Thermal noise
History from Brownian motion until interferometer

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Advanced Interferometer Configuration lecture
6 September 2011 @Institute for Cosmic Ray Research, Kashiwa, Japan

References

Peter R. Saulson,

Thermal noise in mechanical experiments,

Physical Review D 42 (1990) 2437.

Good review before revolution

(on the end of 20th century)

S. Rowan, J. Hough, and D.R.M. Crooks,

Thermal noise and material issues for gravitational wave detectors

Physics Letters A 347 (2005) 25.

One of the special articles for 100th anniversary of Annus Mirabilis (year of miracle) of A. Einstein

References

Erick D. Black

Notes on thermal noise, with a bibliography

LIGO-T030142- 01- R (2004)

http://www.ligo.caltech.edu/~blacke/T030142-01.pdf

From Brown and Einstein to Yamamoto

G.M. Harry, T. Bodiya, and R. DeSalvo (Editors)

Optical Coatings and Thermal Noise in Precision Measurements

Cambridge University Press, Cambridge (in press)

It will appear on January of 2012.

0.Abstract

I would like to explain …

(1) History until Fluctuation-Dissipation Theorem

What is the Fluctuation-Dissipation Theorem ?

(2) Thermal noise of resonant gravitational wave detector

(3) Thermal noise of interferometric gravitational wave detector before revolution (drastic progress in research of thermal noise) on the end of 20th century

(4) Thermal noise of interferometric gravitational wave detector after revolution on the end of 20th century

Contents

1. Until Fluctuation-Dissipation Theorem

Robert Brown investigated random motion of small particles (~1 μm) in water.
R. Brown, Philosophical Magazine 4 (1828) 161.

At first, he thought that this motion of particles from pollens stems from activity of life. However, he discovered that particles from inorganic materials also move at random.

Trivia : R. Brown observed motion of small particles from pollens, not pollens themselves ! Since pollens are too large (25 μm~100 μm), it is difficult to observe Brownian motion.

1. Until Fluctuation-Dissipation Theorem

Robert Brown investigated random motion of small particles (~1 μm) in water.

R. Brown, Philosophical Magazine 4 (1828) 161.

Mechanism was unknown.
Many ideas were proposed and rejected.

Random collisions with atoms of water ?
For example,

G. Cantoni, Nuovo Ciment 27 (1867) 156.
J. Delsaulx

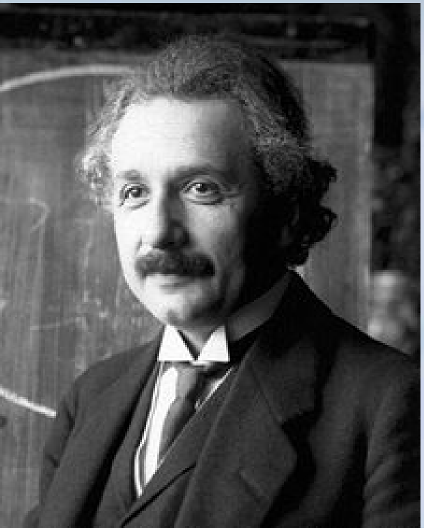
They are not proof but guesses.

1. Until Fluctuation-Dissipation Theorem

Albert Einstein showed theory of Brownian motion.
A. Einstein, Annalen der Physik 17 (1905) 549.

Why is so important this result ?

(1)Evidence of existence of atom
Avogadro constant derived from observation and his theory is consistent with those from other methods.



Wikipedia (A. Einstein, English)

1. Until Fluctuation-Dissipation Theorem

(2) Relation between diffusion (thermal motion) of particles and viscosity (dissipation) of water

He assumed that the law of physics of macroscopic body is the same as that of microscopic one.

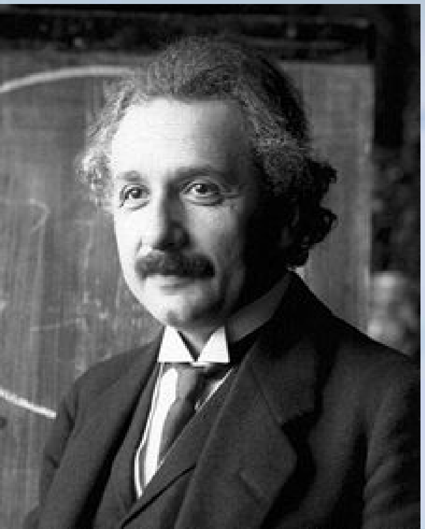
Einstein's relation

diffusion

$$D = \frac{RT}{N}\frac{1}{6\pi\eta a}$$

Avogadro constant

viscosity



Wikipedia (A. Einstein, English)

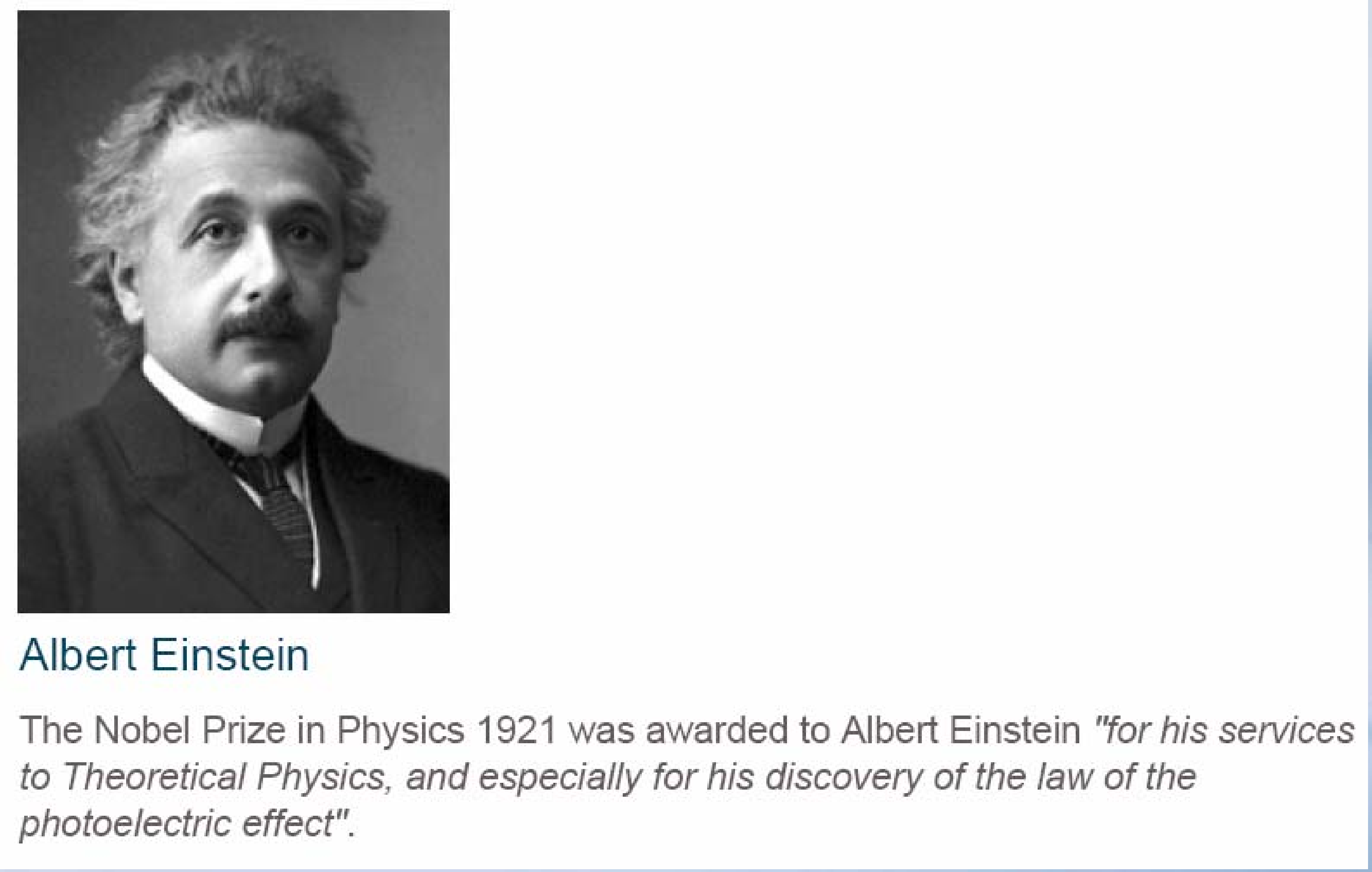
1. Until Fluctuation-Dissipation Theorem

Jean Baptiste Perrin's experiment proved that Einstein's theory is correct.
J. Perrin, Ann. Chim. Phys. 18 (1909) 1.

Perrin checked and confirmed Einstein's assumption (the law of physics of macroscopic body is the same as that of microscopic one) experimentally.

Perrin observed Brownian motion and derived Avogadro constant using Einstein's theory. The result is consistent with those of other methods.

1. Until Fluctuation-Dissipation Theorem



Albert Einstein

The Nobel Prize in Physics 1921 was awarded to Albert Einstein *"for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"*.

Web site of Nobel foundation

1. Until Fluctuation-Dissipation Theorem

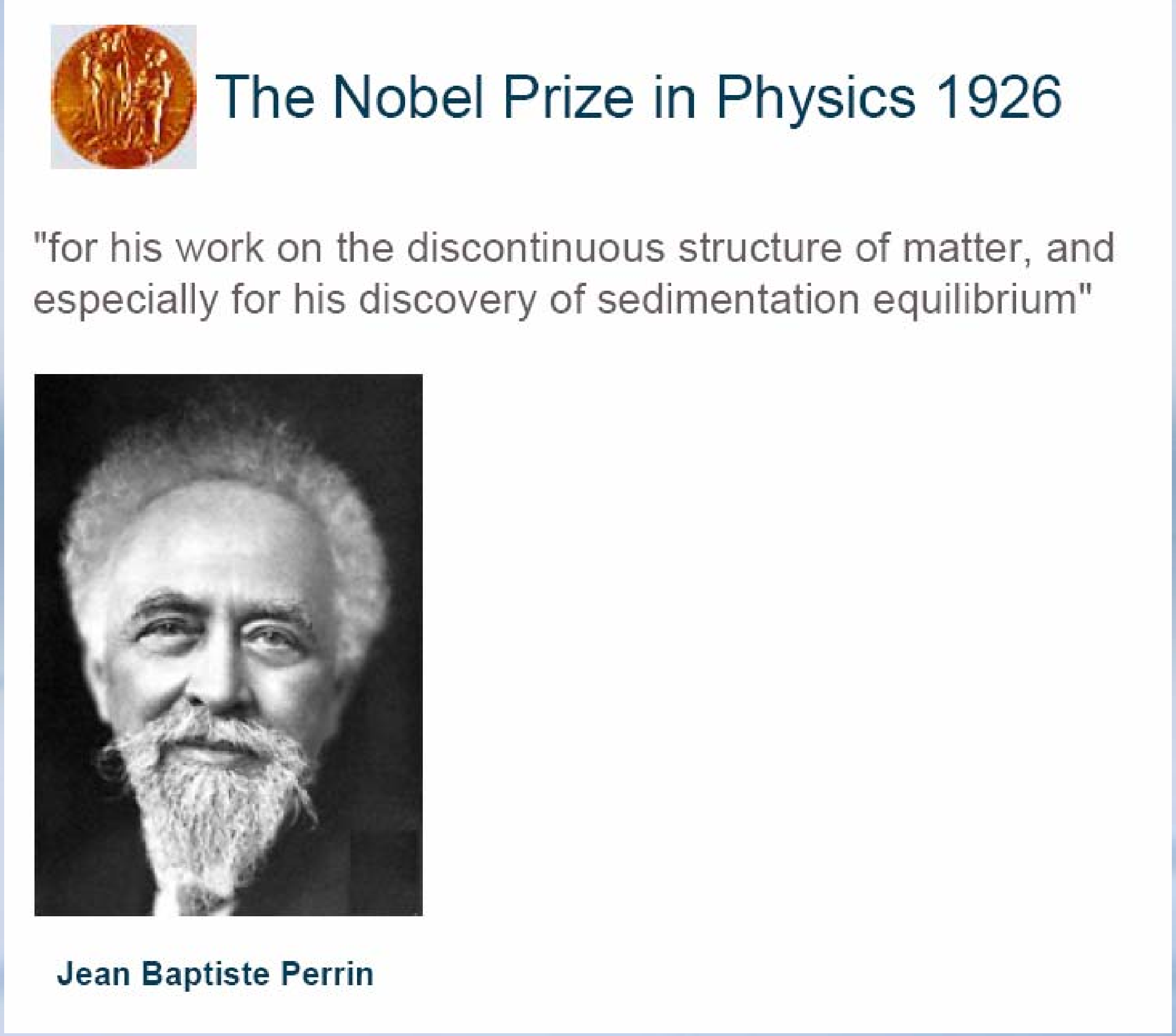
Presentation speech of Nobel prize in Physics 1921

(Laureate is A. Einstein)

Throughout the first decade of this century the so-called Brownian movement stimulated the keenest interest. In 1905 Einstein founded a kinetic theory to account for this movement by means of which he derived the chief properties of suspensions, i.e. liquids with solid particles suspended in them. This theory, based on classical mechanics, helps to explain the behaviour of what are known as colloidal solutions, a behaviour which has been studied by Svedberg, Perrin, Zsigmondy and countless other scientists within the context of what has grown into a large branch of science, colloid chemistry.

Web site of Nobel foundation

1. Until Fluctuation-Dissipation Theorem



The Nobel Prize in Physics 1926

"for his work on the discontinuous structure of matter, and especially for his discovery of sedimentation equilibrium"

Jean Baptiste Perrin

Web site of Nobel foundation

1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of electrical voltage (or current)

J.B. Johnson, Physical Review 32 (1928) 97.

Measurement

H. Nyquist, Physical Review 32 (1928) 110.

Theory

Nyquist’s theorem

$$G_{\mathrm{V}} = 4k_{\mathrm{B}}TR$$

Relation between electrical voltage fluctuation and resistance

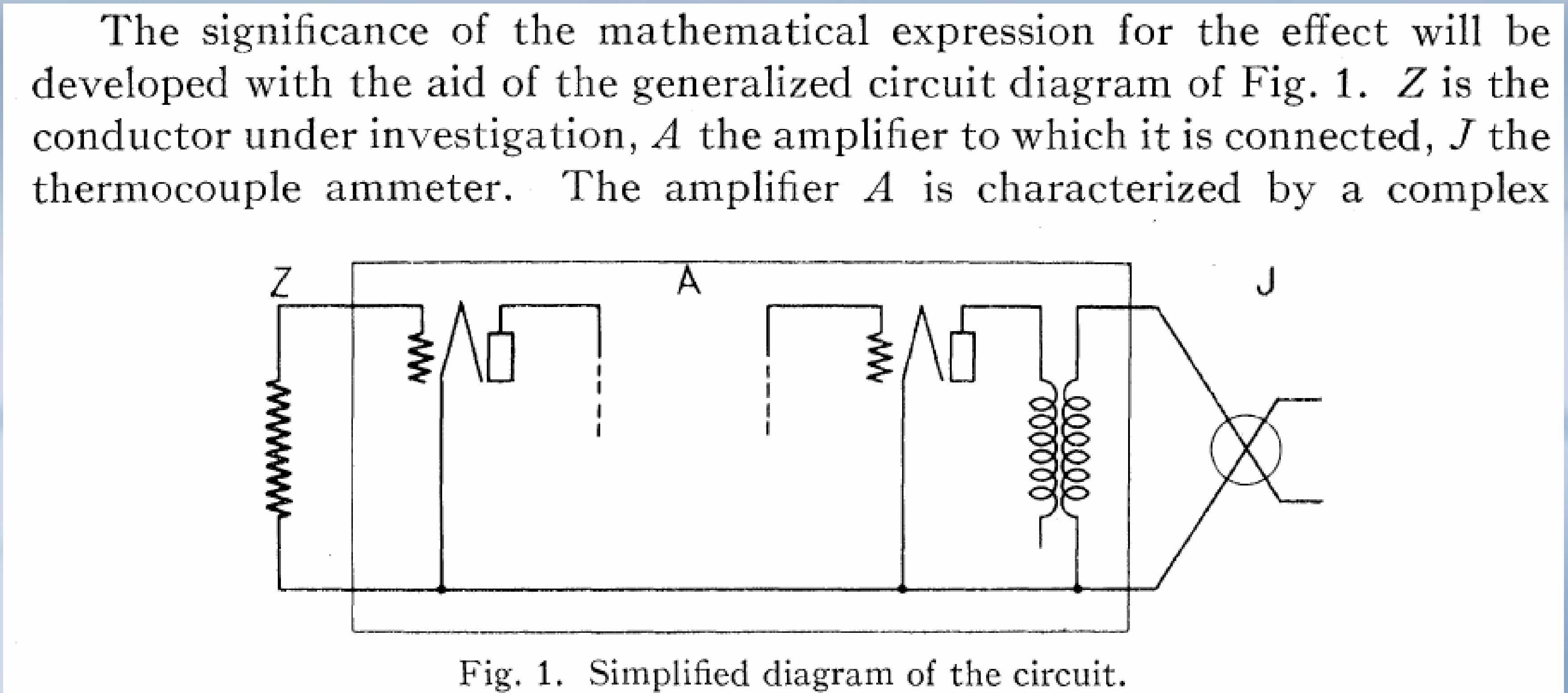
1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of electrical voltage (or current)

J.B. Johnson, Physical Review 32 (1928) 97.

He measured thermal current of resistance

using (resonant type or band pass type) amplifier.



The significance of the mathematical expression for the effect will be developed with the aid of the generalized circuit diagram of Fig. 1. Z is the conductor under investigation, A the amplifier to which it is connected, J the thermocouple ammeter. The amplifier A is characterized by a complex

Fig. 1. Simplified diagram of the circuit.

1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of electrical voltage (or current)

J.B. Johnson, Physical Review 32 (1928) 97.

He confirmed formula for thermal fluctuation.

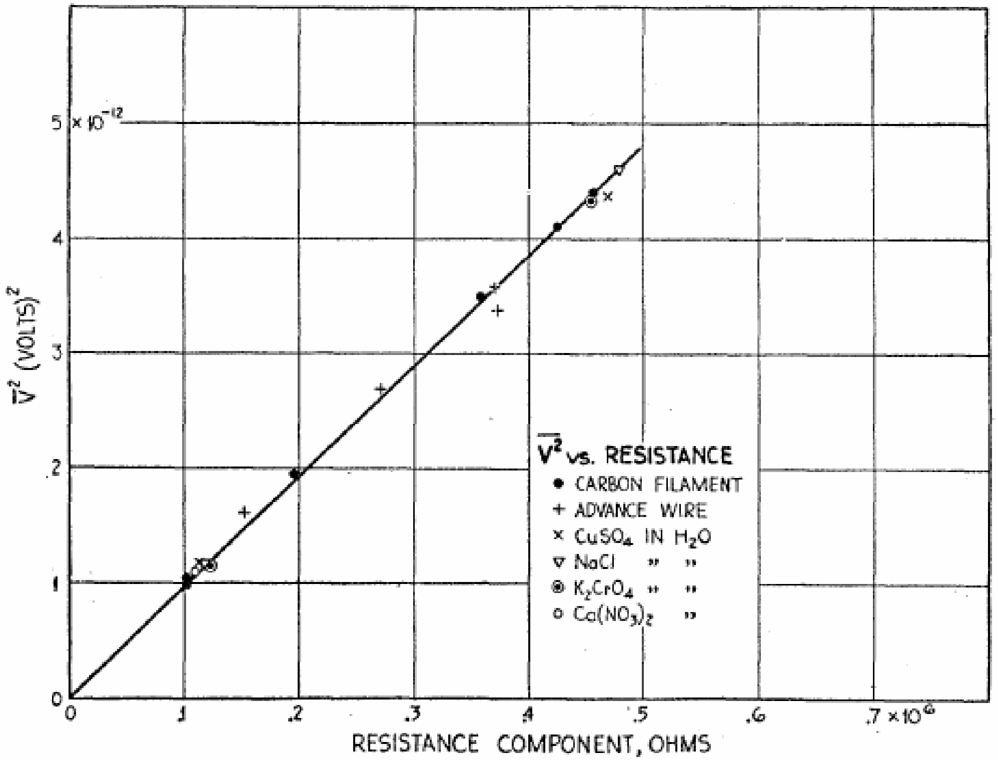


Fig. 4. Voltage-squared vs. resistance component for various kinds of conductors.

