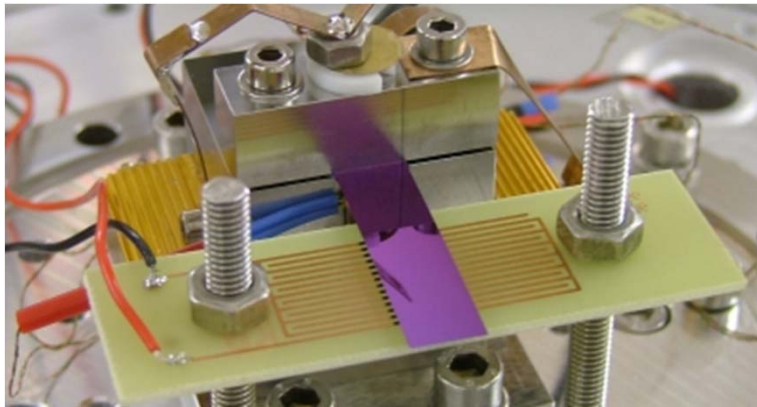
Coating thermal noise



- Coating thermal noise expected to limit achievable sensitivity of future GW detectors at their most sensitive frequencies

Coating loss measurements

- First cryogenic measurement of silica/tantala coating by Yamamoto et al, showed possible slight increase in loss at low temperature
- Cryogenic loss studies of mono-layers of individual coating materials carried out in collaboration between Glasgow, Jena, LMA
 - Study individual materials in isolation
 - Identify microscopic dissipation mechanisms
 - Test coating performance at cryogenic temperatures

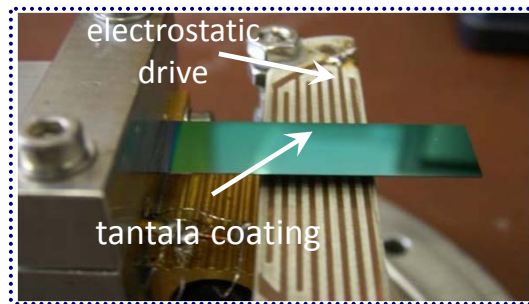
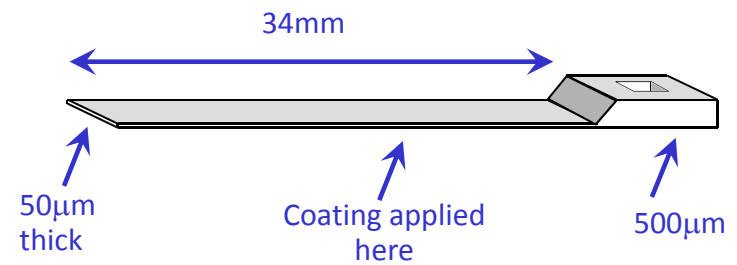


Single layer coatings of silica (left) and tantala (right), clamped for loss measurements

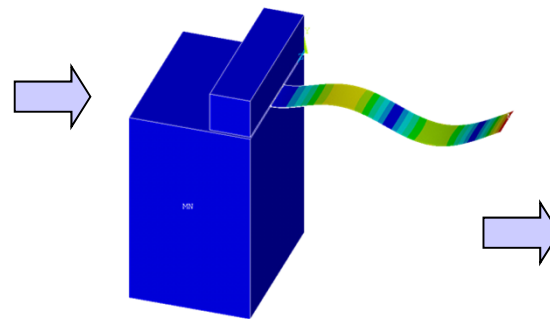
Measuring coating loss - 1

- Single layers of a coating material applied to silicon cantilever substrates
- Loss measured from exponential ring-down of bending modes

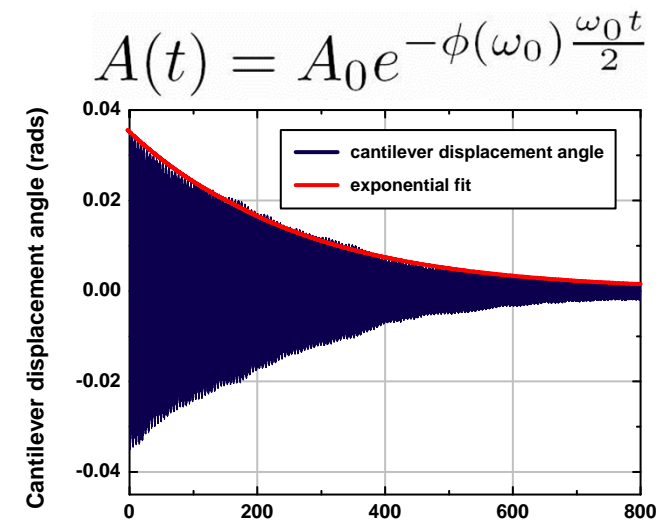
$$\phi(f_0) = \frac{\Delta f}{f_0} = \frac{E_{\text{lost per cycle}}}{2\pi E_{\text{stored}}}$$



coated cantilever in clamp within cryostat



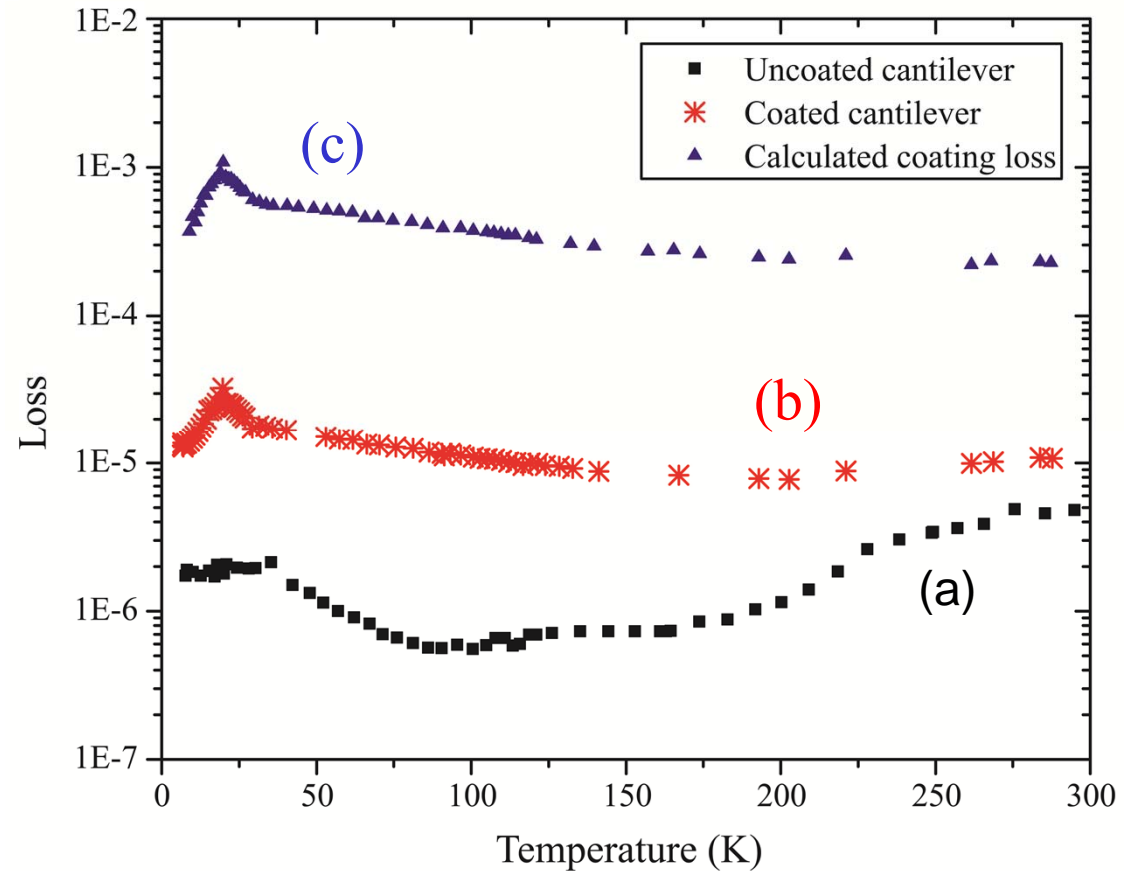
bending modes excited via electrostatic drive



loss from free decay of amplitude

Measuring coating loss - 2

- Loss of coating layer calculated from difference in loss of a coated and un-coated cantilever
- Scaling factor accounts for fraction of total elastic energy stored in the coating