1. Until Fluctuation-Dissipation Theorem

Typical thermal voltage fluctuation (100 ohm, 300 K)

$$1.3\ \mathrm{nV}/\sqrt{\mathrm{Hz}}$$

Typical thermal current fluctuation (100 ohm, 300 K)

$$13\ \mathrm{pA}/\sqrt{\mathrm{Hz}}$$

1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of electrical voltage (or current)

H. Nyquist, Physical Review 32 (1928) 110.

His theory is based on
(1)Principle of energy equipartition
(2)Assumption that ohm law is correct
even if we consider voltage (current) fluctuation.
(Law for small fluctuation
is the same as that of macroscopic voltage or current)
This assumption is similar to Einstein's.

1. Until Fluctuation-Dissipation Theorem

Trivia

We can found three technical terms named after Nyquist.

Nyquist’s Theorem : Thermal noise

Nyquist criterion of stability : Stability of control

Nyquist sampling theorem : Sampling rate of measurement

Are all of them work by same person ?

The answer is yes !

Wikipedia (H. Nyquist, English)

1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of mechanical harmonic oscillator

Many people measured and analyzed fluctuation of angle of torsion pendulum using optical lever around 1925.

W. Einthoven et al., Physica 5 (1925) 358.
J. Tinbergen, Physica 5 (1925) 361.
W.J.H. Moll et al., Philosophical Magazine 50 (1925) 626.
G. Ising, Philosophical Magazine 1 (1926) 827.
F. Zernike, Zeitschrift fuer Physik 40 (1926) 628.
A.V. Hill, Journal of Scientific Instruments 4 (1926) 72.

Probably, this is not perfect list.

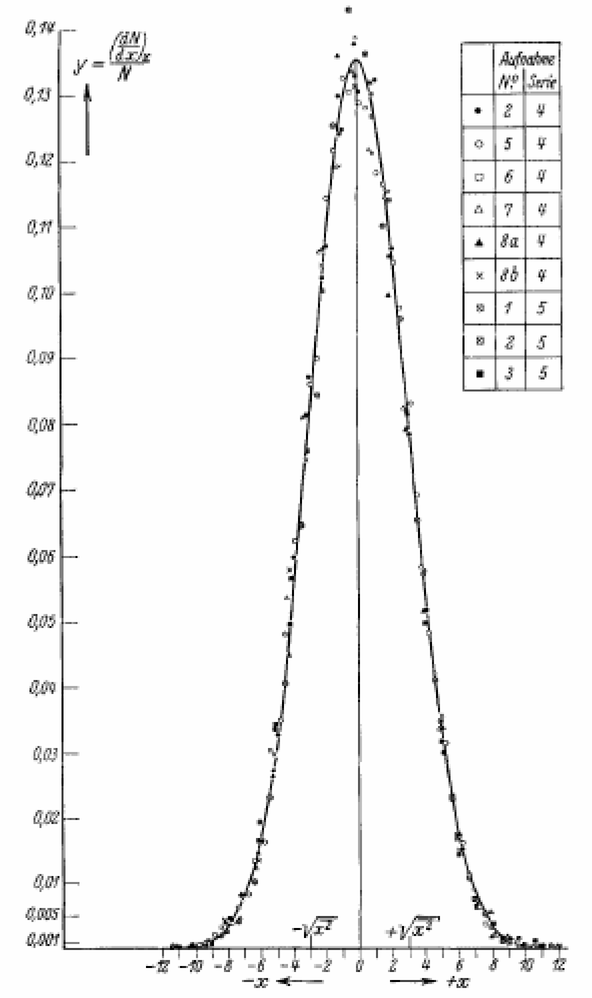
1. Until Fluctuation-Dissipation Theorem

Thermal fluctuation of mechanical harmonic oscillator

E. Kappler, Annalen der Physik 11-3 (1931) 233.

Torsion pendulum

He evaluated Avogadro constant and it is consistent with those of other experiment.



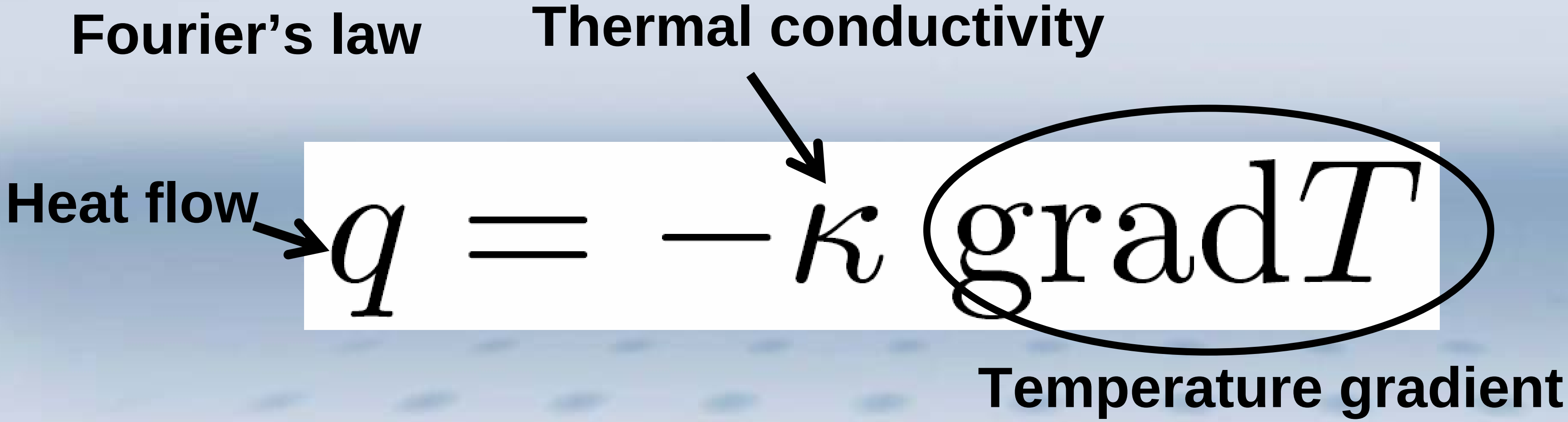
Experimentelle Verteilungskurve aus 9 Schwankungsaufnahmen von durchschnittlich je 11-stündiger Beobachtungsdauer

1. Until Fluctuation-Dissipation Theorem

Onsager reciprocity theorem

L. Onsager, Physical Review 37 (1931) 405.

Fourier's law



Thermal conductivity

Heat flow

$$q = -\kappa \, \mathrm{grad} T$$

Temperature gradient

In general case, κ is tensor.
According to Onsager reciprocity theorem, this tensor should be symmetric even if the material is not isotropic (like sapphire !).

1. Until Fluctuation-Dissipation Theorem

Onsager reciprocity theorem

L. Onsager, Physical Review 37 (1931) 405.

Onsager's assumption

(1)Microscopic reversibility

Symmetry of cross correlation function in time reflection

$$< \alpha_1(t)\alpha_2(t+\tau) >=< \alpha_1(t)\alpha_2(t-\tau) >$$

$$< \alpha_1(t)\alpha_2(t+\tau) >=< \alpha_2(t)\alpha_1(t+\tau) >$$

1. Until Fluctuation-Dissipation Theorem

Onsager reciprocity theorem

L. Onsager, Physical Review 37 (1931) 405.

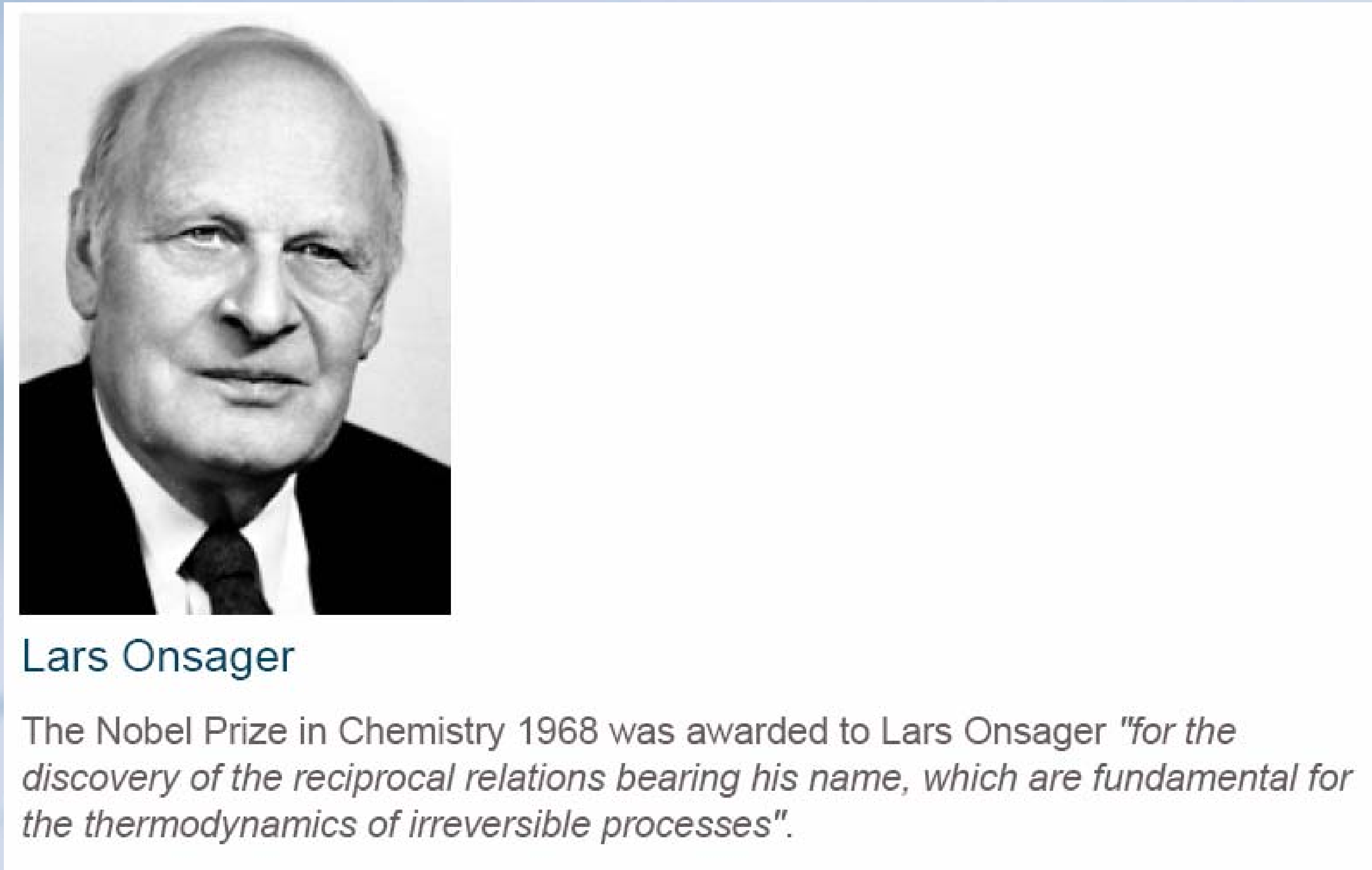
Onsager's assumption

(2)The average decay of fluctuations will obey the ordinary laws.

Law for average decay of small fluctuation is the same as that of macroscopic motion (ordinary law).

This assumption is similar to Einstein's and Nyquist's.

1. Until Fluctuation-Dissipation Theorem



Lars Onsager

The Nobel Prize in Chemistry 1968 was awarded to Lars Onsager *"for the discovery of the reciprocal relations bearing his name, which are fundamental for the thermodynamics of irreversible processes"*.

Web site of Nobel foundation

1. Until Fluctuation-Dissipation Theorem

Finally, general theorem appeared.

Fluctuation-Dissipation Theorem (FDT)

H.B. Callen and R.F. Greene, Physical Review 86 (1952) 702.
R.F. Greene and H.B. Callen, Physical Review 88 (1952) 1387.

Relation between thermal fluctuation and dissipation

Fluctuation : Energy from heat bath
Dissipation : Energy to heat bath
Interaction between system and heat bath

1. Until Fluctuation-Dissipation Theorem

Fluctuation-Dissipation Theorem is valid if
(1)system is linear.
(2)system is in thermal equilibrium.

$$G(f) = -\frac{4k_{\mathrm{B}}T}{\omega}\mathrm{Im}[H(\omega)]$$

Power spectrum
of thermal fluctuation

Imaginary part of susceptibility
(dissipation)

1. Until Fluctuation-Dissipation Theorem

Fluctuation-Dissipation Theorem is valid if
(1)system is linear.
(2)system is in thermal equilibrium.

$$G_{ij}(f) = -\frac{4k_{\mathrm{B}}T}{\omega}\boxed{\mathrm{Im}[H_{ij}(\omega)]}$$

Cross correlation spectrum
of thermal fluctuation

1. Until Fluctuation-Dissipation Theorem

(a)Einstein's relation

Relation between Brownian motion (fluctuation)
and viscosity (dissipation) of water

FDT in the case with free mass
with viscous damping at low frequency

(b)Nyquist's theorem

Relation between thermal voltage fluctuation
and resistance (dissipation)

FDT in electric circuit

(c)Onsager reciprocity theorem

Cross correlation spectrum at low frequency in FDT

All these formulae are examples of FDT !

1. Until Fluctuation-Dissipation Theorem

Assumption of fluctuation dissipation theorem

Onsager's assumption

The average decay of fluctuations will obey the ordinary laws.

Law for average of small fluctuation is the same as that of macroscopic motion with dissipation.

Relation between fluctuation and dissipation is assumed, not proved !

1. Until Fluctuation-Dissipation Theorem

Fluctuation : Energy from heat bath
Dissipation : Energy to heat bath
Interaction between system and heat bath

In the case of Brownian motion ….

Fluctuation : Random collision of atoms
Dissipation : Collision of atoms

Even in dissipation, collision is at random.
In some cases of dissipation process, atoms give particle energy. However, in average, particle gives atoms energy. Therefore, the dissipation process is the average of fluctuation.

→ **Onsager's assumption**

1. Until Fluctuation-Dissipation Theorem

How to derive fluctuation dissipation theorem ?

Onsager’s assumption implies that
time development of auto (or cross) correlation
function of thermal fluctuation is the same as that
of step function response
which is the decay motion to new equilibrium position
after applied constant force vanished.

The amplitude of auto correlation function is derived from principle of energy equipartition.

Power (or cross correlation) spectrum is
Fourier transform of auto (or cross) correlation function.
Wiener-Khinchin relation

1. Until Fluctuation-Dissipation Theorem

FDT in quantum mechanics
H.B. Callen and T.A. Welton, Physical Review 83 (1951) 34.

When we should take quantum mechanics into account ?

$$\hbar\omega \gg k_{\mathrm{B}}T$$

Smallest energy
in quantum mechanics

Average energy
in classical statistical mechanics

At room temperature, if the frequency is more than $6*10^{12}$ Hz, we should consider quantum mechanics.

1. Until Fluctuation-Dissipation Theorem

Fluctuation Dissipation Theorem in Quantum mechanics

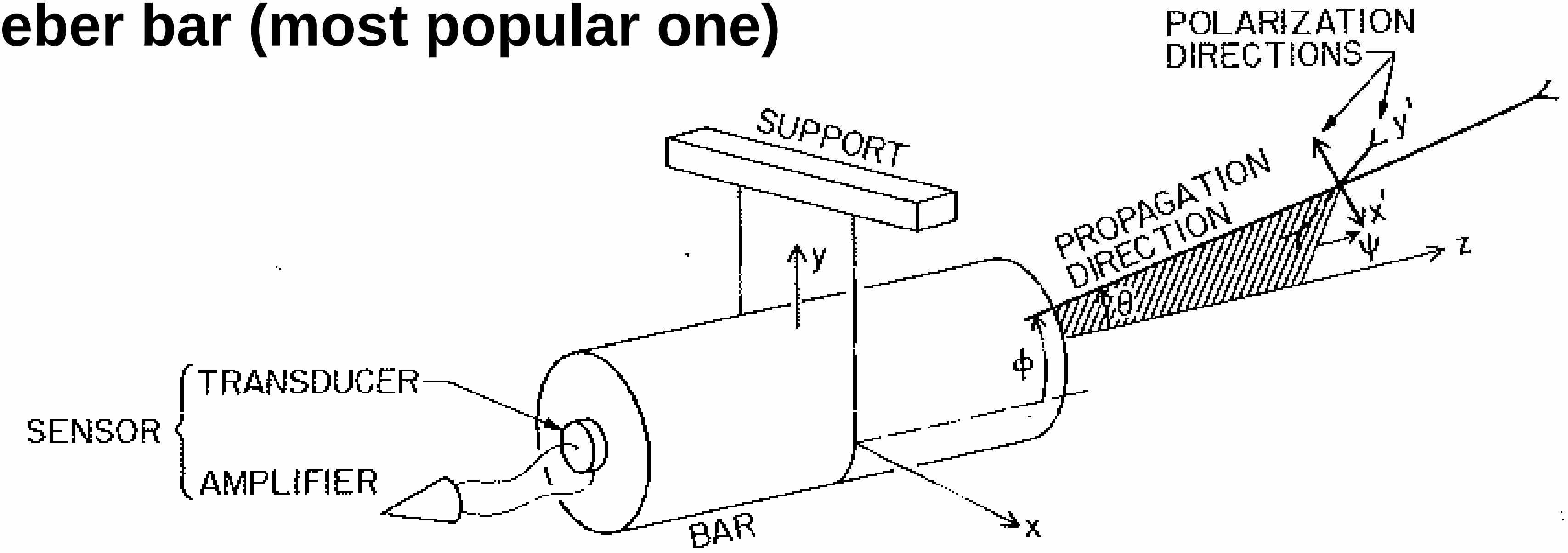
Kubo formula
R. Kubo,
Journal of the Physical Society of Japan 12 (1957) 570.

2. Resonant detector

Resonant detector

Gravitational wave excites resonant motion of elastic body.

Weber bar (most popular one)



**“*300 years of gravitation*”
(1987) Cambridge University Press
Fig. 9.8**

**Diameter : several tens cm
Length : a few meters
Resonant frequency : about 1 kHz**

2. Resonant detector

Resonator : tidal force of gravitational wave

Thermal fluctuation force must be considered.

Observation of thermal fluctuation of torsion pendulum

Displacement, not force, was monitored.

Formula of thermal fluctuation force (on resonance)

$$G_{\mathrm{F}}(f_0) = \frac{4k_{\mathrm{B}}Tm\omega_0}{Q}$$

***T*/*Q* should be small.**

Low temperature (low *T*), Small mechanical loss (large *Q*)

2. Resonant detector

First generation (room temperature)

Weber bar (University of Maryland, U.S.A.) …

Second generation (4 K) Liquid helium

Explorer (Italy, CERN), Allegro (U.S.A.),

Niobe (Australia), Crab (Japan) …

Third generation (< 100 mK) Dilution refrigerator

Nautilus (Italy), Auriga (Italy),

Mini-Grail (Netherlands),

Mario Schenberg (Brazil) …

This is not a perfect list !


NAUTILUS
INFN - LNF

G. Pizzella, ET first general meeting (2008)

2. Resonant detector

High Q-value (low mechanical loss) material

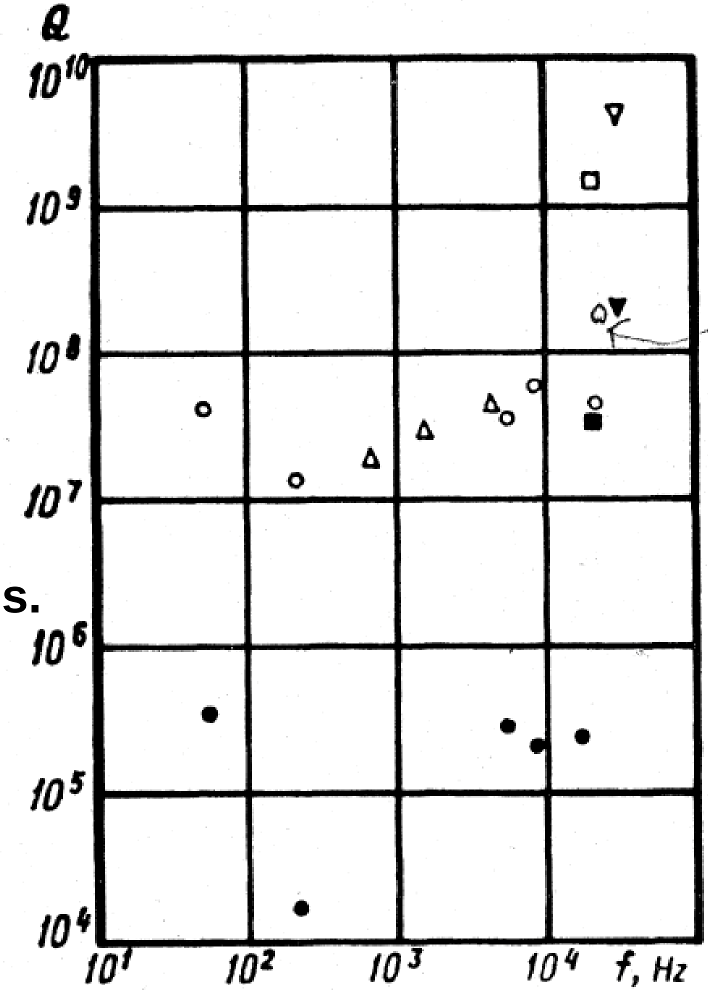
Small dissipation at low temperature

Sapphire and Silicon (Moscow)
Niobium (Australia)
CuAl6% (Mini-Grail (Netherlands), Mario Schenberg (Brazil))

K.S. Thorne, Chapter 9 of "*300 years of gravitation*"(1987) Cambridge University Press p409.
A de Waard *et al*., Classical and Quantum Gravity 21 (2004) S465.
O.D. Aguiar *et al*., Classical and Quantum Gravity 25 (2008) 114042.

Almost all resonators (Weber bar also) are made from aluminum.
Large bulk, low cost ...

V.B. Braginsky,
V.P. Mitrofanov,
K.S. Thorne
"*Systems*
***with Small Dissipation*"**
(1986)
University of Chicago Press.



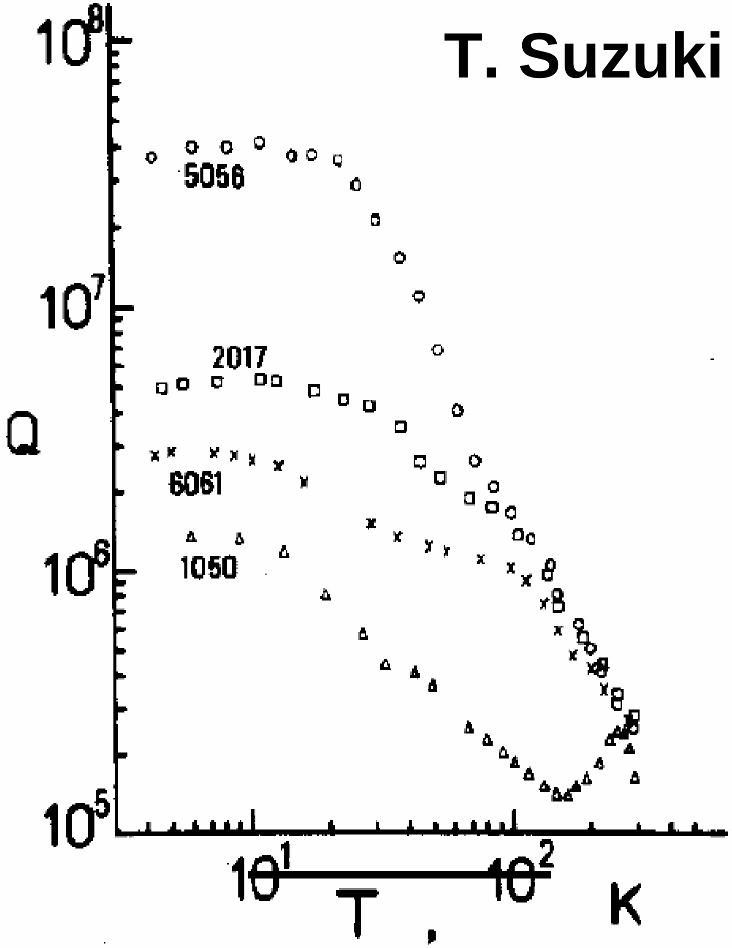
coccia et al
PLA 219 (19
35 KHz c
40mK

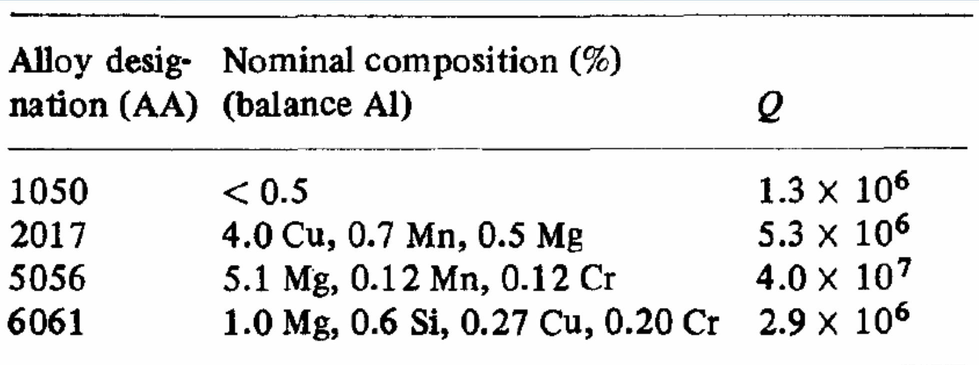
Fig. 6 Maximum quality factors for mechanical resonators made
different materials. ○ = Al-5056 ~~(Oide, Tsubono, and Hirakawa 1~~
Δ = Nb (Blair *et al.* 1980b), □ = Si (McGuigan *et al.* 1978), ▽ = A
(Bagdasarov *et al.* 1977). Solid symbols correspond to $T = 300$ K,
symbols to $T = 4.2$ K. Note added in press: Recently Veitch *e*
(1985) have reported a Q of 2×10^8 at 4 K in a niobium bar.

2. Resonant detector

What is kinds of aluminum alloy best ?
T. Suzuki *et al.* discovered that Al5056 has high Q-value.
Almost all resonators are made from Al5056.

T. Suzuki *et al.*, Physics Letters 67A (1978) 2.





Alloy designation (AA)	Nominal composition (%) (balance Al)	Q
1050	< 0.5	1.3×10^6
2017	4.0 Cu, 0.7 Mn, 0.5 Mg	5.3×10^6
5056	5.1 Mg, 0.12 Mn, 0.12 Cr	4.0×10^7
6061	1.0 Mg, 0.6 Si, 0.27 Cu, 0.20 Cr	2.9×10^6

3. Interferometer before revolution

Interferometric gravitational wave detector

Mirrors must be free and are suspended.

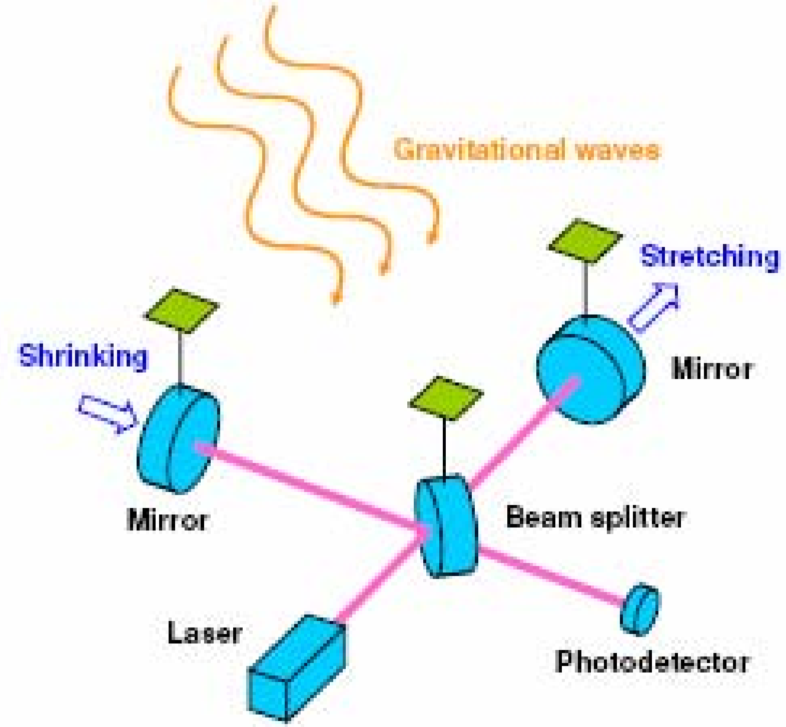


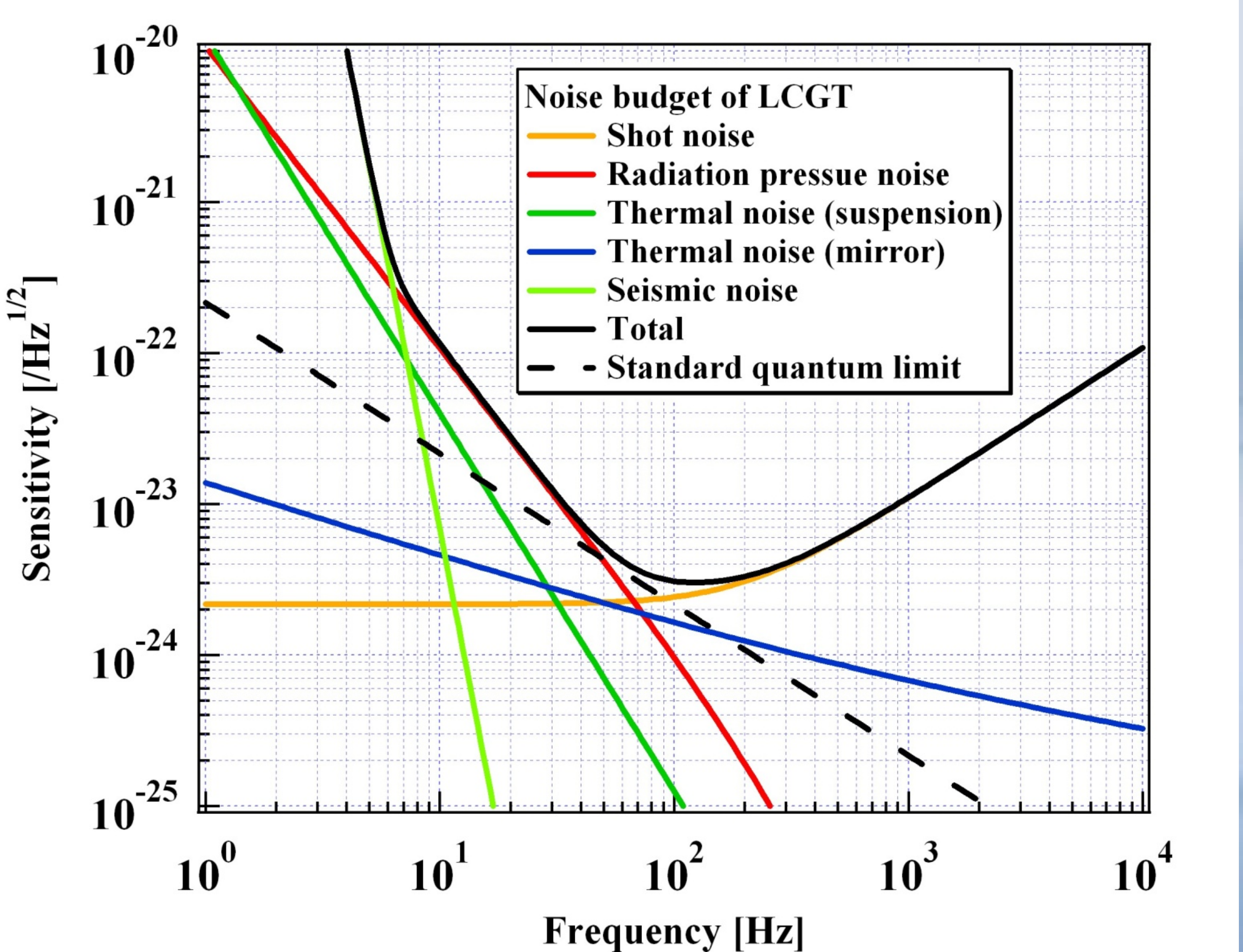
Figure 1. Michelson laser interferometer with suspended mirrors for the detection of gravitational waves.

S. Kawamura, Classical and Quantum Gravity 27 (2010) 084001.

3. Interferometer before revolution

Typical example of sensitivity of interferometer

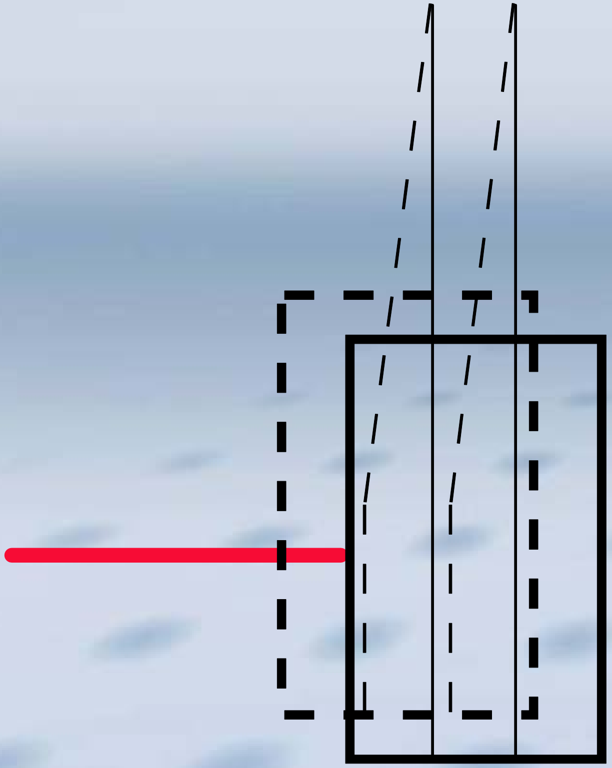
(Old version of LCGT)

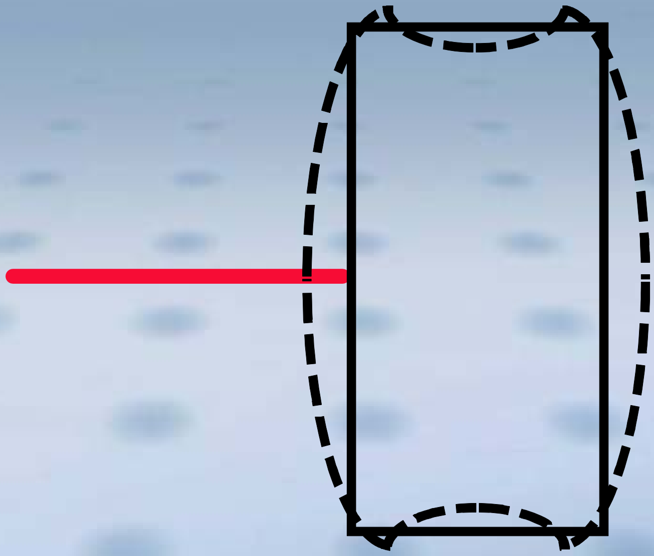


http://spacefil

3. Interferometer before revolution

Thermal noise of suspension and mirror





3. Interferometer before revolution

Suspension and mirror : Mechanical harmonic oscillator

Resonant frequency : suspension : ~ 1 Hz
mirror : > 10 kHz
Target frequency of gravitational wave : ~ 100 Hz
Off resonance thermal fluctuation of displacement

Resonant detector : Force on resonance
Torsion pendulum : Displacement on resonance

Residual gas damping is not a problem because interferometer in vacuum (< 10^{-7} mbar).
Mechanical loss in suspension and mirror is crucial.

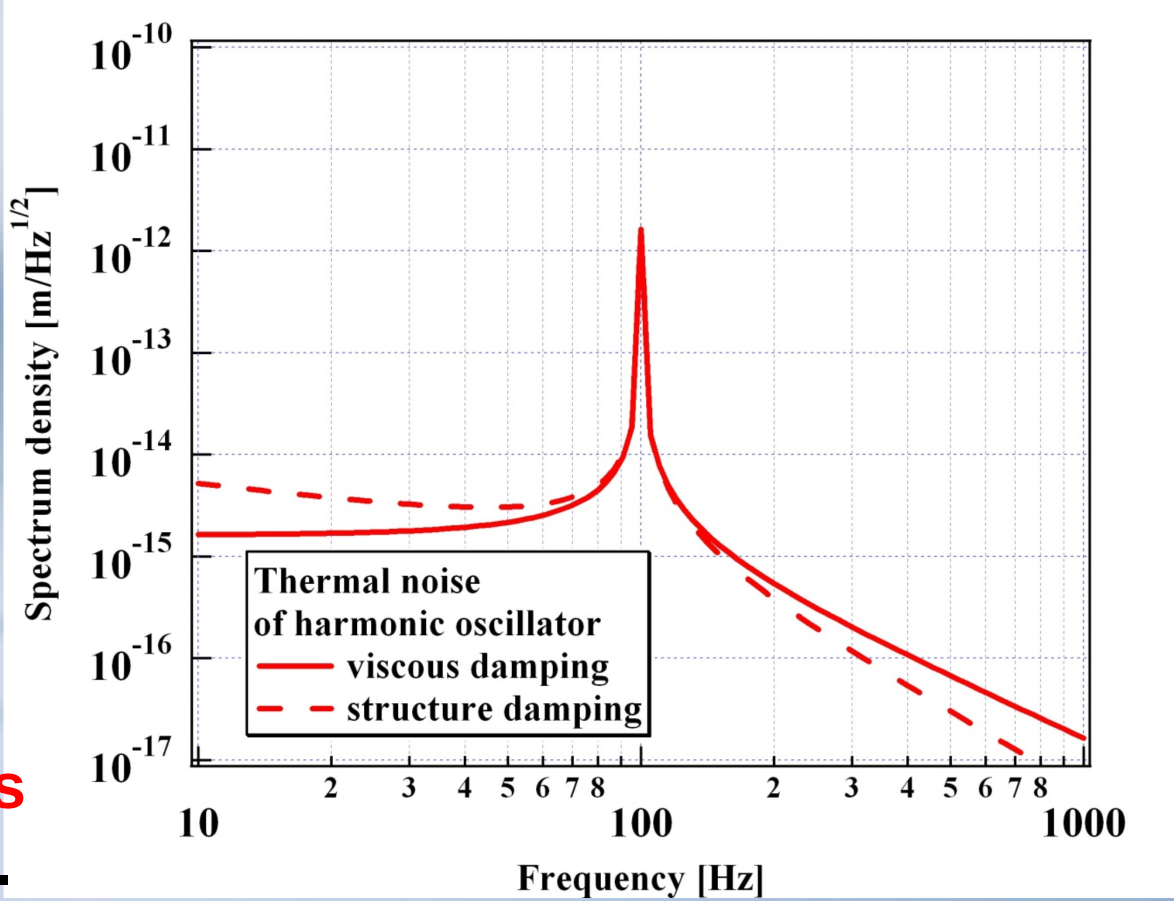
3. Interferometer before revolution

Spectrum density of thermal noise of harmonic oscillator

Viscous damping :
Friction force is
proportional
to velocity.