Loss of (a) uncoated silicon cantilever with thermal oxide layer, (b) cantilever coated with 500 nm of TiO₂-doped Ta₂O₅ (14.5% Ti) and (c) the calculated loss of the coating layer

$$\phi_{\text{coating}} = \frac{Y_{\text{cantilever}} t_{\text{cantilever}}}{3Y_{\text{coating}} t_{\text{coating}}} (\phi_{\text{coated}} - \phi_{\text{un-coated}})$$

Loss peak analysis - tantala

- Debye-like mechanical loss peaks

$$\phi(\omega) = \Delta \frac{\omega\tau}{1 + (\omega\tau)^2}$$

Δ ... relaxation strength

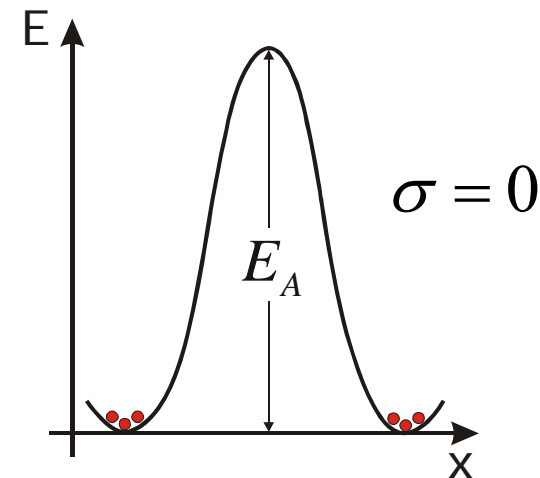
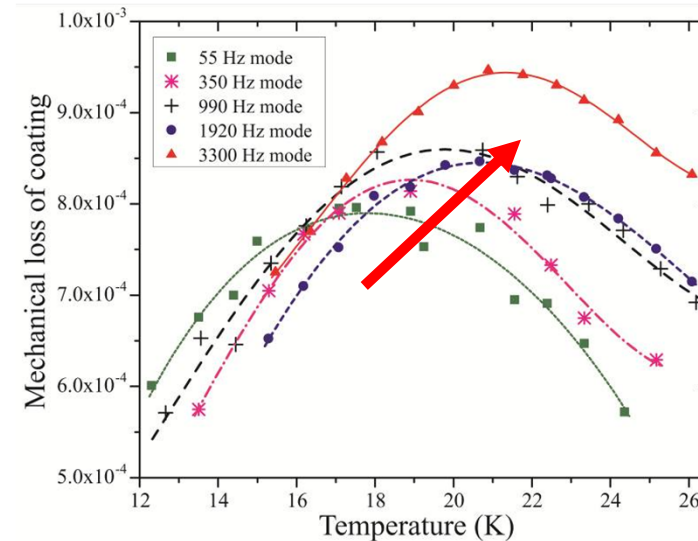
τ ... relaxation time

- thermally activated process

$$\tau = \tau_0 e^{\frac{E_A}{k_B T}}$$

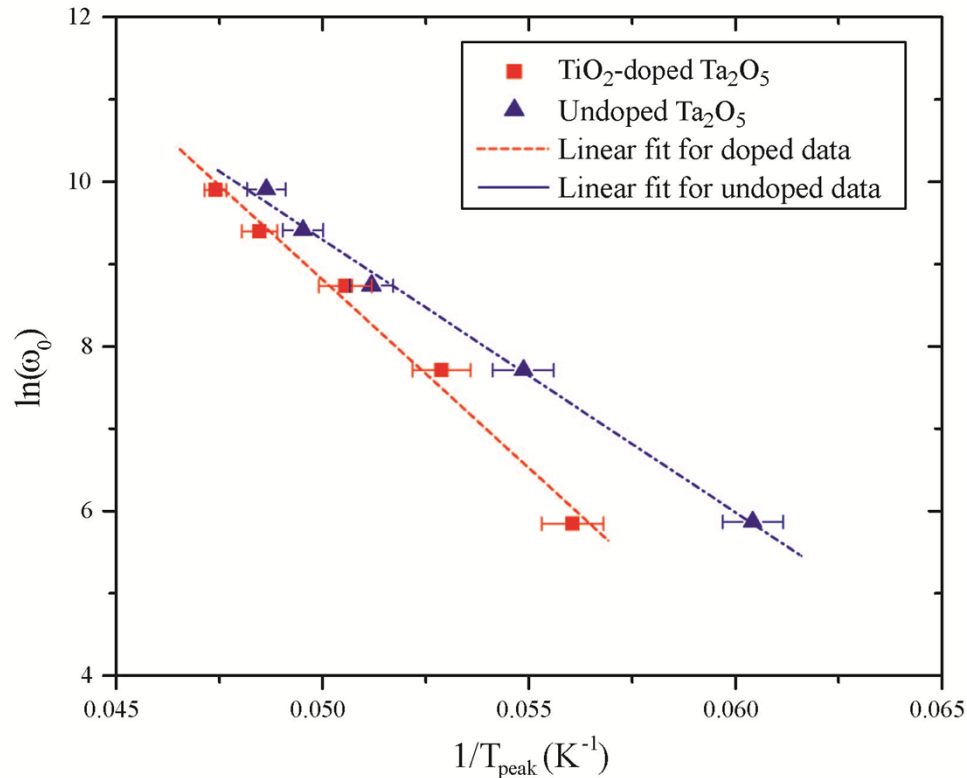
E_A ... activation energy

τ_0 ... relaxation constant



double well potential
without external stresses

Loss mechanism parameters – Arrhenius plot

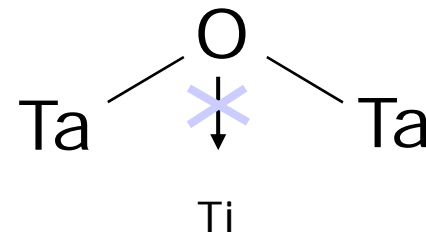
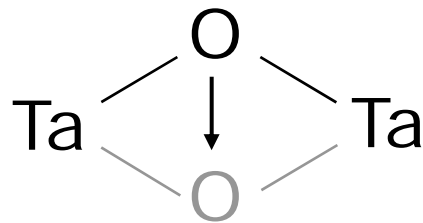


- Activation energy
 - (40 ± 3) meV for TiO₂ doped Ta₂O₅
 - (29 ± 2) meV for undoped Ta₂O₅

- Doping with TiO₂ increases the activation energy.
- Transition between two stable states appears to be hindered

Possible microscopic processes

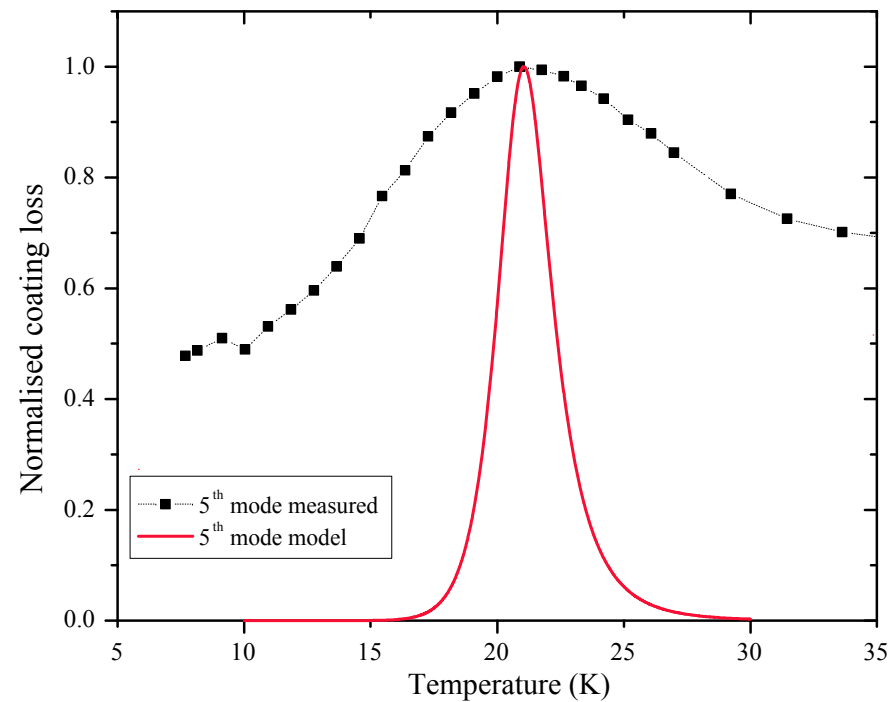
- no long-distance order in coating materials (amorphous)
- possible transitions of atoms / atom groups



- doping might block possible positions → increase of activation energy

Distribution of model parameters

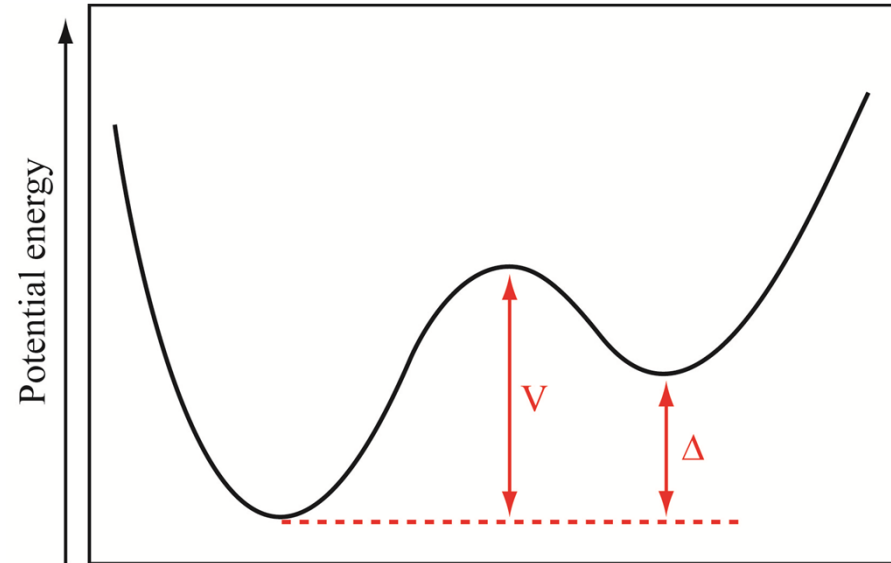
- Debye loss peak plotted using calculated activation energy and relaxation constant



- Much narrower than experimental peak
- Amorphous structure results in a **distribution** of activation energies.

Distribution of parameters

- refined model:
asymmetric double-well
potential
- barrier height distribution $g(V)$
- asymmetry distribution $f(\Delta)$



$$\phi = \frac{\gamma^2}{k_B T C_{ii}} \int_0^\infty \int_0^\infty \frac{\omega \tau}{1 + (\omega \tau)^2} \operatorname{sech}^2 \left(\frac{\Delta}{2k_B T} \right) f(\Delta) g(V) d\Delta dV$$

[Gilroy, Phillips 1981]

- γ represents the coupling between strain and the dissipation mechanism
- C_{ii} is the elastic constant of the material

Distribution of barrier heights

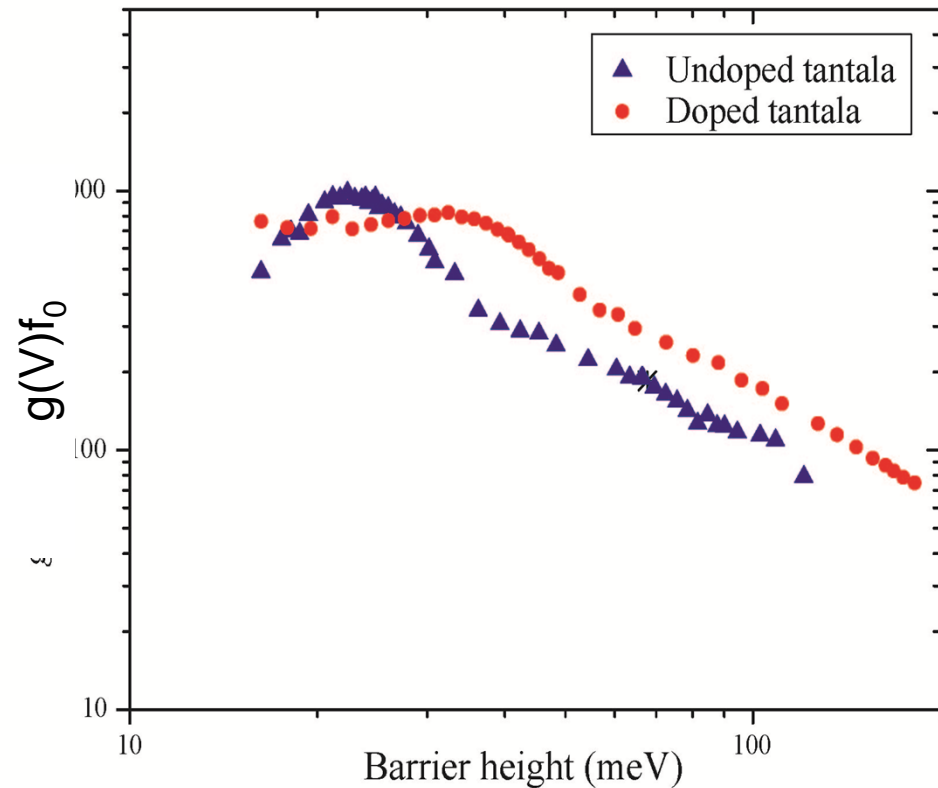
- asymmetric double well potential:

$$\phi = \frac{\pi\gamma^2 f_0}{C_{ii}} k_B T g(V)$$

[Gilroy, Phillips 1981]

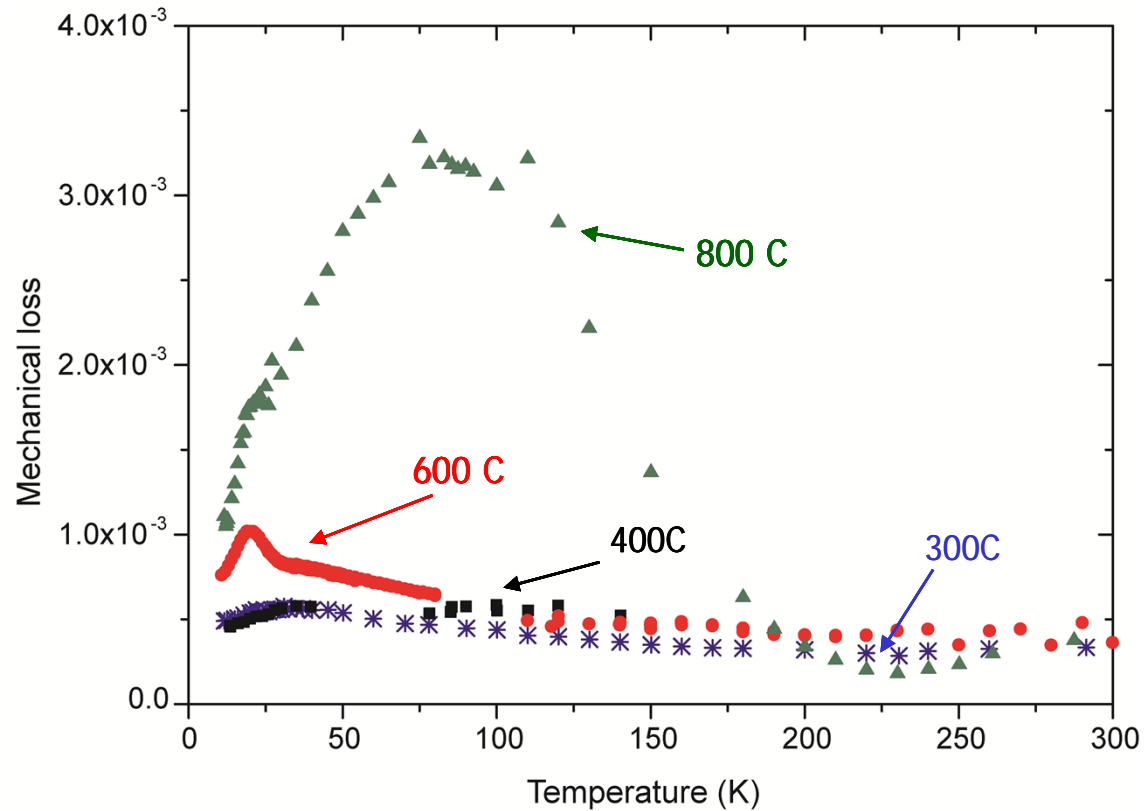
$g(V)$... barrier height distribution

- barrier height distribution contains information about the microscopic structure
- doping changes height and distribution of barrier heights