Structure damping :
Loss is
independent
of frequency.

Loss in many materials
are structure damping.



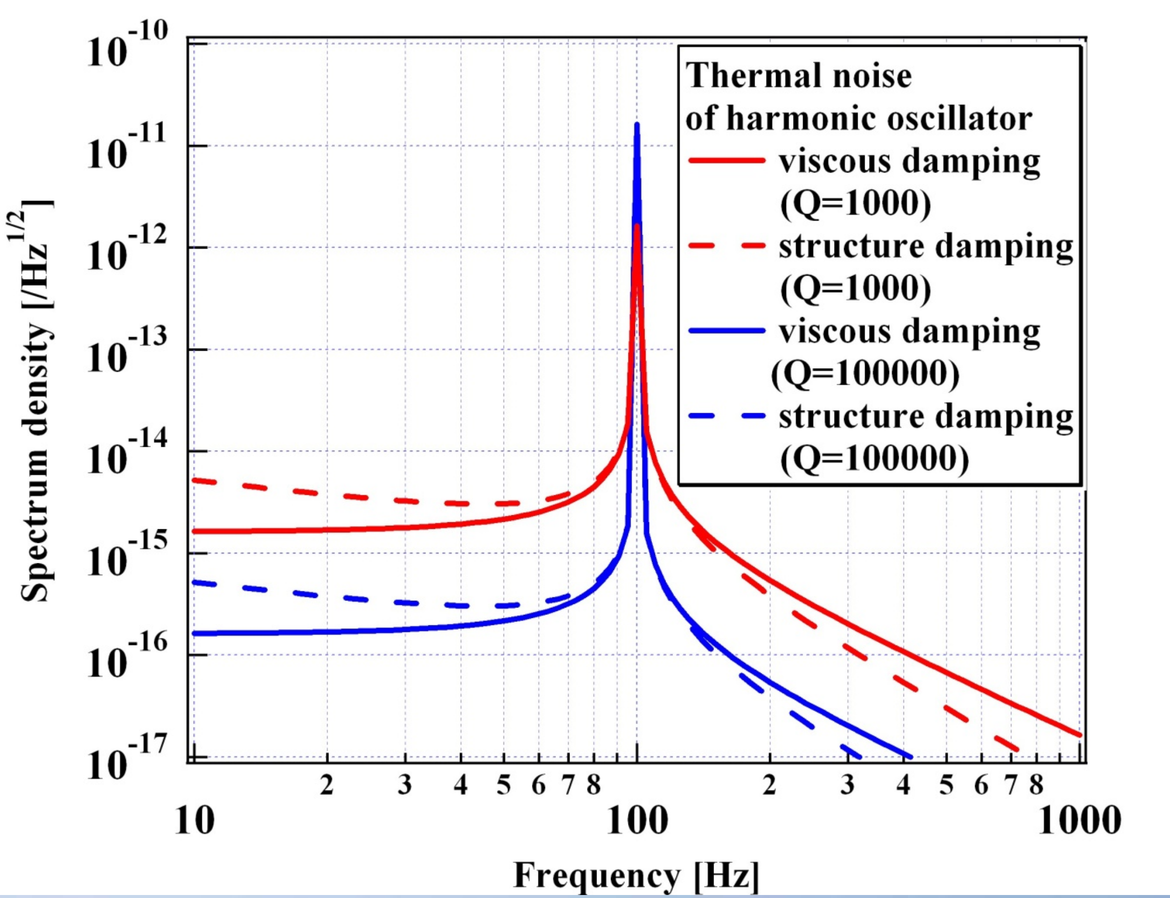
3. Interferometer before revolution

Spectrum density of thermal noise of harmonic oscillator

Q-value :
Magnitude of loss

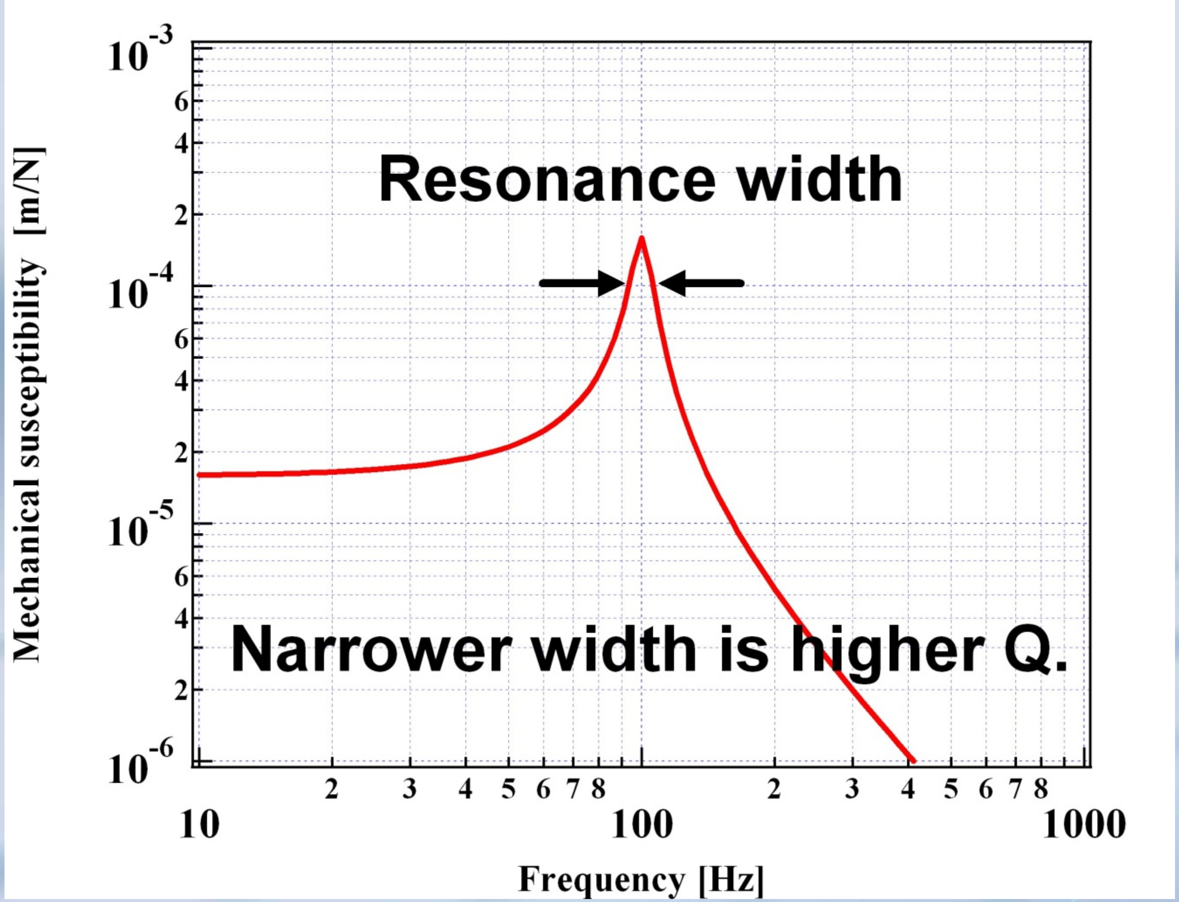
Higher Q is
smaller loss.

Higher Q is
smaller off resonance
thermal noise
and better.



3. Interferometer before revolution

Measurement of Q-value (Width of resonance peak)

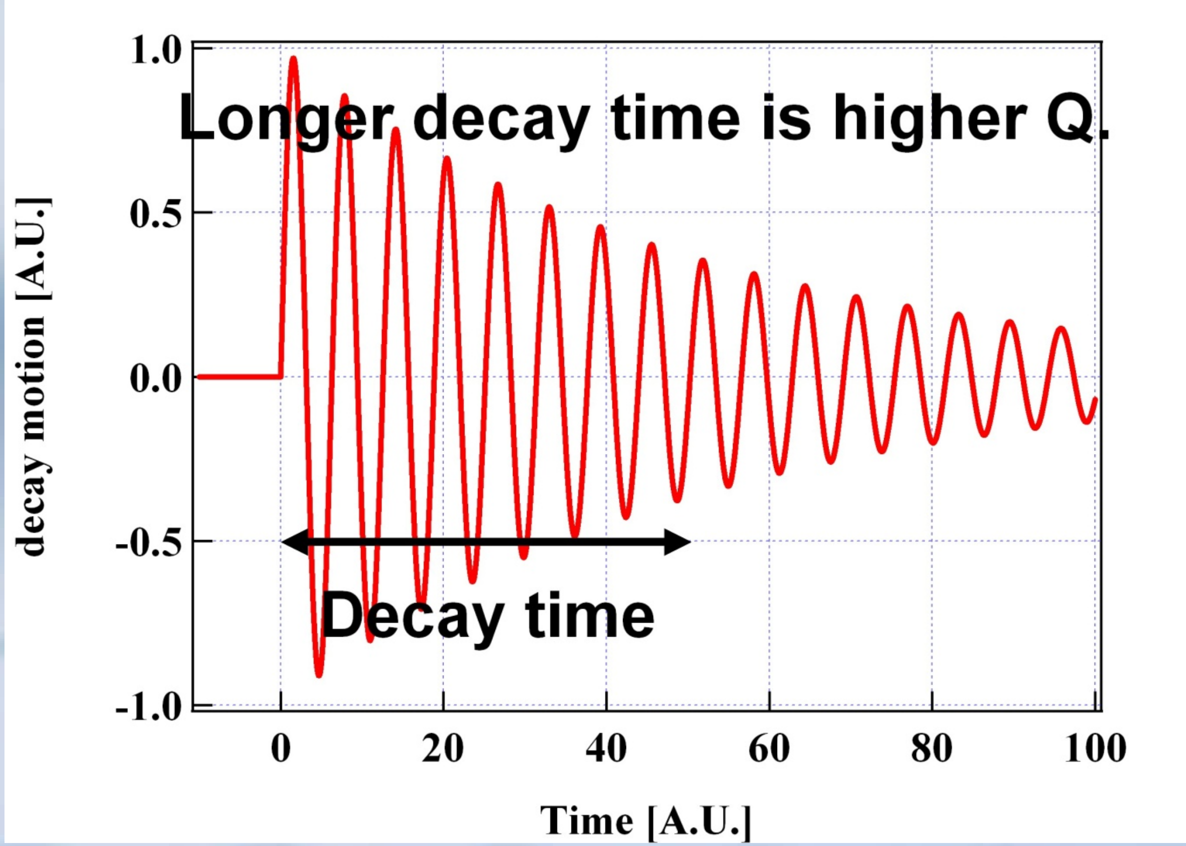

Resonance width

Narrower width is higher Q.

If Q-value is too high, measurement is difficult.

3. Interferometer before revolution

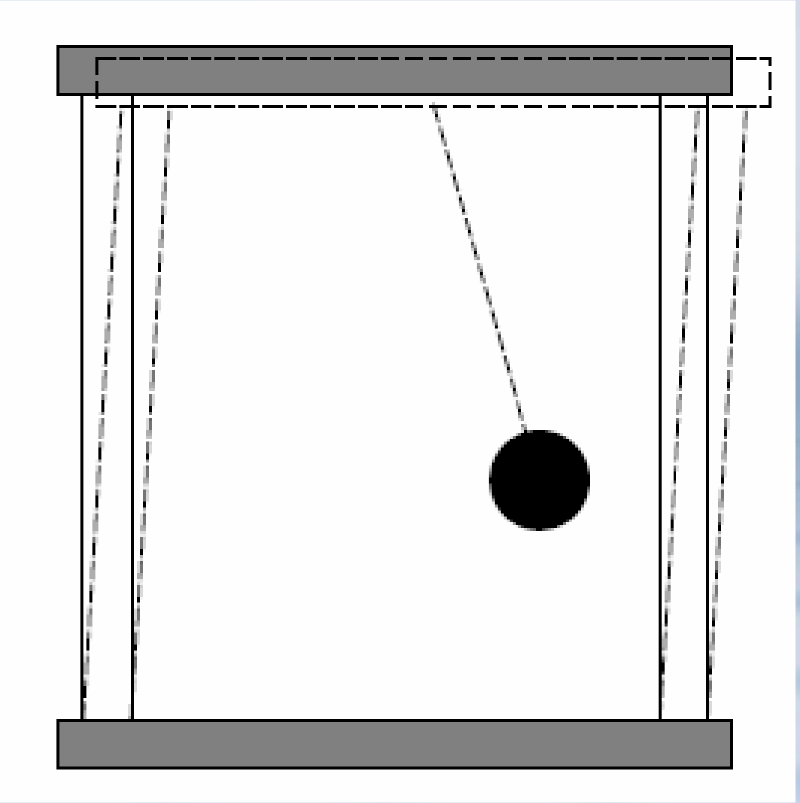
Measurement of Q-value (Decay time of resonance motion)



In (our) usual cases, we adopt this method.

3. Interferometer before revolution

Recoil loss (problem in measurement of decay time)

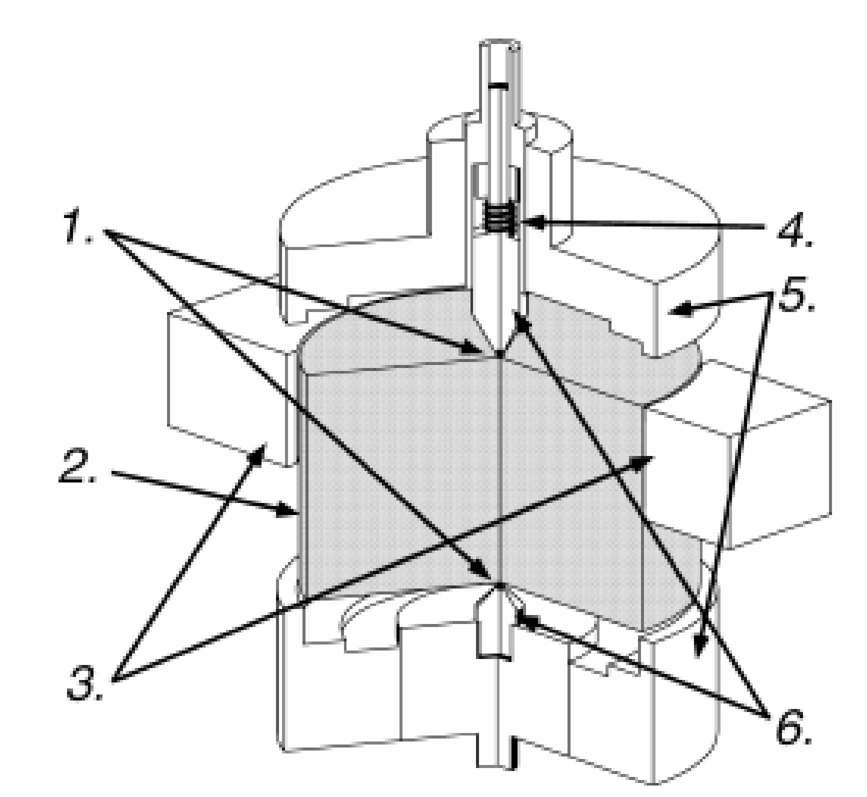


Contamination of loss in support system

Suspension : Rigid and heavy support system

3. Interferometer before revolution

Recoil loss (problem in measurement of decay time)



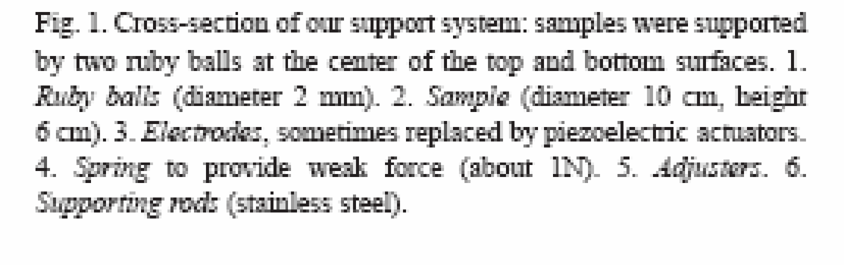


Fig. 1. Cross-section of our support system: samples were supported by two ruby balls at the center of the top and bottom surfaces. 1. *Ruby balls* (diameter 2 mm). 2. *Sample* (diameter 10 cm, height 6 cm). 3. *Electrodes*, sometimes replaced by piezoelectric actuators. 4. *Spring* to provide weak force (about 1N). 5. *Adjusters*. 6. *Supporting rods* (stainless steel).

Mirror : Nodal support system
(Center of flat surface is node for many modes)
K. Numata *et al*., Physics Letters A 276 (2000) 37.

3. Interferometer before revolution

Measurement of Q-value of pendulum

(monolithic fused silica)

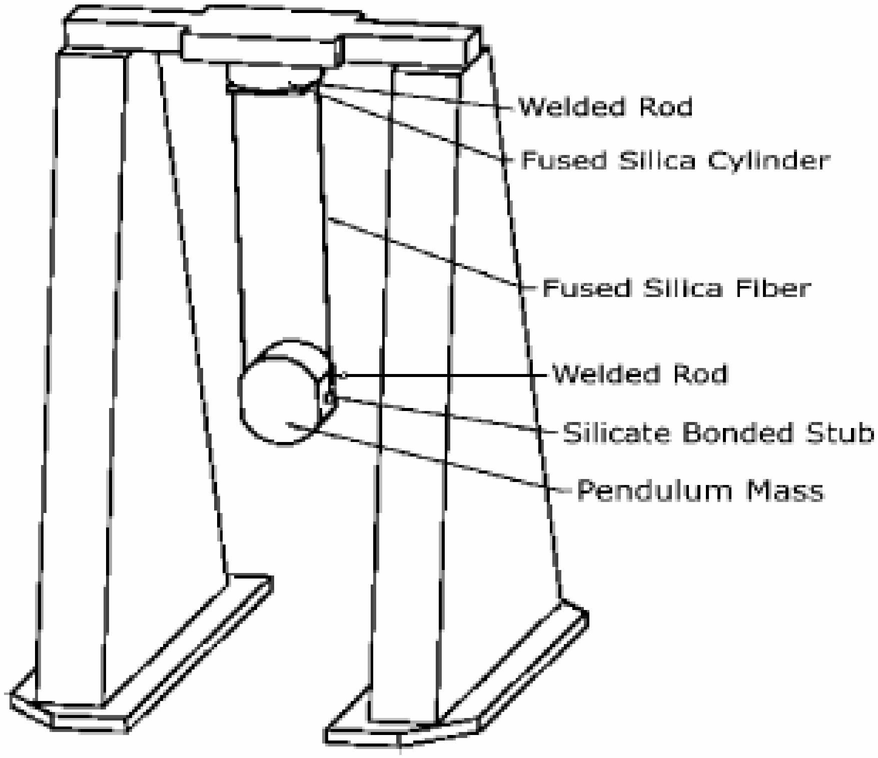


FIG. 1. Sketch of experimental setup.