[Martin et al. 2009]

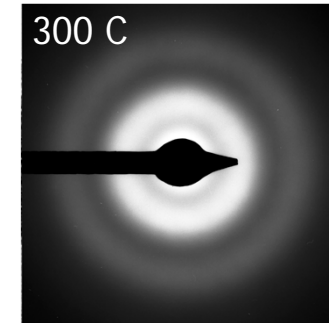
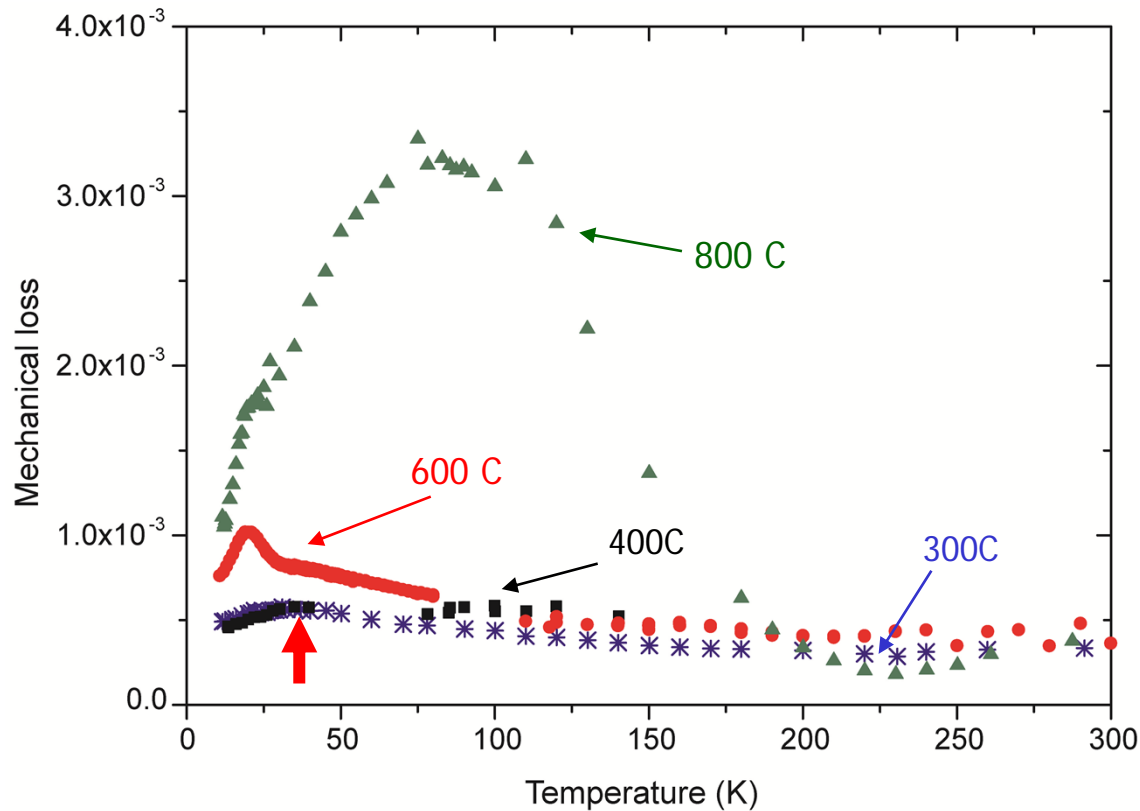
Effect of heat treatment temperature on Ta₂O₅ loss



Loss at 1.9 kHz of 0.5 μm thick un-doped Ta₂O₅ coatings heat treated at 300, 400, 600 and 800 C. (Coatings from CSIRO)

- Three loss peaks, triggered at different post-deposition heat-treatment temperatures

Effect of heat treatment temperature on Ta₂O₅ loss

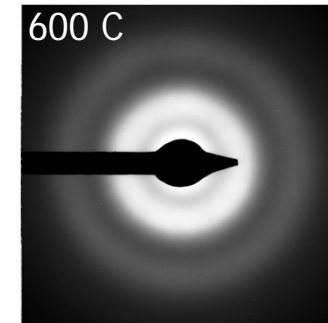
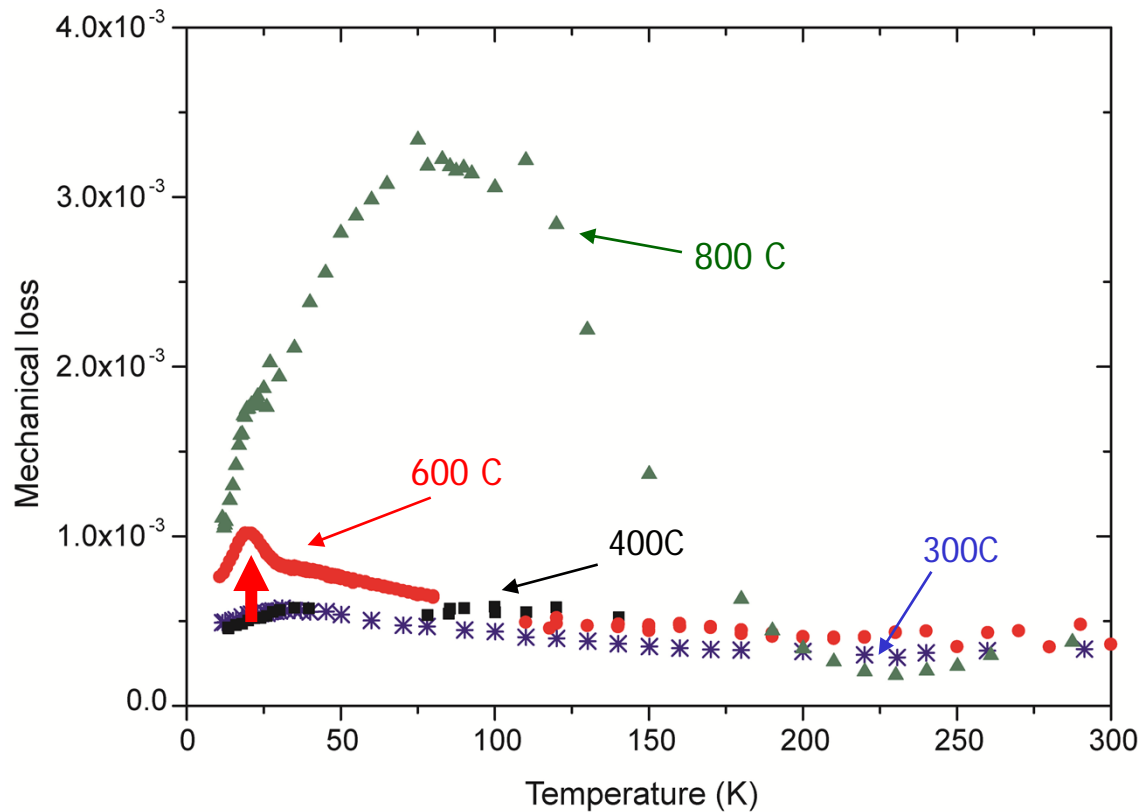


Above: Electron diffraction pattern of Ta₂O₅ heat treated at 600 C

Left: Loss at 1.9 kHz of 0.5 μm Ta₂O₅ coatings annealed at 300, 400, 600 and 800 C.