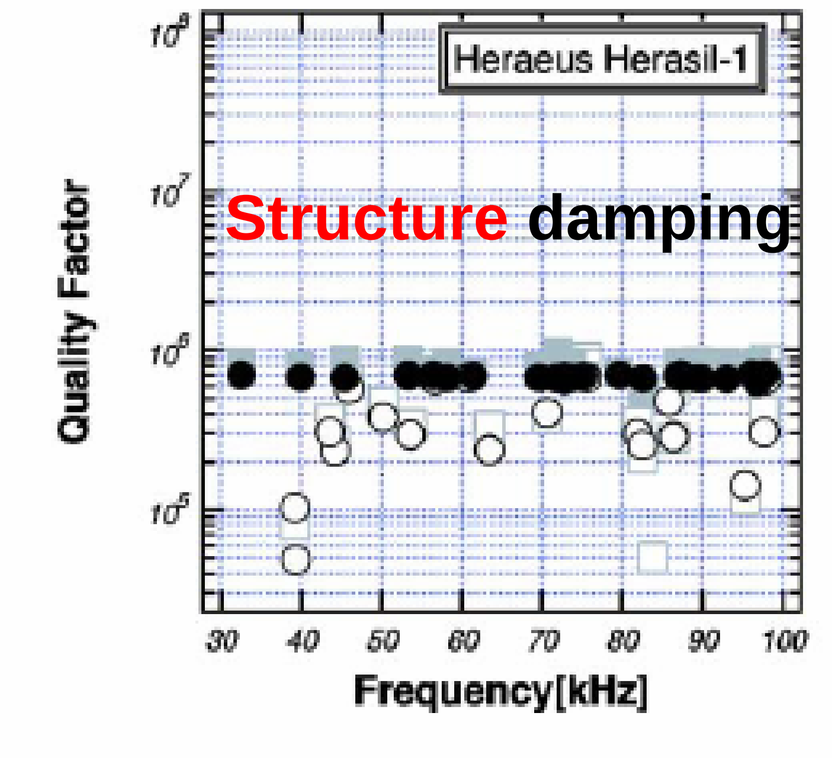
$Q = 2.3 * 10^7$

G. Cagnoli *et al*., Physical Review Letters 85 (2000) 2442.

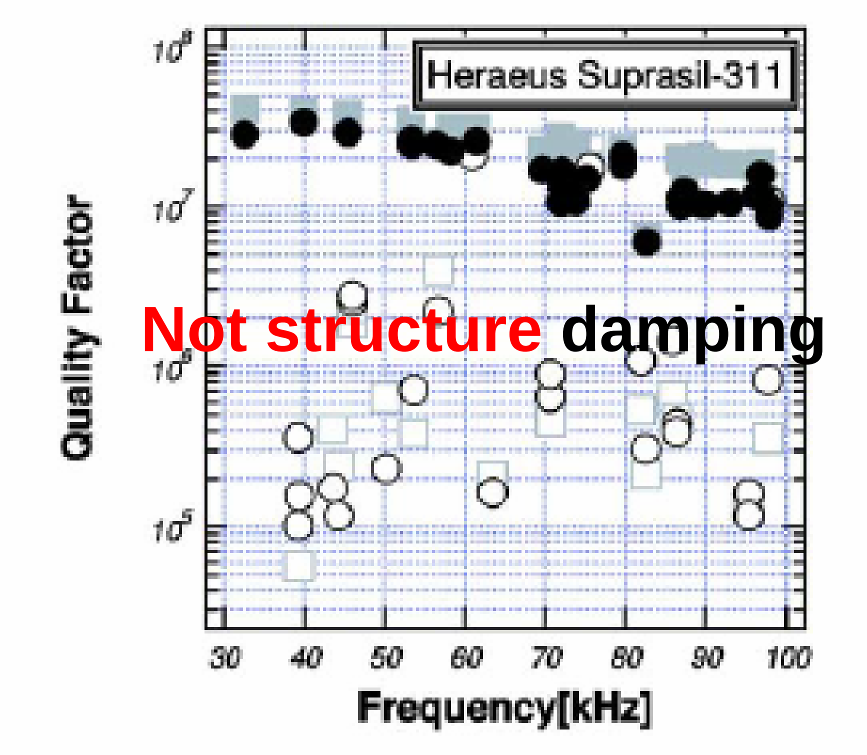
3. Interferometer before revolution

Measurement of Q-value of fused silica mirror

$Q = 4 * 10^7$



Structure damping

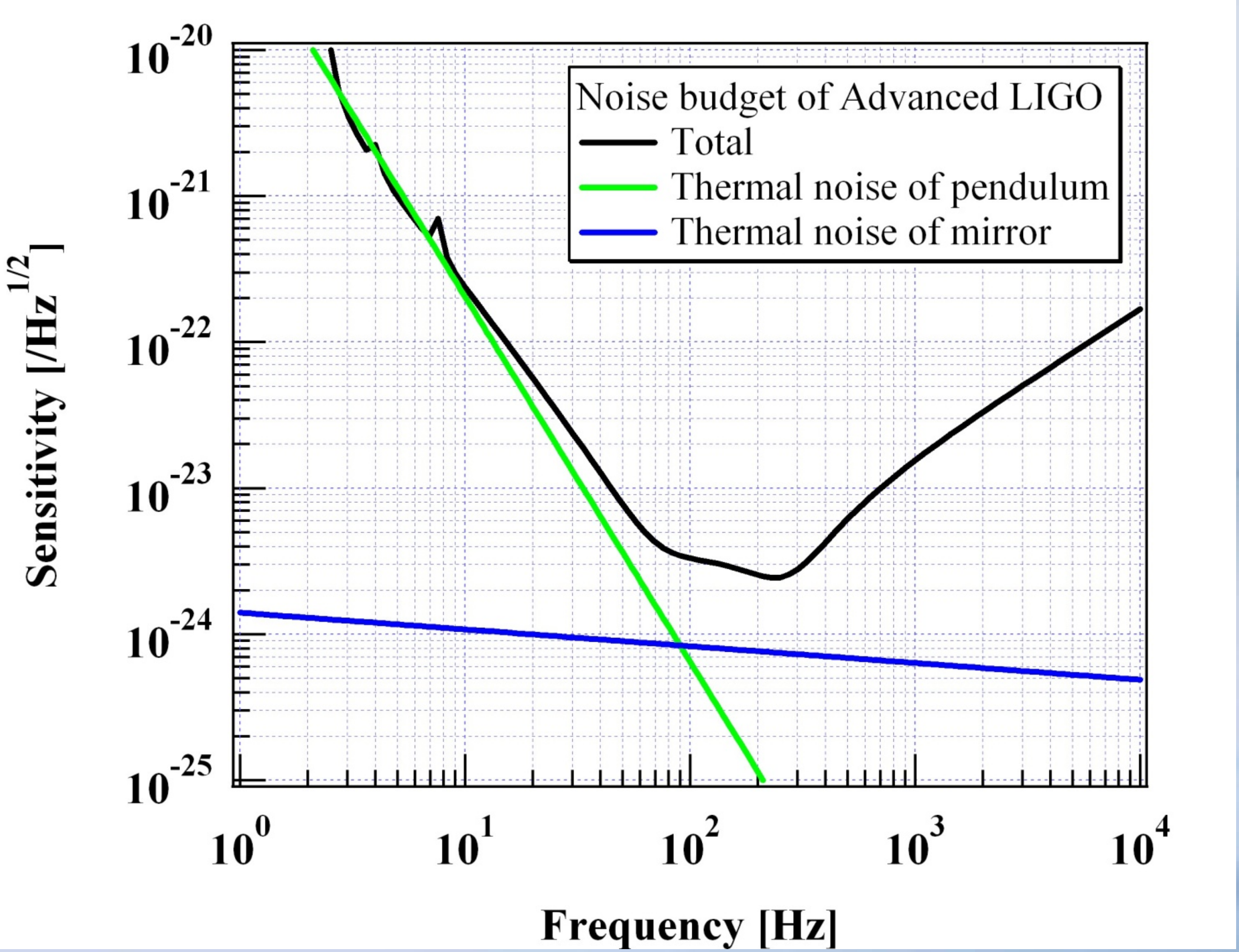


Not structure damping

K. Numata *et al*., Physics Letters A 327 (2004) 263.

3. Interferometer before revolution

Evaluated thermal noise based on Q-value measurement



This sensitivity is not latest one.

4. Interferometer after revolution

(1) Thermoelastic noise

(2) Thermal noise caused by inhomogeneous loss

(3) Direct measurement of thermal noise

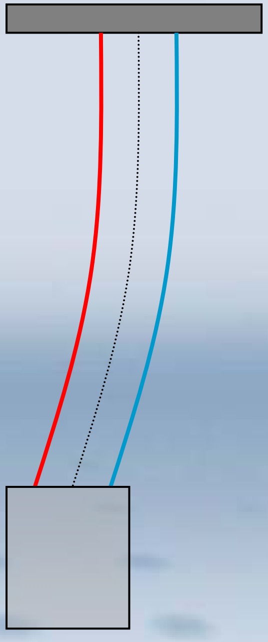
(4) Direct measurement of off resonance dissipation

(5) Reduction of thermal noise

(6) Impact on other fields

4. Interferometer after revolution

(1) Thermoelastic noise



Drawing by
Tobias Westphal

Thermoelastic damping : a kind of loss

Inhomogeneous strain

Thermal expansion coefficient

Temperature gradient

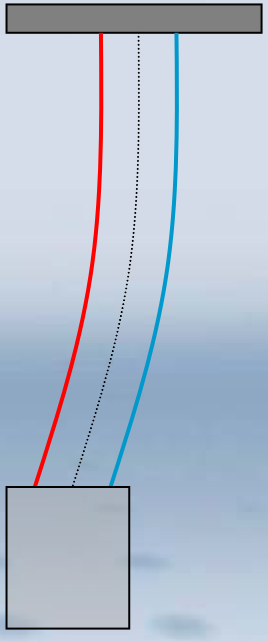
Heat flow

We can calculate thermoelastic noise

using only material properties.

4. Interferometer after revolution

(1) Thermoelastic noise



Thermoelastic noise :

thermal noise by thermoelastic damping

Other interpretation

Temperature fluctuation

Thermal expansion coefficient

Deformation of elastic body

4. Interferometer after revolution

(1) Thermoelastic noise

Long, long history of research of thermoelastic damping

1 dimension (bar, ribbon)

C. Zener, Physical Review 52 (1937) 230; 53 (1938) 90.

2 dimension (disk)

L.D. Landau and E.M. Lifshitz

"*The Theory of Elasticity*" , 1953.

A.S. Nowick and B. Berry

"*Anelastic Relaxation in Crystalline Solids*" , 1972.

3 dimension (mirror) 1999

4. Interferometer after revolution

(1) Thermoelastic noise

Thermoelastic noise of 3 dimension mirror without coating

V. B. Braginsky *et al.*, Physics Letters A 264 (1999) 1.

We can calculate thermoelastic noise using only substrate material properties.

This noise is larger than expectation !

4. Interferometer after revolution

(1) Thermoelastic noise

Fused silica vs. Sapphire

Current interferometric gravitational wave detector

Fused silica mirror

Future interferometer

Sapphire was a candidate.

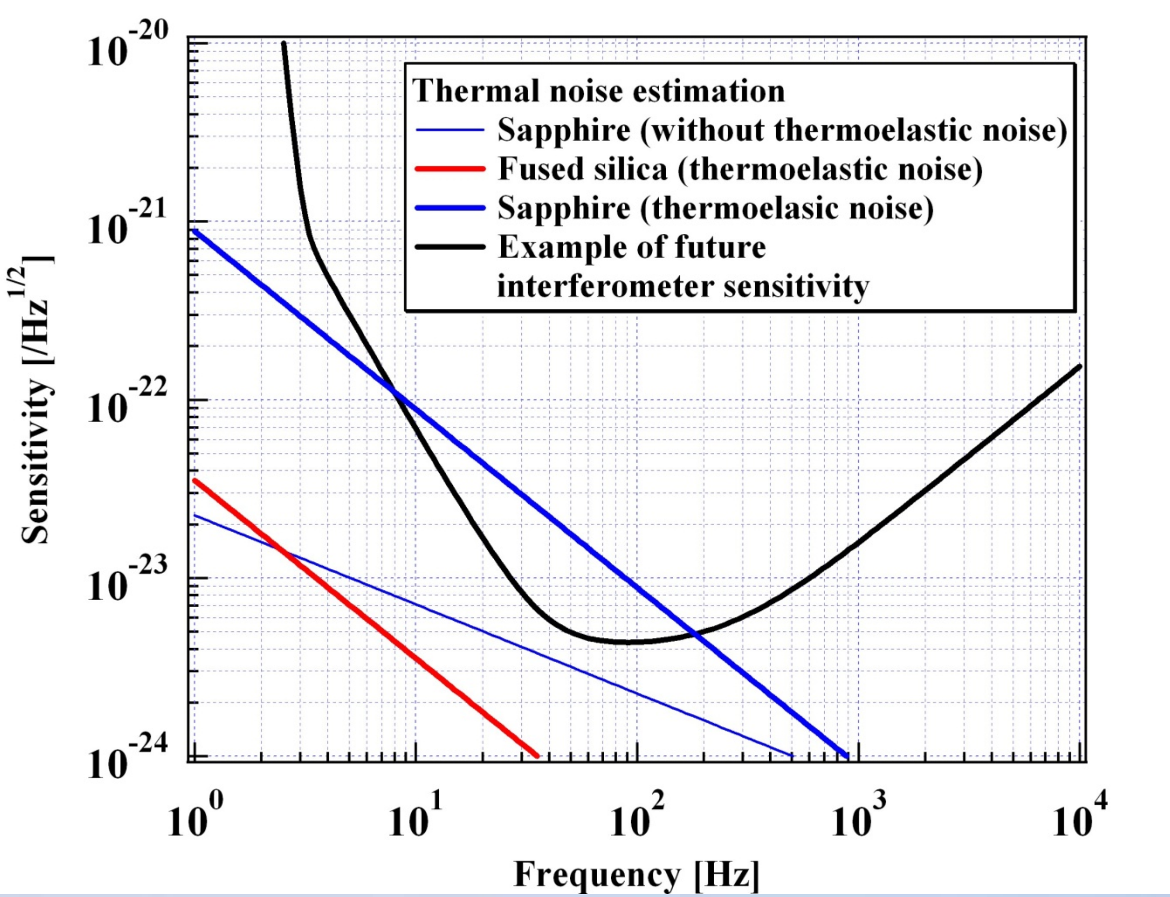
Optical properties of fused silica is better.

It was expected that thermal noise of sapphire is smaller.

However …

4. Interferometer after revolution

(1) Thermoelastic noise



One of advantages of sapphire was lost.
Future (room temperature) interferometer will use fused silica mirror.

4. Interferometer after revolution

(1) Thermoelastic noise

Thermo-optic noise

Temperature fluctuation in reflective coating

↓

Thermal expansion (α)
Temperature coefficient of refractive index (β)

↓

Fluctuation of phase of reflected light

Material properties of reflective coating are important issues.

V. B. Braginsky *et al.*, Physics Letters A 312 (2003) 244.
V. B. Braginsky *et al.*, Physics Letters A 271 (2000) 303.
M. Evans *et al*., Physical Review D 78 (2008) 102003.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Mirror is not a harmonic oscillator.
It has a lot of resonant modes.

Modal expansion
Thermal noise of mirror is the summation of those of resonant modes.
Thermal noise of resonant mode is the same as that of a harmonic oscillator .

Peter R. Saulson, Physical Review D 42 (1990) 2437.

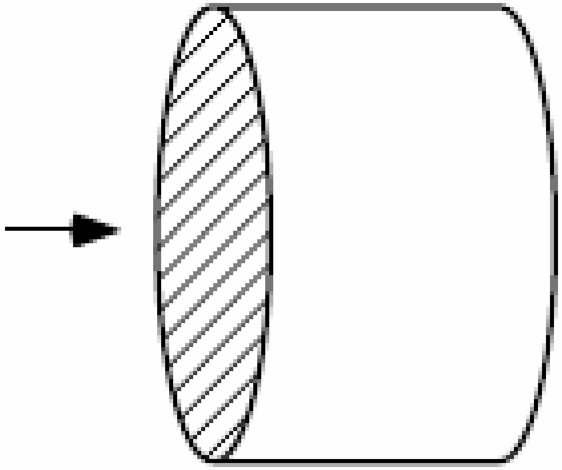
Same Q-values implies same thermal noise.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

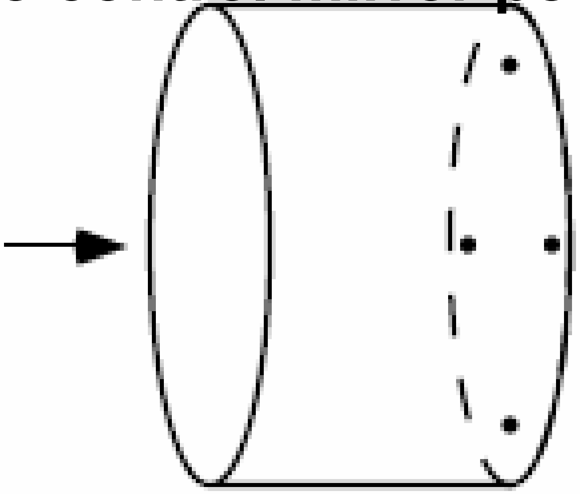
Mirror consists of not only bulk !

Reflective coating



Thickness of coating : ~ 5 μm
Thickness of mirror : ~ 10 cm
Coating do not decrease Q-values so much.
Nobody cared.

Magnet
(coil-magnet actuator to control mirror position)



Magnets decrease Q-values.
Serious problem

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Y. Levin, Physical Review D 57 (1998) 659.

Q-value of loss at A is the same as that at B.

Loss at A can shake illuminated surface more largely owing to conservation of momentum.

Thermal noise depends on not only Q-values but also spatial distribution of loss.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Modal expansion predicts that thermal noise is the same if Q-values are same. But Levin's discussion proved that it is invalid.

What is wrong ?

Although some people investigated, K. Yamamoto's four papers provide clear solution for this problem.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

What is wrong in modal expansion ?

Inhomogeneous loss causes couplings between modes.
Inhomogeneous loss destroys the shape of a mode.
Other modes appear.

These couplings generate correlations between thermal fluctuations of resonant modes. This correlation is not taken into account in modal expansion.