- 35 K peak
 - Observed in Ta₂O₅ heat treated at 300, 400 C, and likely in Ta₂O₅ heat treated at 600 C
 - Activation energy 54 meV
 - Analogous to dissipation peak in fused silica, involving thermally activated transitions of oxygen atoms?

Effect of heat treatment temperature on Ta₂O₅ loss

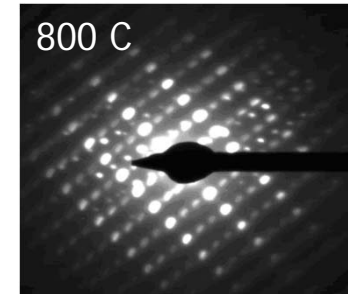
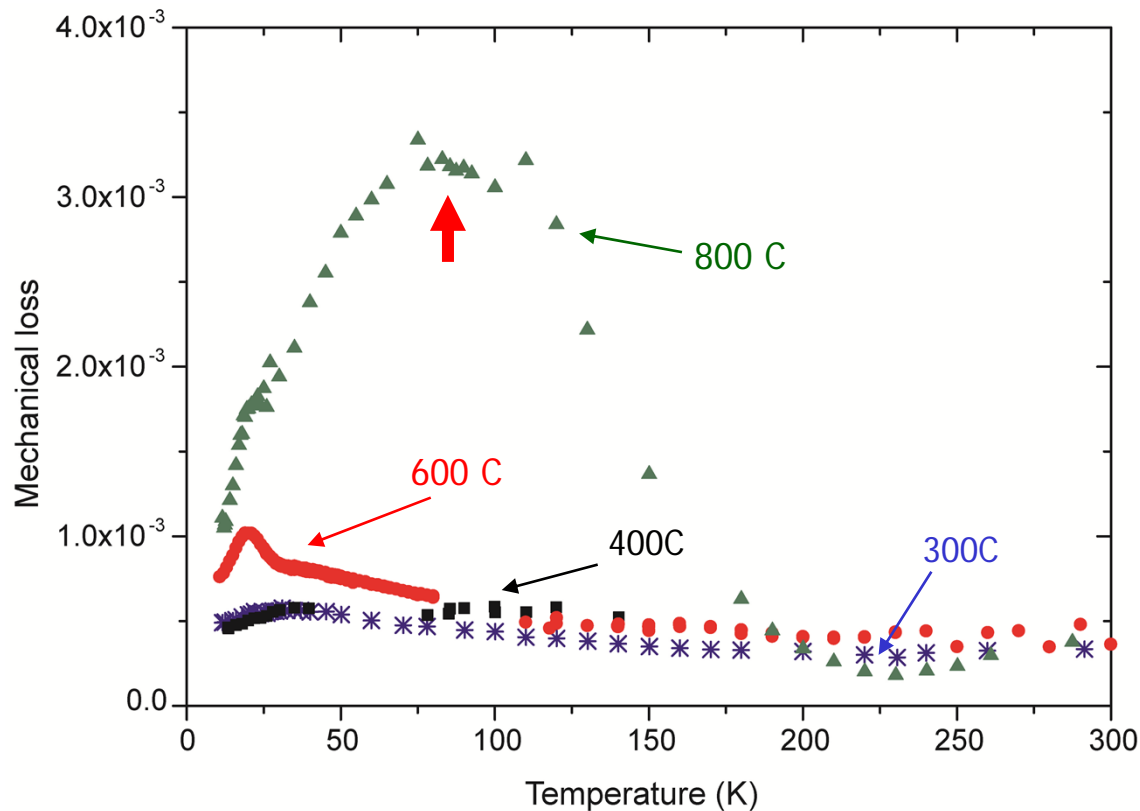


Above: Electron diffraction pattern of Ta₂O₅ heat treated at 600 C

Left: Loss at 1.9 kHz of 0.5 μm Ta₂O₅ coatings annealed at 300, 400, 600 and 800 C.

- 18 K peak
 - Observed in Ta₂O₅ heat treated at 600 C and 800 C
 - Related to structural changes brought on by heat treatment close to crystallisation temperature?

Effect of heat treatment temperature on Ta₂O₅ loss

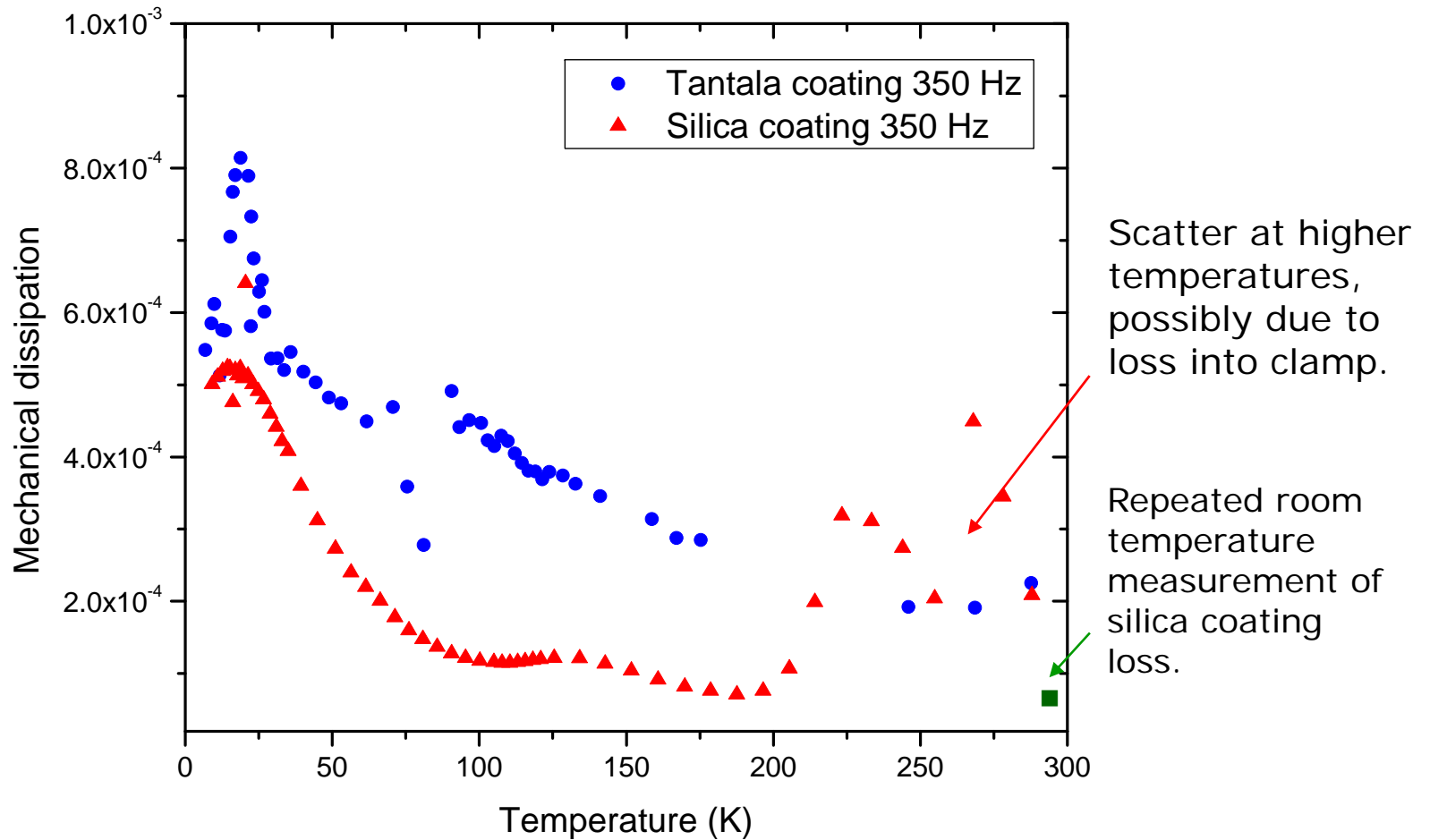


Above: Electron diffraction pattern of Ta₂O₅ heat treated at 800 C

Left: Loss at 1.9 kHz of 0.5 μm Ta₂O₅ coatings annealed at 300, 400, 600 and 800 C.

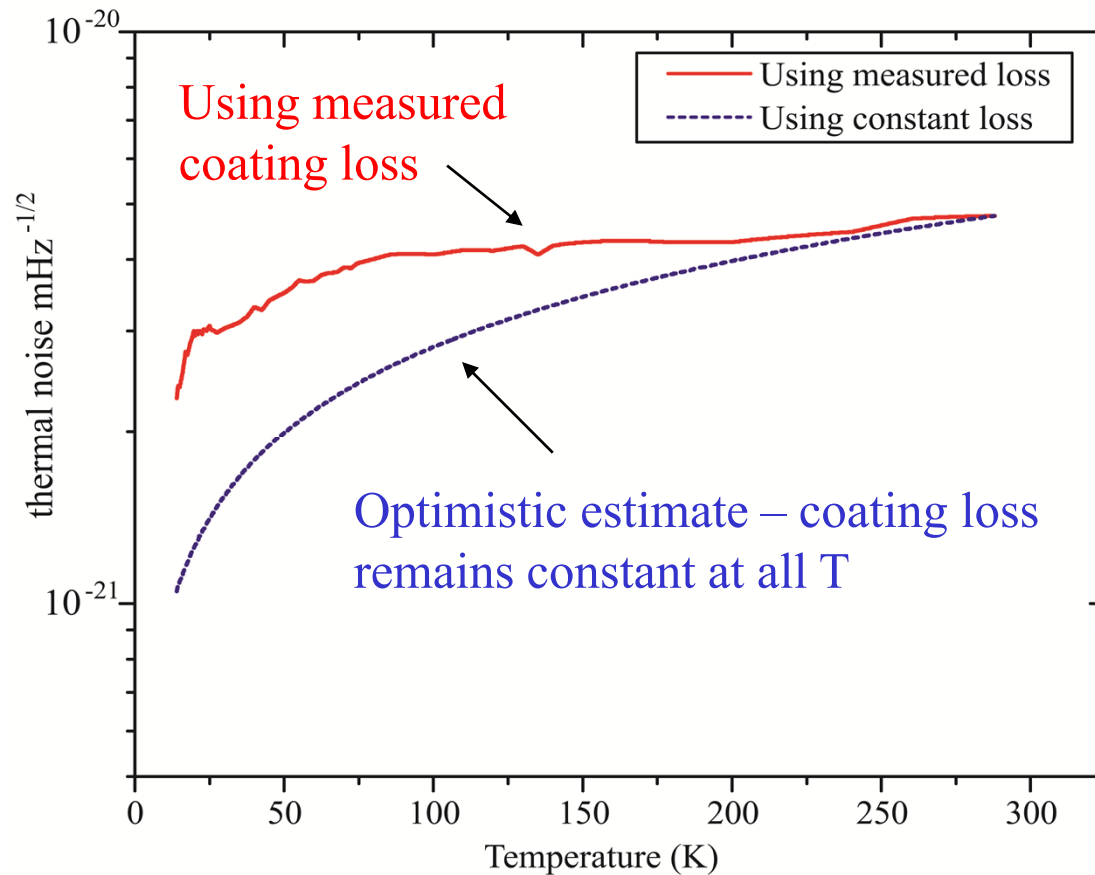
- 90 K peak
 - Observed in coating heat treated at 800 C
 - Large, broad loss peak likely to be related to (expected) **onset of polycrystalline structure** due to high temperature heat treatment. Loss mechanism could be e.g. phonon scattering at grain boundaries

Loss of silica coatings



- Loss of SiO_2 will have a significant contribution to coating thermal noise below 100 K

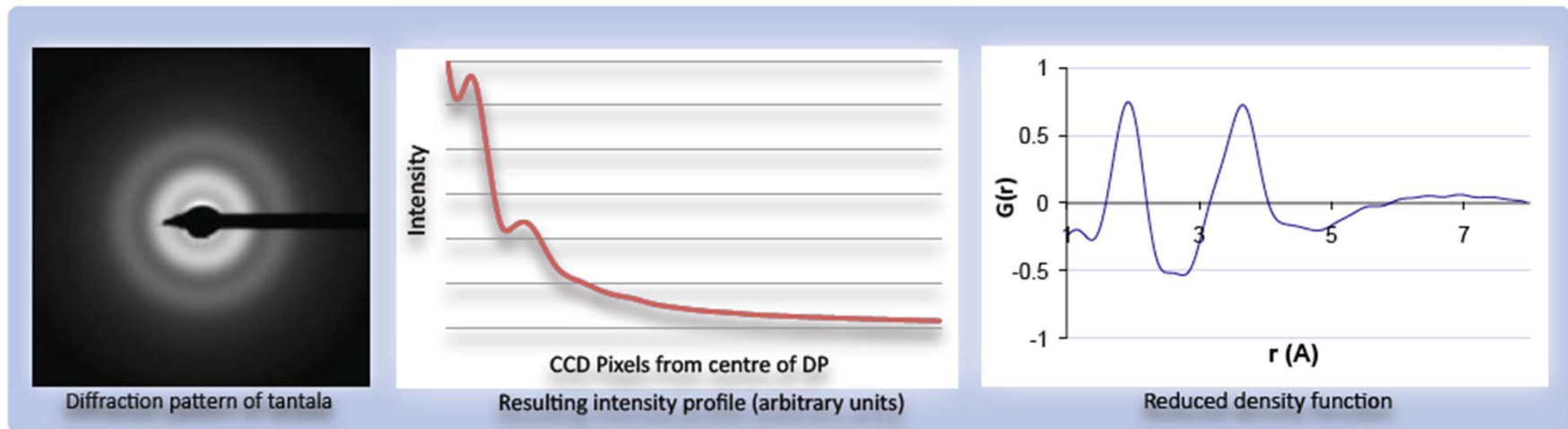
Coating thermal noise at 100 Hz



- If coating loss was constant with temperature, could gain factor of ~ 4 in TN at 18 K
- Measured coating losses imply we can only gain a factor of ~ 1.7 in coating TN by cooling to 18 K

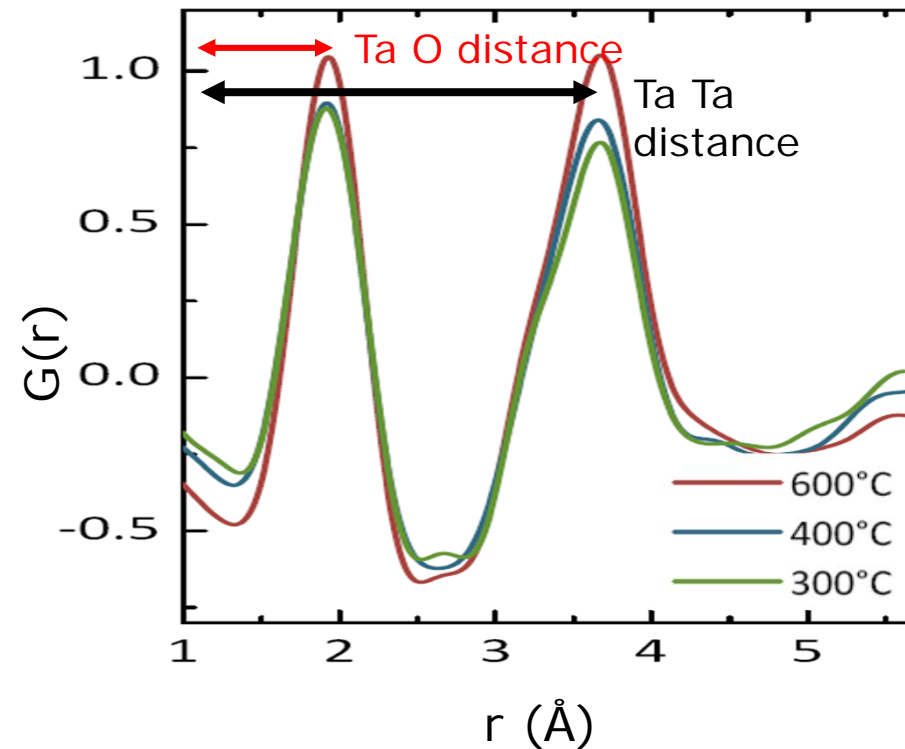
Probing links between atomic structure and loss