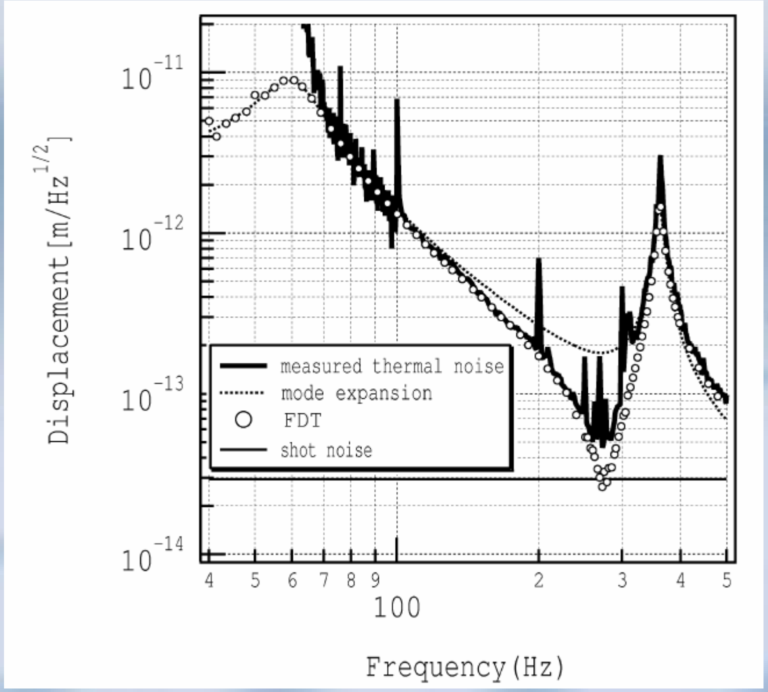
If we consider these correlations, modal expansion can provide correct evaluation (advanced modal expansion).

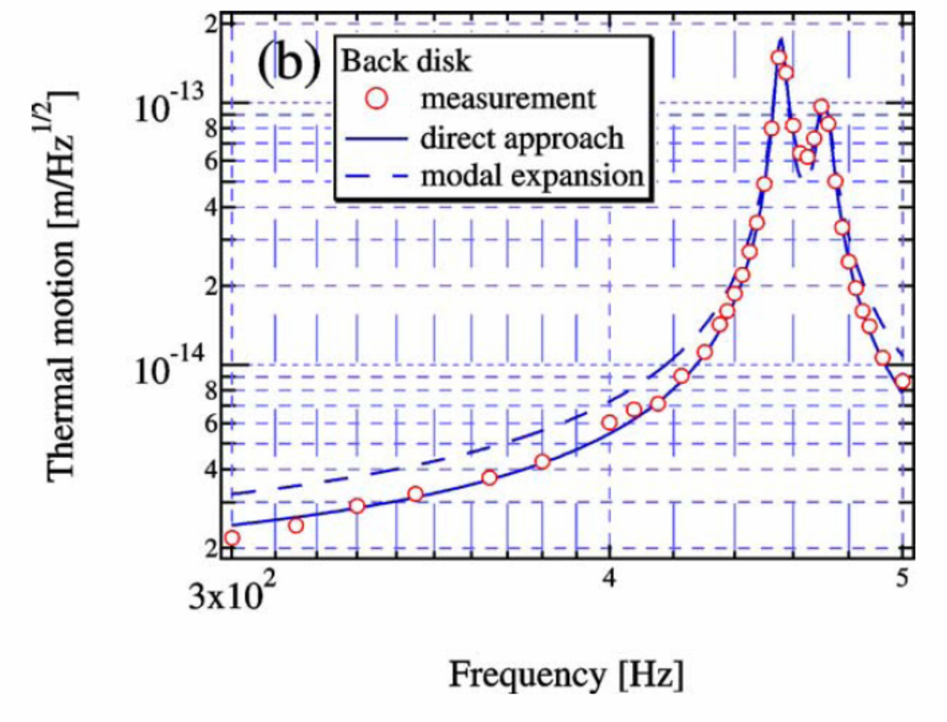
K. Yamamoto *et al*., Physical Review D 75 (2007) 082002.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Experimental checks : (Traditional) modal expansion breaks down and advanced modal expansion and Levin’s method are correct if loss is inhomogeneous.





K. Yamamoto *et al*., Physics Letters A 280 (2001) 289.
K. Yamamoto *et al*., Physics Letters A 321 (2004) 79.

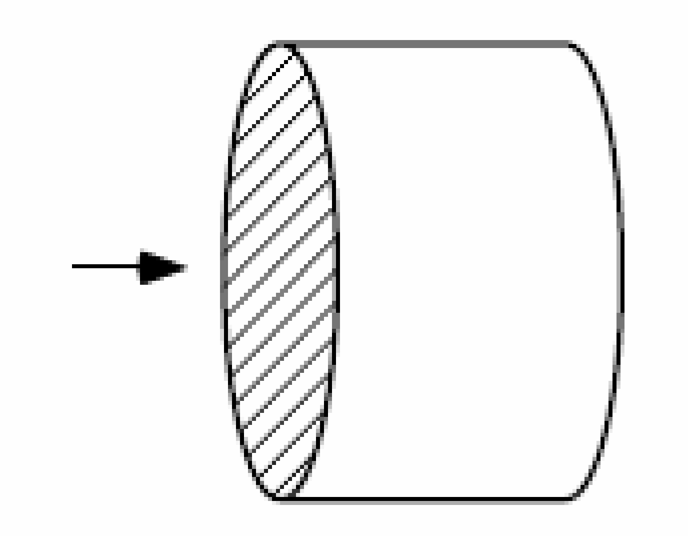
4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Quantitative discussion

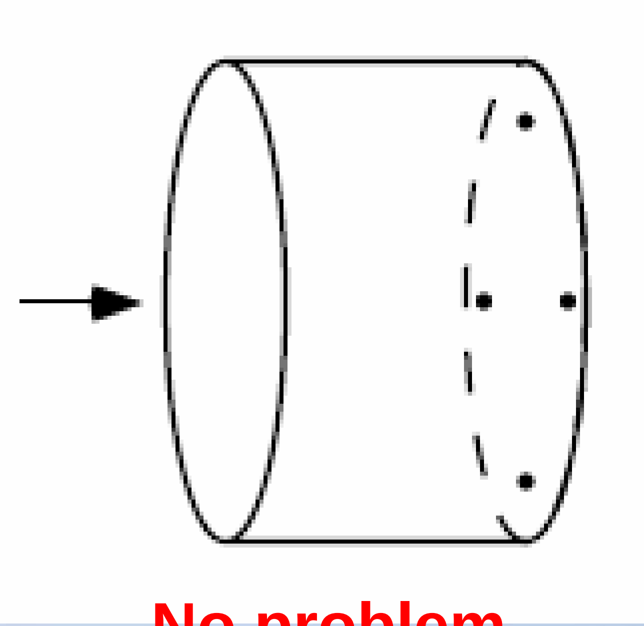
K. Yamamoto *et al*., Physics Letters A 305 (2002) 18.

Reflective coating



It can be a problem.

Magnet



No problem

The previous expectation is perfectly wrong.

The strategy of research of thermal noise must be changed.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Formula about coating thermal noise

G. Harry *et al*., Classical and Quantum Gravity 19 (2002) 897.
N. Nakagawa *et al*., Physical Review D 65 (2002) 102001.

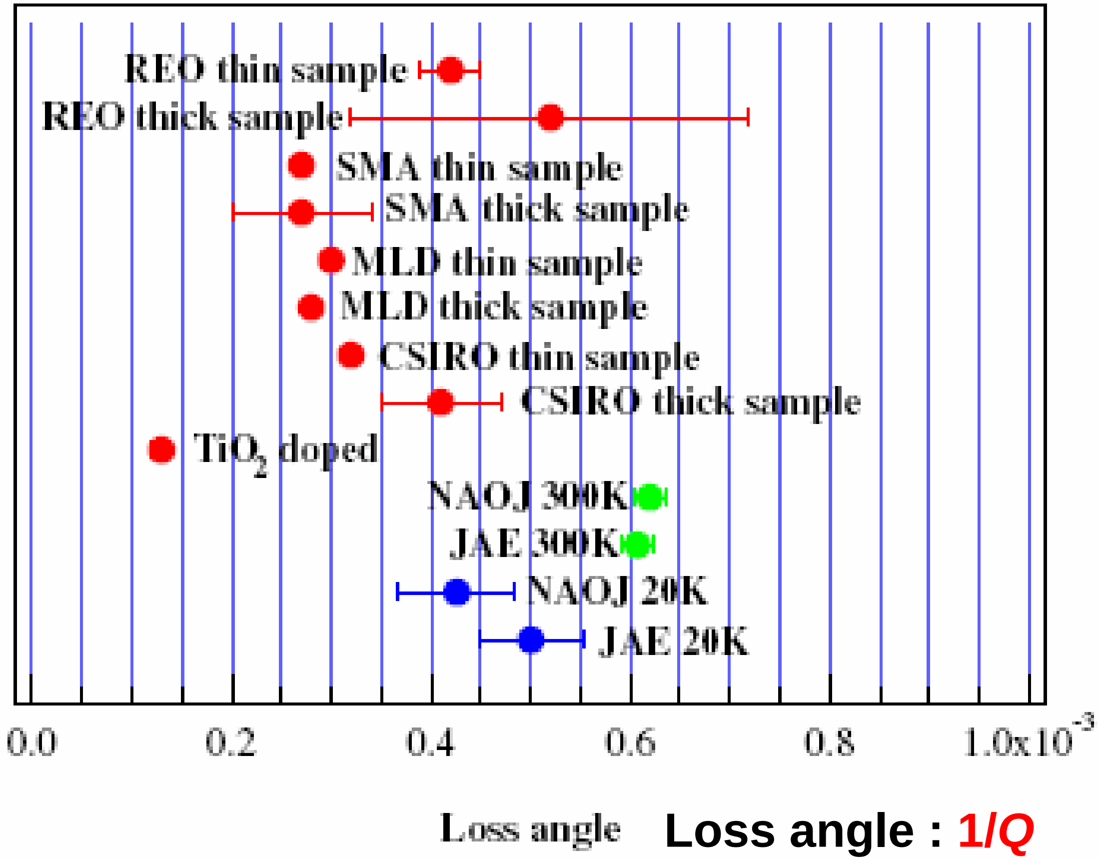
The details are in

G.M. Harry, T. Bodiya, and R. DeSalvo (Editors)
Optical Coatings and Thermal Noise in Precision Measurements
Cambridge University Press, Cambridge (in press)

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Old summary of coating mechanical loss



Loss angle : $1/Q$

Similar results (same order of magnitude)

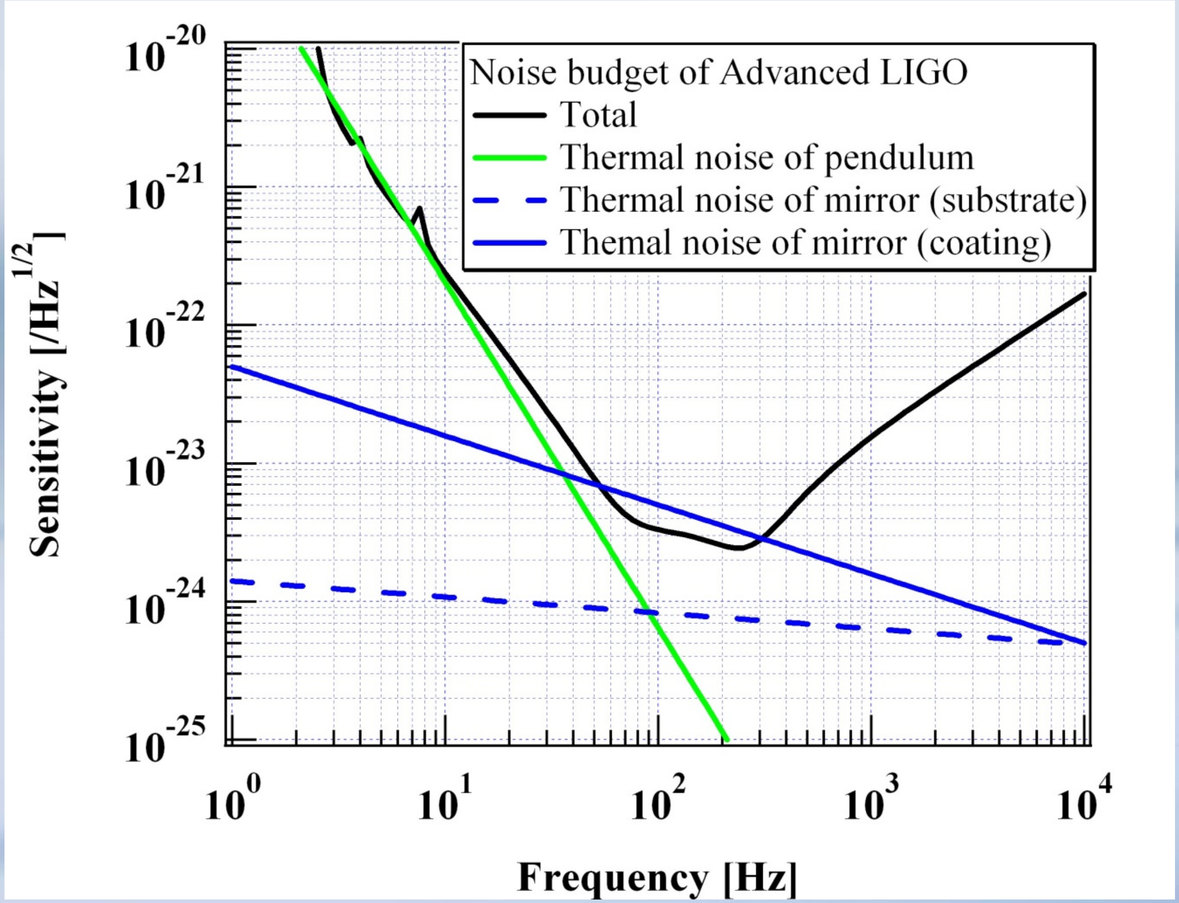
Structure damping

K. Yamamoto *et al*., Physical Review D 74 (2006) 022002.

4. Interferometer after revolution

(2) Thermal noise caused by inhomogeneous loss

Evaluated thermal noise based on measurement



Coating thermal noise is the most serious problem in future!

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Are formulae of thermal noise correct ?

Direct measurement of thermal noise of mirror

University of Tokyo, California Institute of Technology

Small beam radius (about 0.1 mm)

to enhance thermal noise

Fabry Perot cavity length ~ 1 cm

Direct measurement of thermal noise of suspension

University of Tokyo

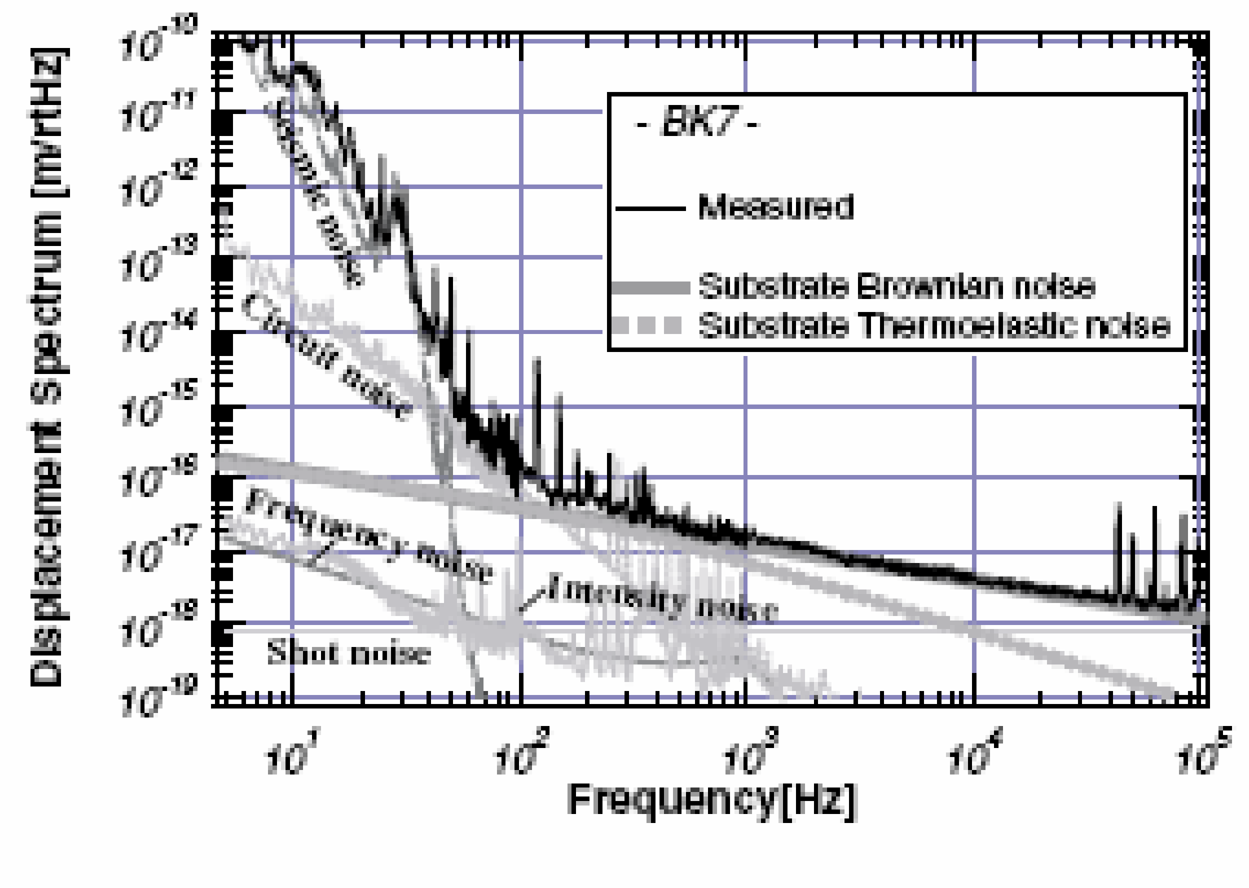
Underground site with small seismic motion

Cavity length ~ 100 m

4. Interferometer after revolution

(3) Direct measurement of thermal noise

BK7 : Structure damping in substrate

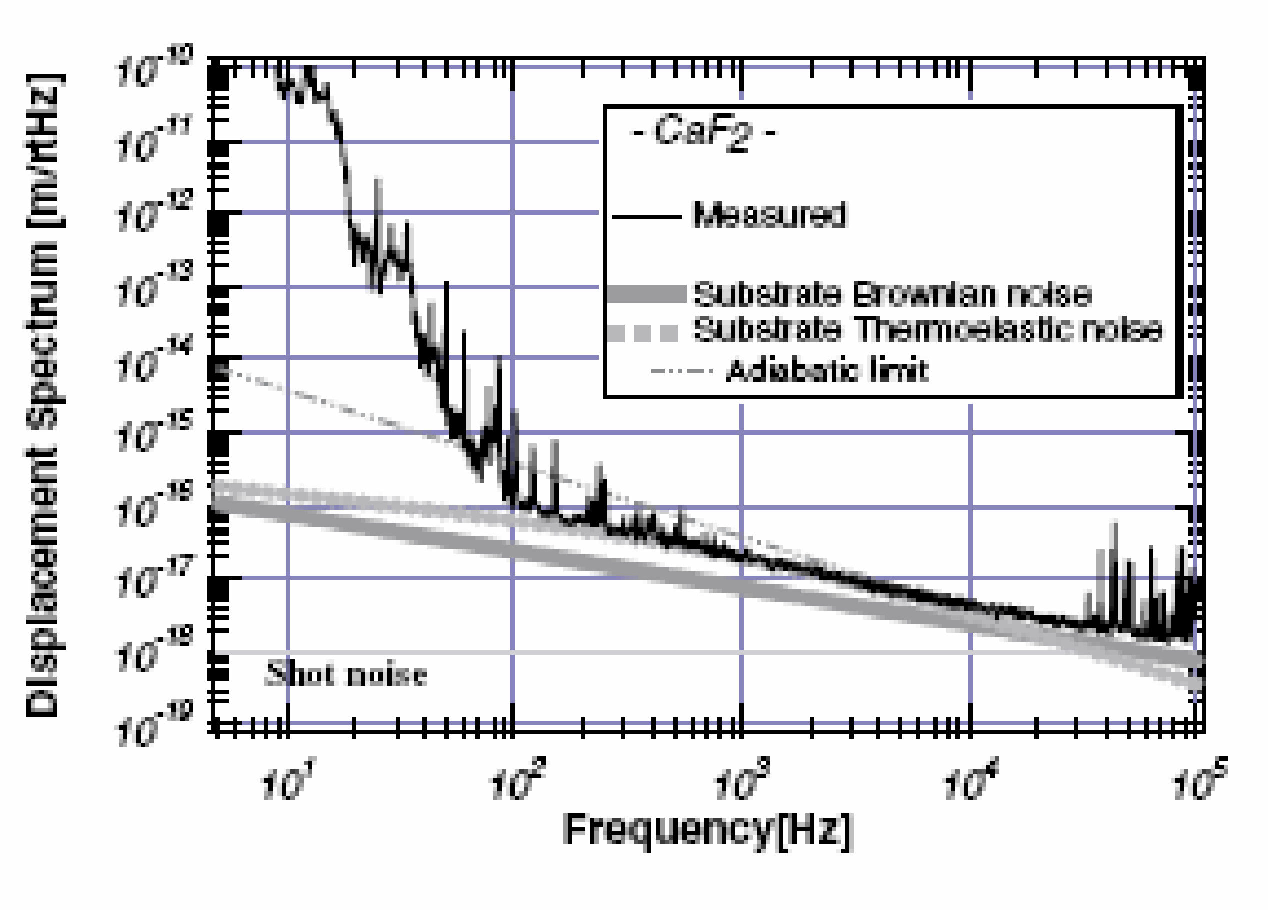


K. Numata *et al*., Physical Review Letters 91 (2003) 260602.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Calcium fluoride : Thermoelastic damping in substrate

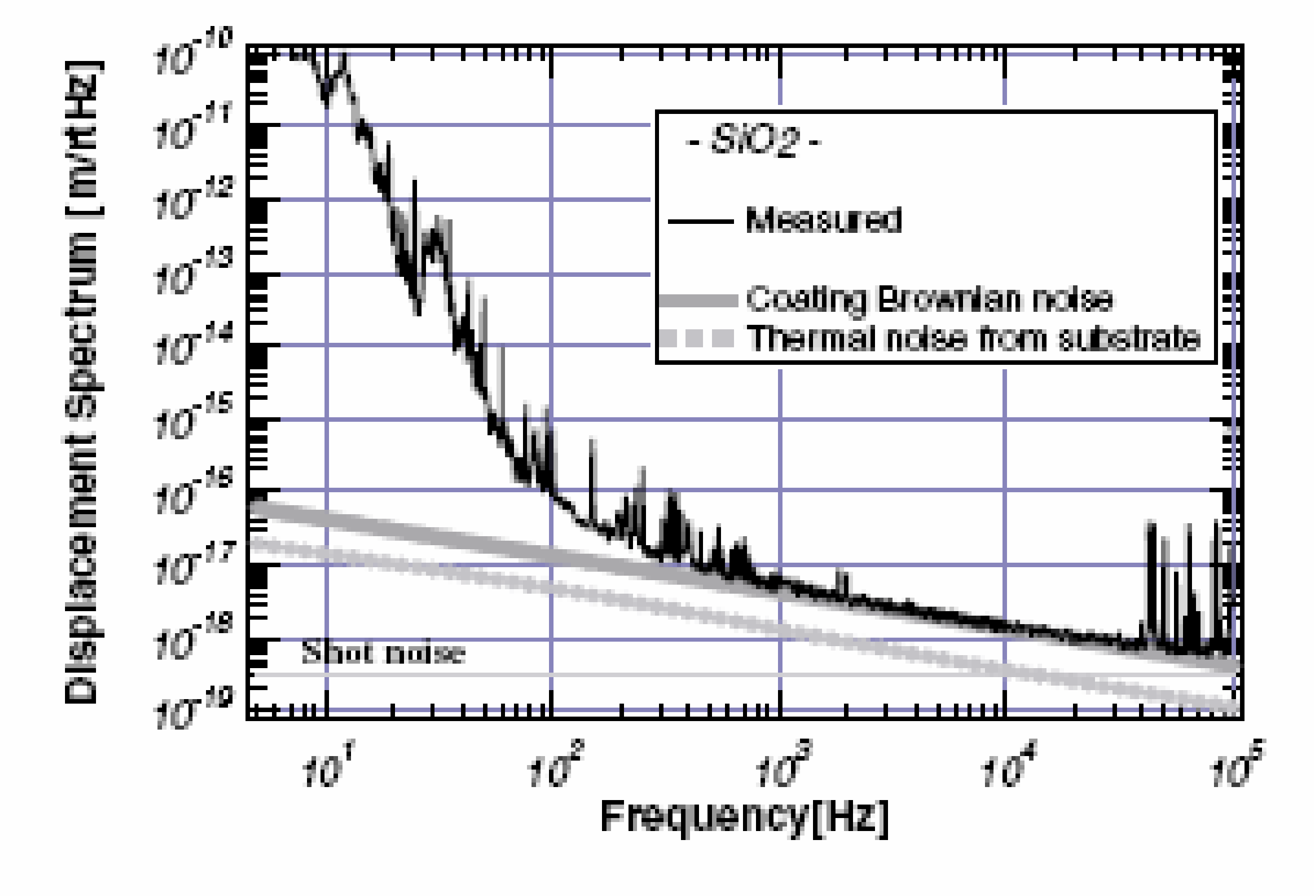


K. Numata *et al*., Physical Review Letters 91 (2003) 260602.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Fused silica : Structure damping in coating

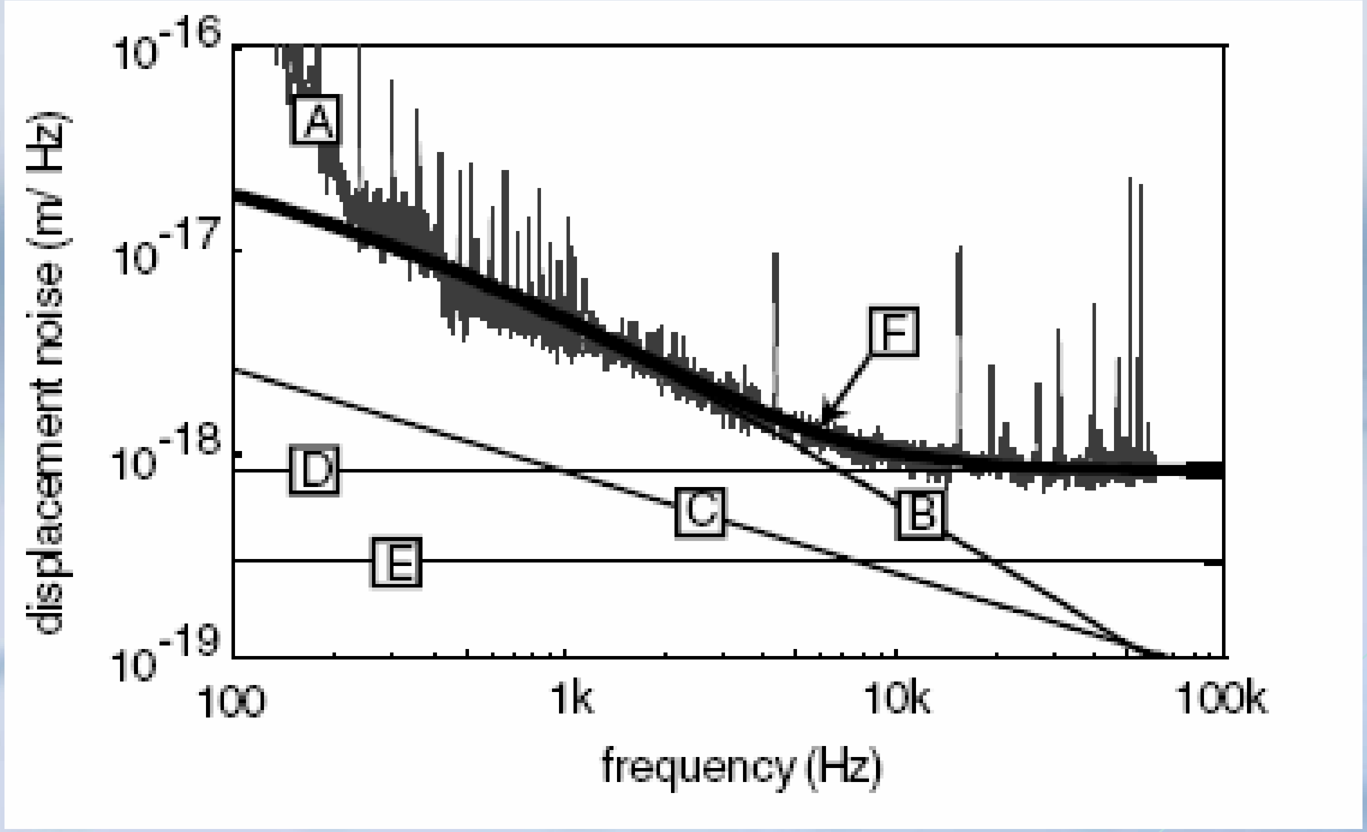


K. Numata *et al*., Physical Review Letters 91 (2003) 260602.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Sapphire : Thermoelastic damping in substrate

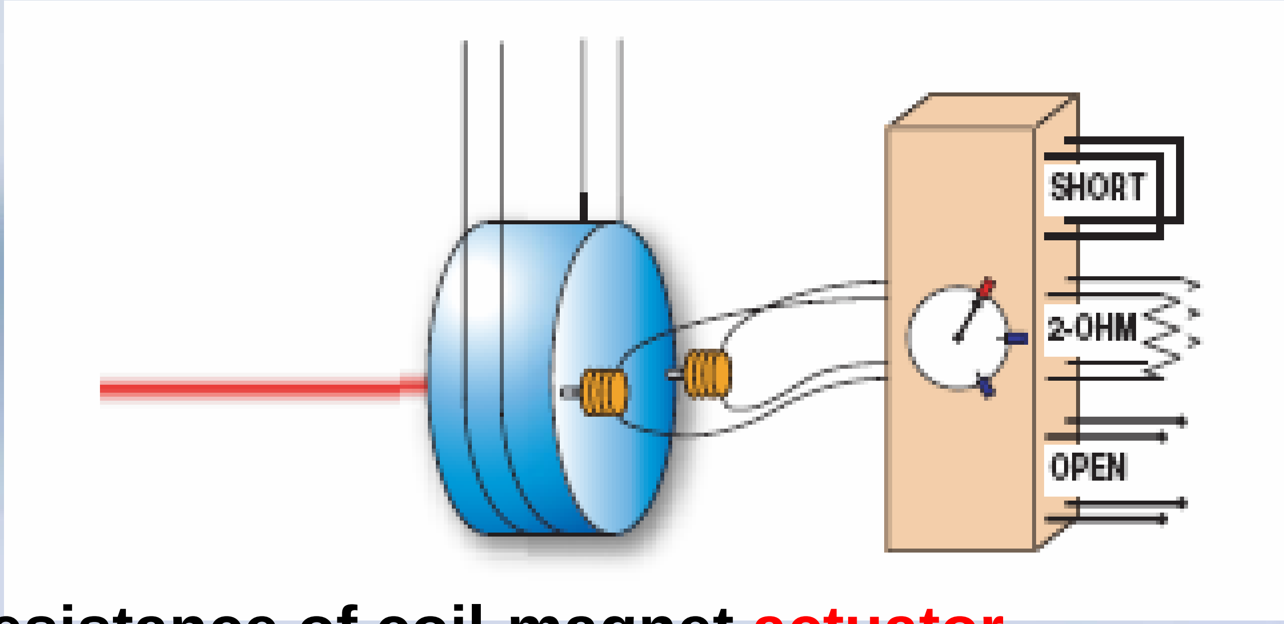


E.D. Black *et al*., Physical Review Letters 93 (2004) 241104.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Direct measurement of thermal noise of suspension



Loss : Resistance of coil-magnet actuator

(not material or clamp loss)

We can change this resistance.

Q-value is still enough high (~10^5).

K. Agatsuma *et al*., Physical Review Letters 104 (2010) 040602.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

Cryogenic Laser Interferometer Observatory (CLIO,Japan)

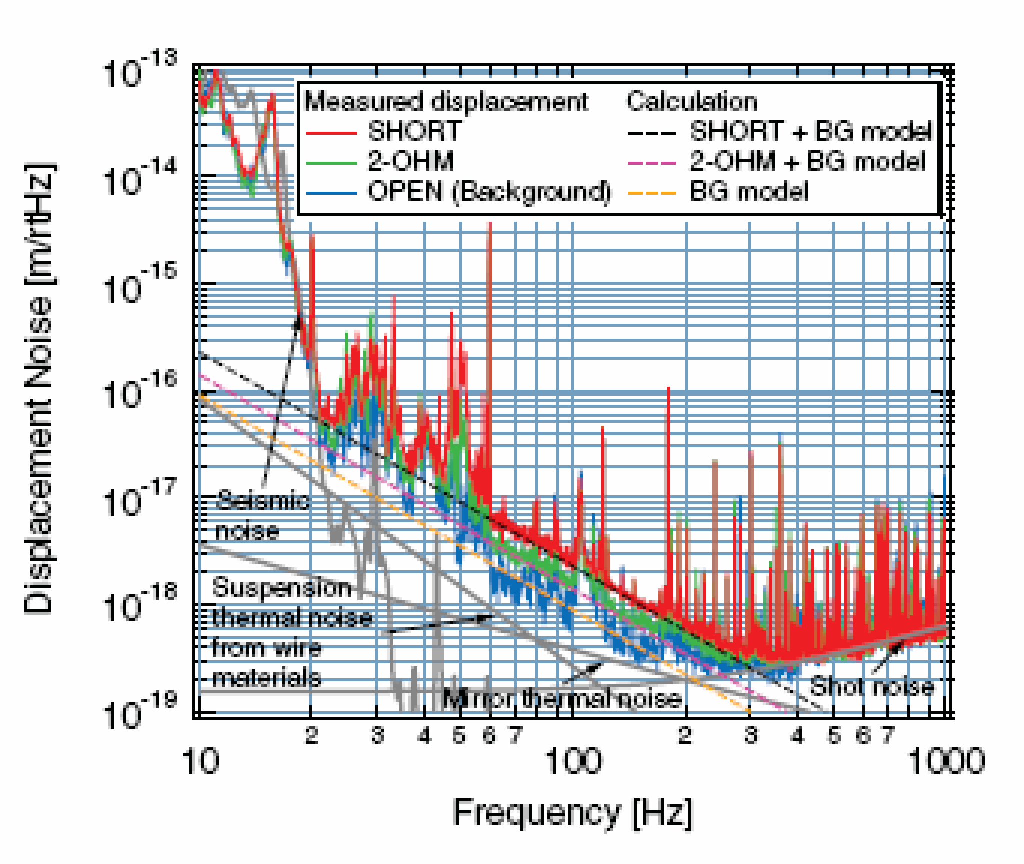
100 m, Kamioka (Japan)

Prototype for LCGT ; cryogenic interferometer
in underground site with small seismic motion

S. Kawamura, Classical and Quantum Gravity 27 (2010) 084001.

4. Interferometer after revolution

(3) Direct measurement of thermal noise



Change of resistance

↓

Change of observed sensitivity as theoretical prediction

Next step : Thermal noise by loss in suspension itself

K. Agatsuma *et al*., Physical Review Letters 104 (2010) 040602.

4. Interferometer after revolution

(3) Direct measurement of thermal noise

I expect that the sensitivities of LIGO(U.S.A.), Virgo (Italy and France) and GEO (Germany and U.K.) are comparable with suspension thermal noise.

However, I can not find refereed papers which claim observation of suspension thermal noise.

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

Now we observe thermal fluctuation directly.

Fluctuation dissipation theorem

$$G(f) = -\frac{4k_{\mathrm{B}}T}{\omega}\,\mathrm{Im}[H(\omega)]$$

Power spectrum of thermal fluctuation

Imaginary part of susceptibility (dissipation)

How about direct measurement of dissipation (imaginary part of susceptibility) ?

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

How about dissipation (imaginary part of susceptibility) ?
It is not so easy.
Imaginary part is much smaller than real part.

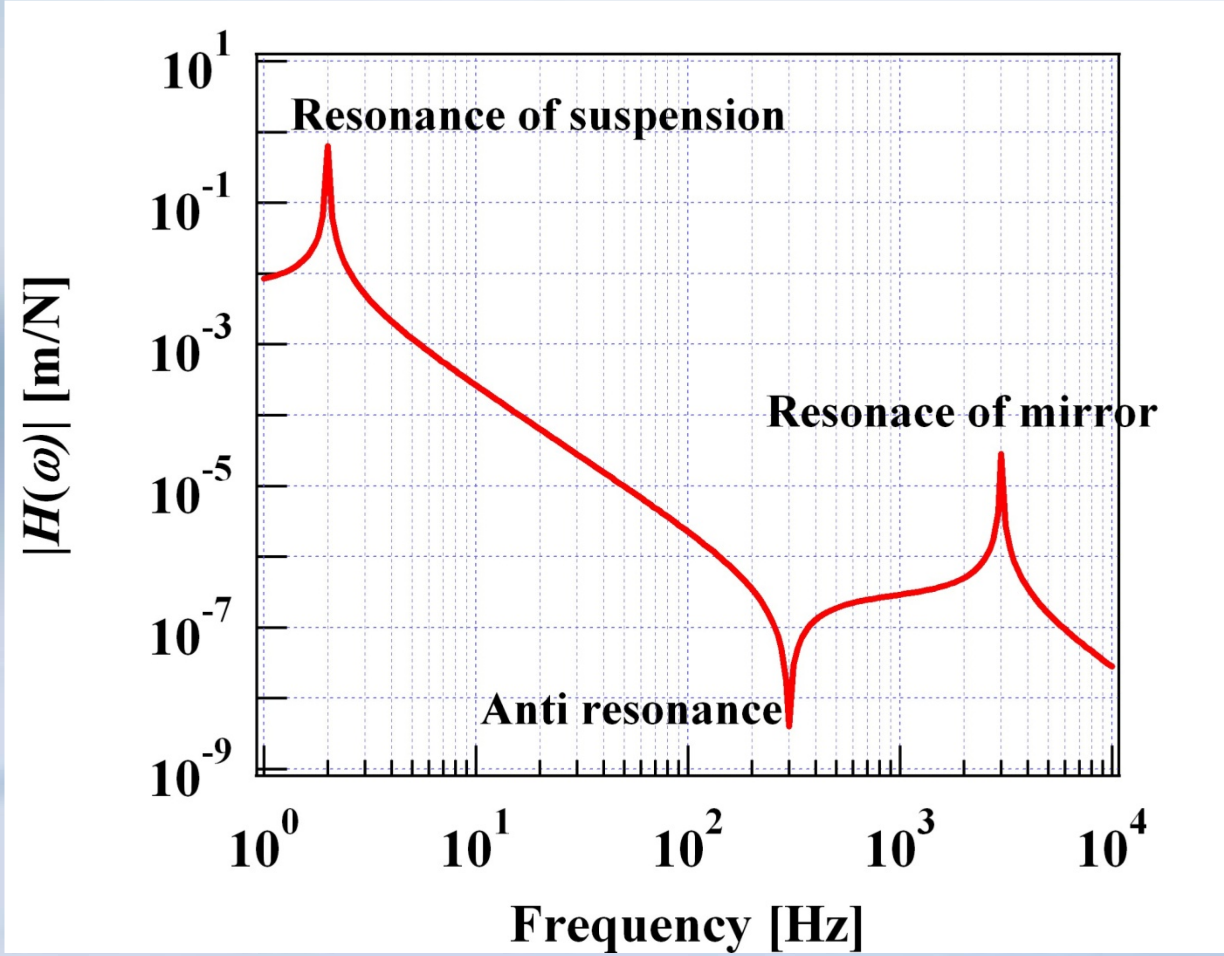
$$\frac{\mathrm{Im}[H(\omega)]}{\mathrm{Re}[H(\omega)]} \sim \frac{1}{Q} < 10^{-6}$$

Off resonance region

Extreme precise measurement is necessary.
Relative error should be smaller than 10^{-6} !