- Short range structure of amorphous materials probed by Reduced Density Function analysis of TEM electron diffraction data
 - RDF is a Fourier transform of the information gained from the intensity profile of a diffraction pattern
 - RDF gives statistical representation of where atoms are located with respect to a central atom



$$G(r) = 4 \int_0^{\infty} \varphi(q) \sin(qr) dq$$

The Reduced Density Function (RDF)

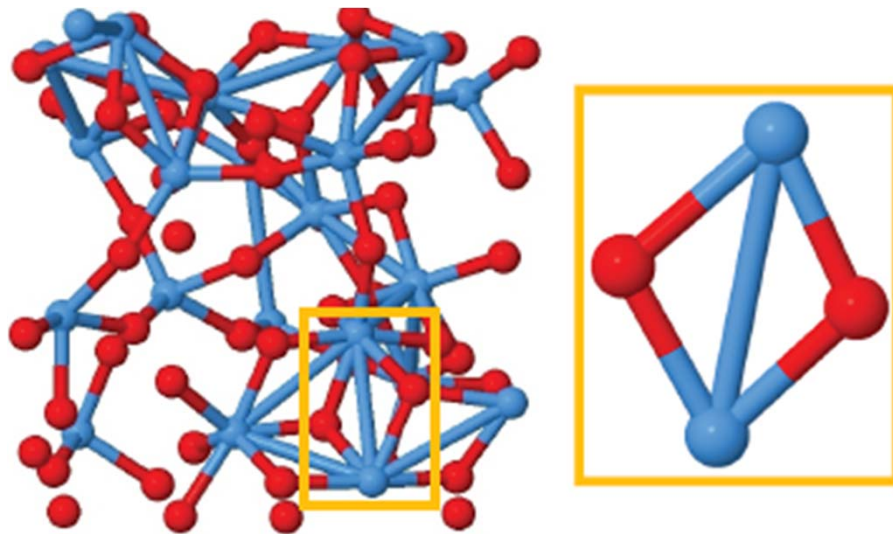


RDFs of heat-treated tantalum coatings¹

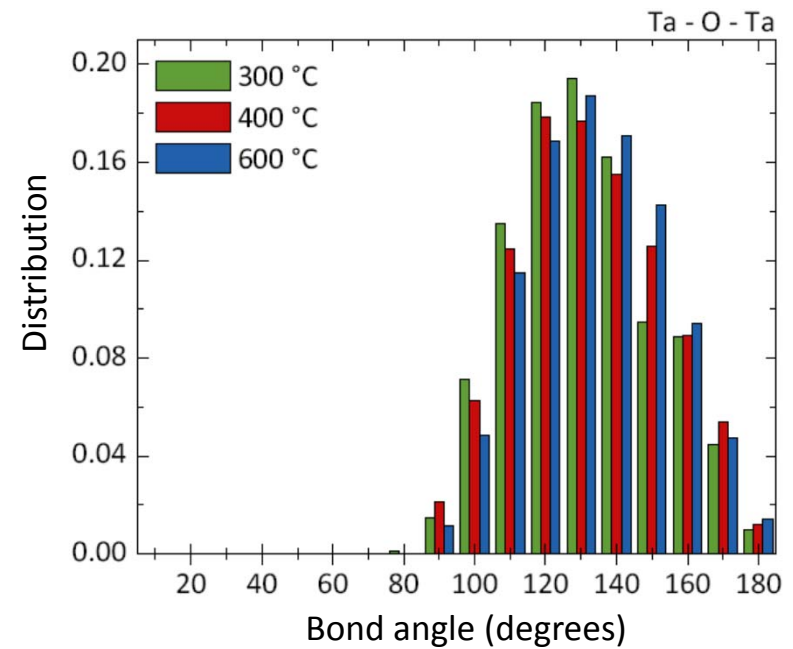
- Interpreting RDFs
 - Peak position - nearest neighbour distances
 - Peak height – nearest neighbour abundances
 - Peak width – indicates level of order in structure

Structural modelling

- RDF can be used as basis for Reverse Monte Carlo models of the microstructure, allowing e.g. bond angle distributions to be extracted
 - Molecular dynamics simulations used to ensure models are energetically stable



Ta – blue O - red



Current / future coatings research

- Alternative high-index coating materials e.g. amorphous Si, hafnia
- Alternative low-index materials e.g. Al_2O_3
- Exploring links between short-range atomic structure and loss
- Reduced coating / coating-free optics – diffractive optics and resonant waveguide mirrors

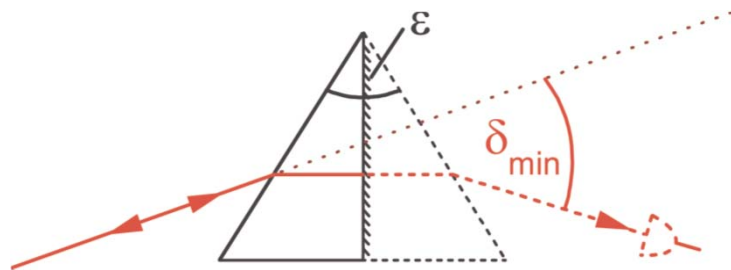
Optical Properties

Important Properties

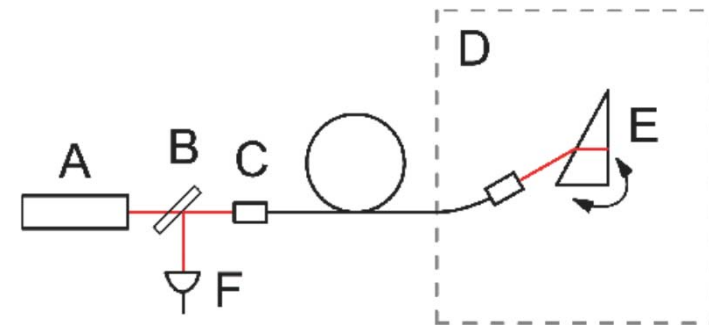
- refractive index n → influences coating selection
- thermo-refractive coefficient dn/dT → governs the thermo-refractive noise
- optical absorption α → limits the operation at low temperatures and at high laser powers
- all these parameters are needed at low temperatures!

Research on the refractive index of Si at low temperatures

- classical prism experiment needed to determine refractive index
- method of minimum deviation with high resolution cryogenic actuators



method of minimum deviation
in Littrow-order



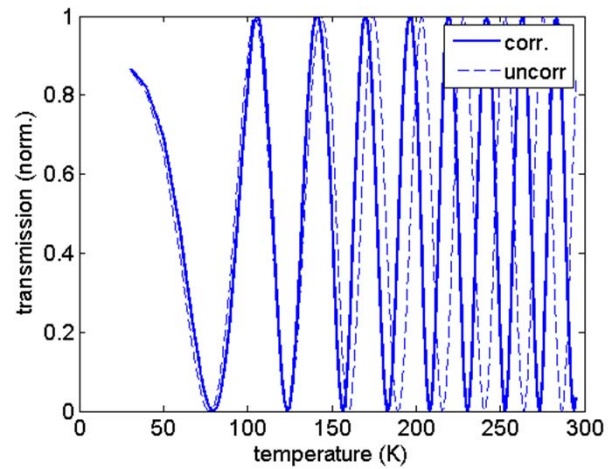
experimental setup for cryogenic
measurements of the refractive index
A – laser, B – beam splitter, C – Faraday
isolator, D – cryostat, E- prism, F- detector

Research on the thermo-refractive coefficient of Si

incoming light



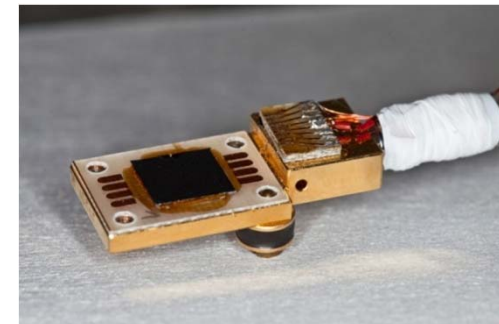
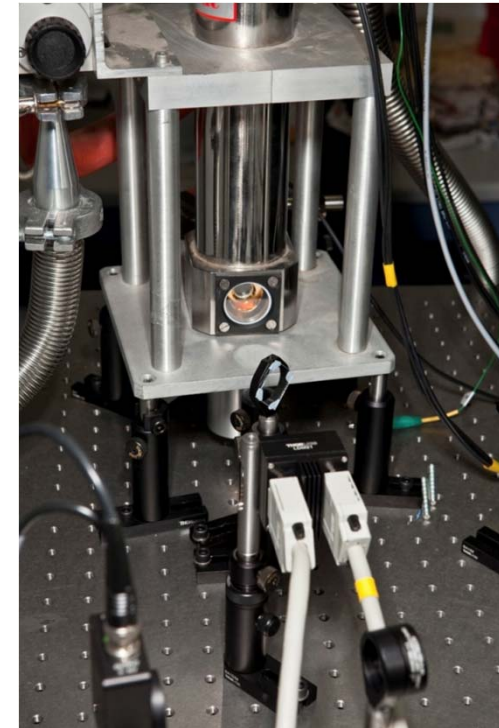
outcoming light



internal reflections

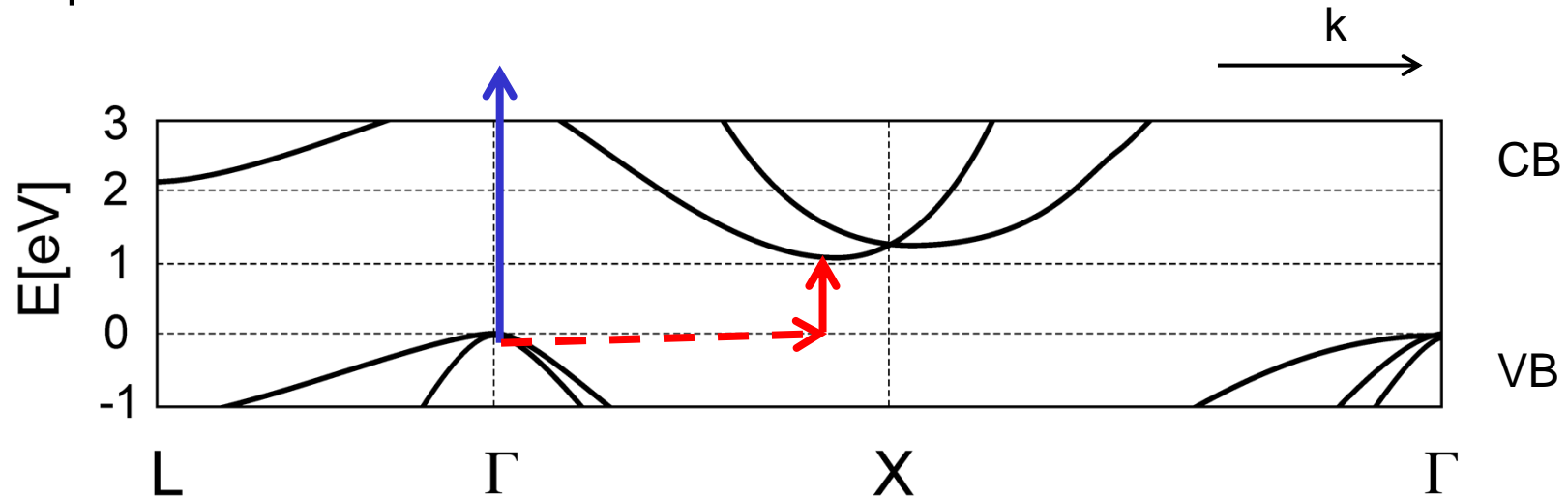


Fabry-Perot-Cavity
filled with silicon, dn/dT
determines optical path
length inside FPC



Research on the optical absorption of silicon at low T.

- simplified electronic band structure



— direct transition

— indirect transition

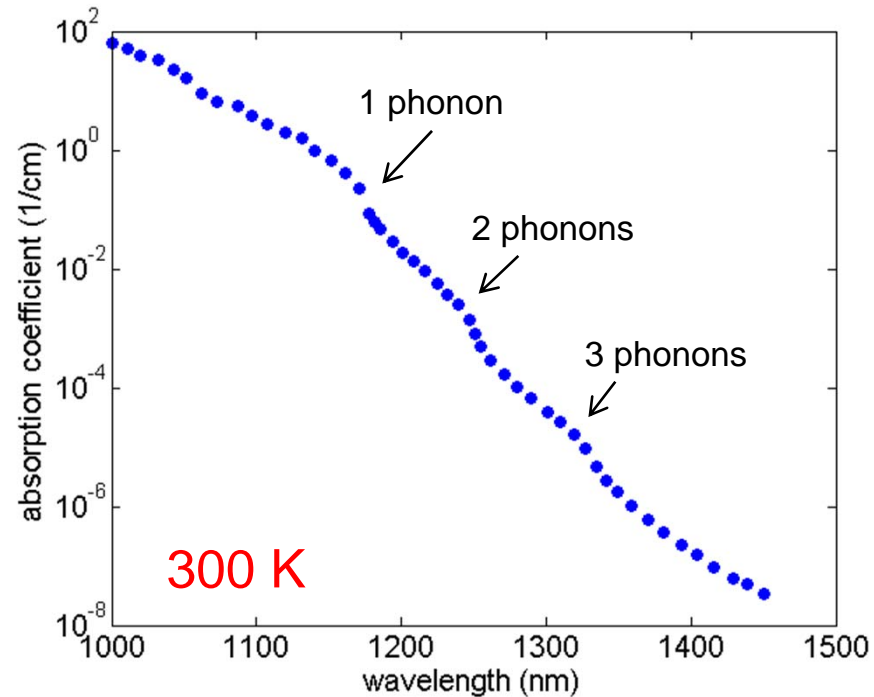
- - - phonon contribution

$\Delta k = 0$ photons do not carry momentum

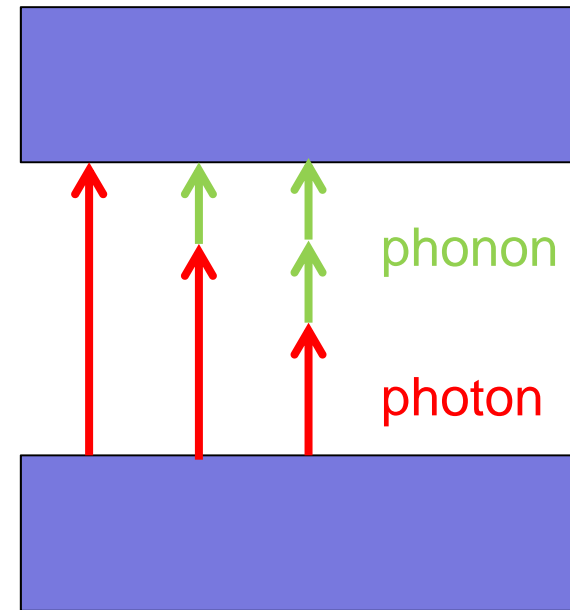
$\Delta k = k_{\text{phonon}}$ $E_{\text{photon}} + E_{\text{phonon}} = E$

→ photons with $E < E_{\text{gap}} = 1.1 \text{ eV}$ can be absorbed by assistance of phonons

Research on the optical absorption of silicon at low T.



[Keeves et al., J. Appl. Phys.]



- density of phonons is strongly temperature dependent
 - much smaller absorption can be expected at low temperatures
- measurements ongoing

Summary

What have we learned?

- material properties strongly determine the thermal noise performance of a detector
- simple temperature scaling is dangerous and leads to wrong results
- impurities determine mechanical losses → strong influence on thermal noise
- material science (understanding temperature behaviour of parameters) is needed to optimise future detectors
- a wide and open field...