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation



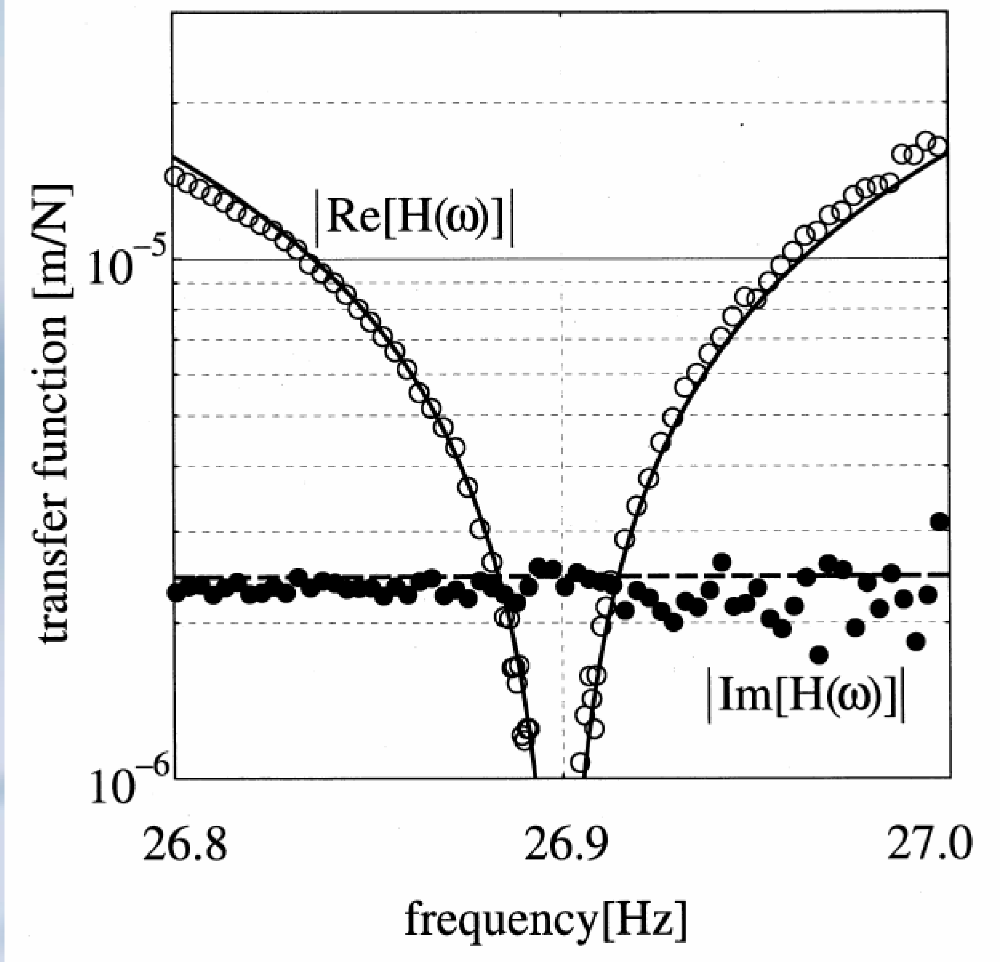
On anti resonance Re[*H*] vanishes.
|*H*| is |Im[*H*]|
(dissipation).

Requirement of relative error is not so serious.

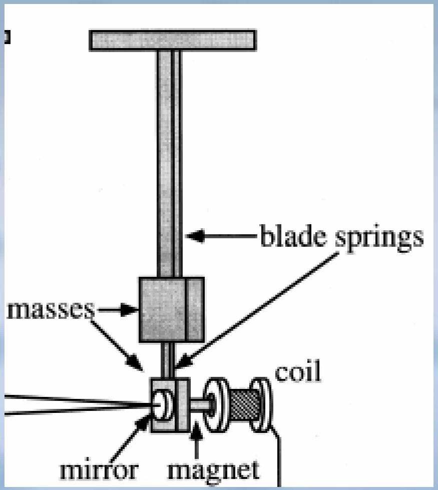
N. Ohishi *et al*., Physics Letters A 266 (2000) 228.

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation



Ohishi confirmed that this method works well using small 2 modes oscillator.

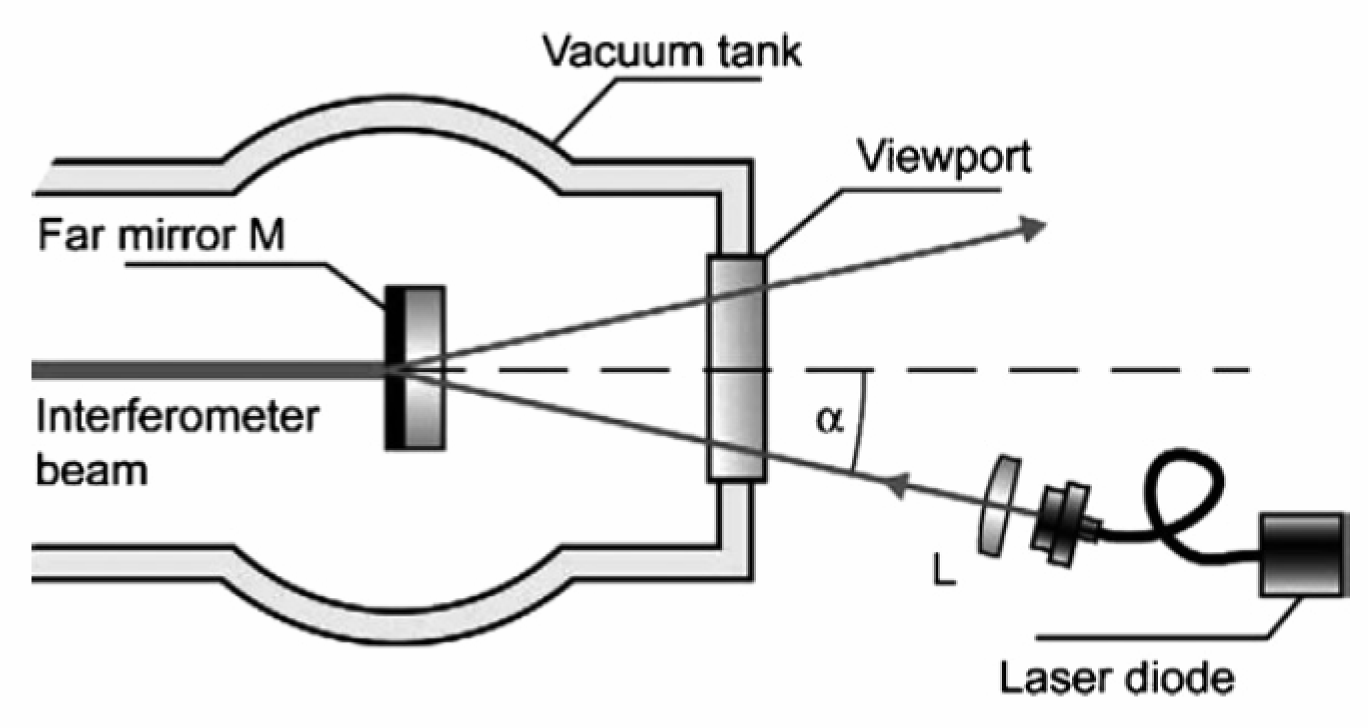


N. Ohishi *et al*., Physics Letters A 266 (2000) 228.

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

In gravitational wave detector, actuator is a problem.
Probably, photon pressure actuator is the best one.

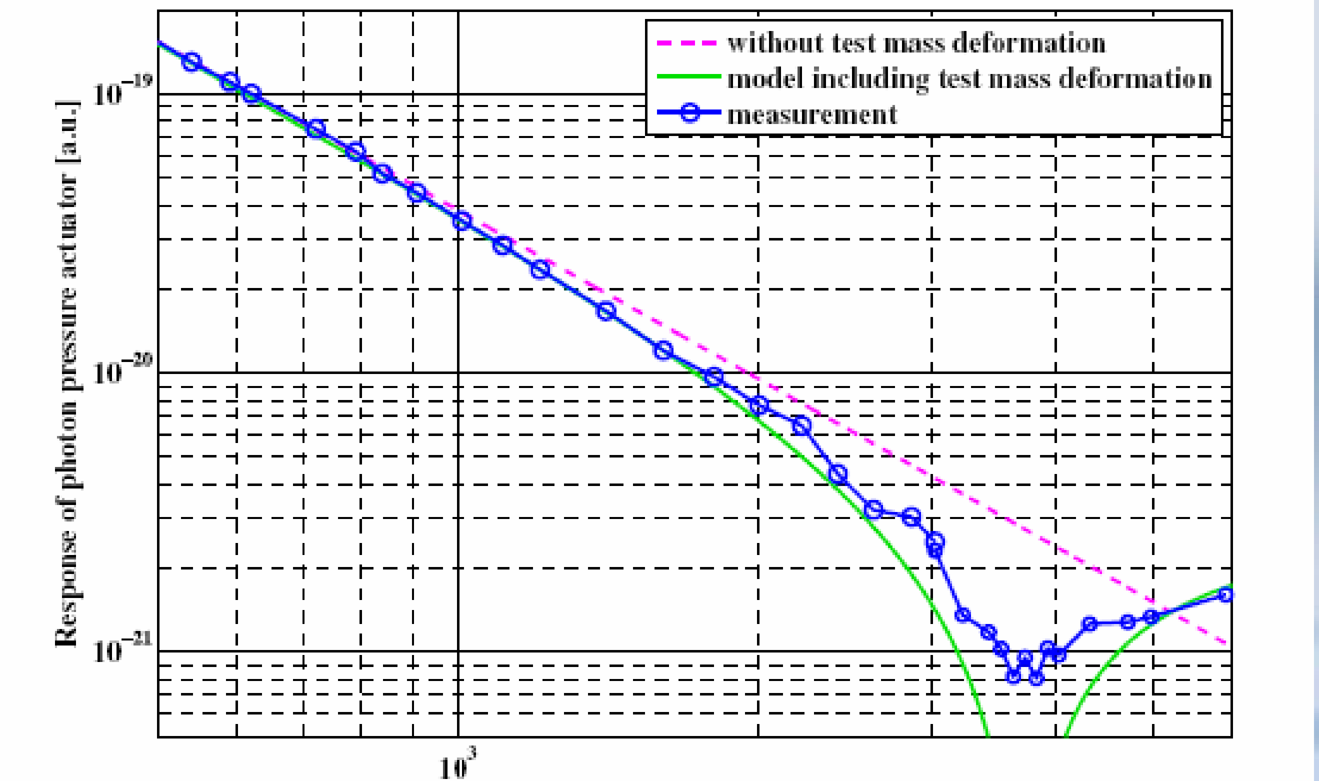


S. Hild *et al*., Classical and Quantum Gravity 24 (2007) 5681.

4. Interferometer after revolution

(4) Direct measurement of off resonance dissipation

Measurement with photon pressure actuator in GEO600



Susceptibility could not be measured at anti resonance.
The progress is necessary.

S. Hild *et al*., Classical and Quantum Gravity 24 (2007) 5681.

4. Interferometer after revolution

(5) Reduction of thermal noise

Coating thermal noise is the most serious problem !
Coating loss reduction is not so easy …

Second generation interferometer

Advanced LIGO (U.S.A.)
and Virgo (Italy and France): Larger beam

LCGT (Japan) : Cooled sapphire mirror

Third generation interferometer (10 times better sensitivity)

Einstein Telescope (ET, Europe) :
Cooled silicon or sapphire mirror and larger beam

4. Interferometer after revolution

(5) Reduction of thermal noise

Larger beam

Mirror radius should be 3 times larger than Gaussian beam radius to avoid large clipping loss.

How about other kinds of beam shape ?

Mesa hat, higher modes…

The details are in Chapter 13 (A. Freise) of

G.M. Harry, T. Bodiya, and R. DeSalvo (Editors)

Optical Coatings and Thermal Noise in Precision Measurements

Cambridge University Press, Cambridge (in press)

4. Interferometer after revolution

(5) Reduction of thermal noise

One of the simplest solutions : Cooling mirrors

First feasibility study

T. Uchiyama *et al*., Physics Letters A 242 (1998) 211.

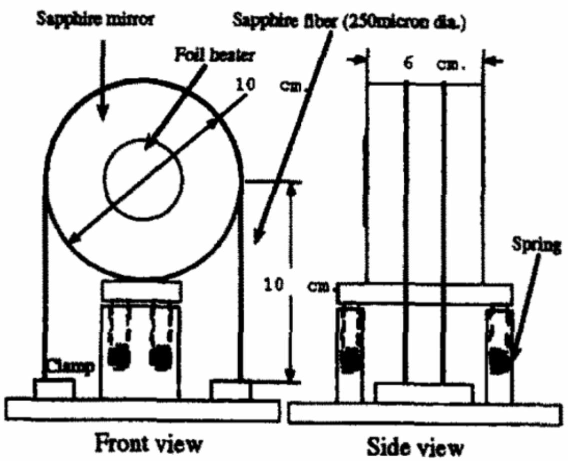


Fig. 1. The experimental setup.

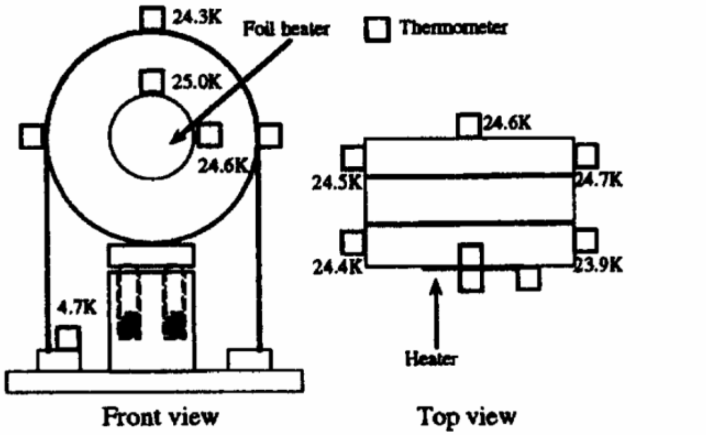


Fig. 3. Distribution of the equilibrium temperature in the case of 29 mW heat power.

4. Interferometer after revolution

(5) Reduction of thermal noise

One of the simplest solutions : Cooling mirrors

In LCGT, sapphire mirror is suspended by sapphire fibers.

Small mechanical loss (small thermal noise)

T. Uchiyama *et al*., Physics Letters A 273 (2000) 310.

Large thermal conductivity (effective cooling)

T. Tomaru *et al*., Physics Letters A 301 (2002) 215.

In ET, silicon is also candidate.

S. Hild *et al*.,

Classical and Quantum Gravity 28 (2011) 094013.

R. Nawrodt *et al*.,

General Relativity and Gravitation 43 (2011) 593.

4. Interferometer after revolution

(5) Reduction of thermal noise

Amplitude of thermal noise is proportional to

$$(T/Q)^{1/2}$$

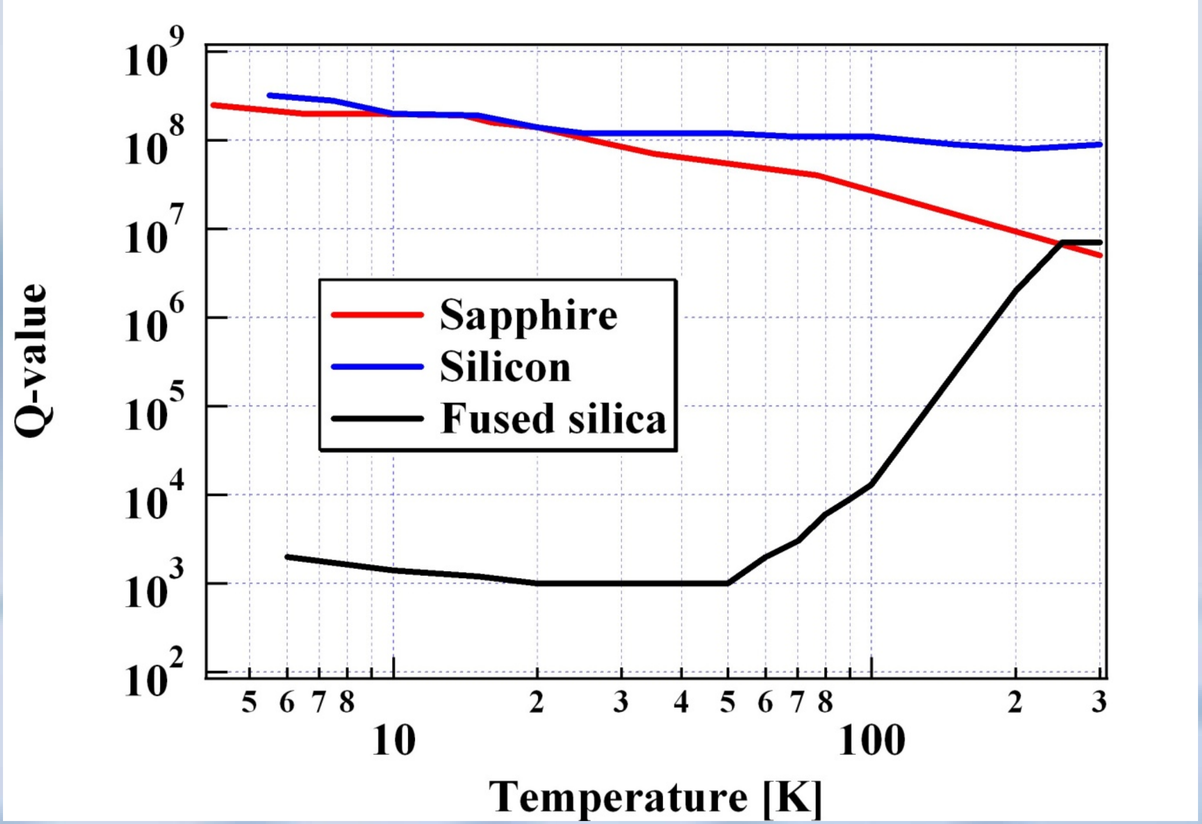
In general, Q-value depends on T (temperature).

We must investigate how dissipation depends on temperature in cryogenic region.

4. Interferometer after revolution

(5) Reduction of thermal noise

Structure damping in substrate

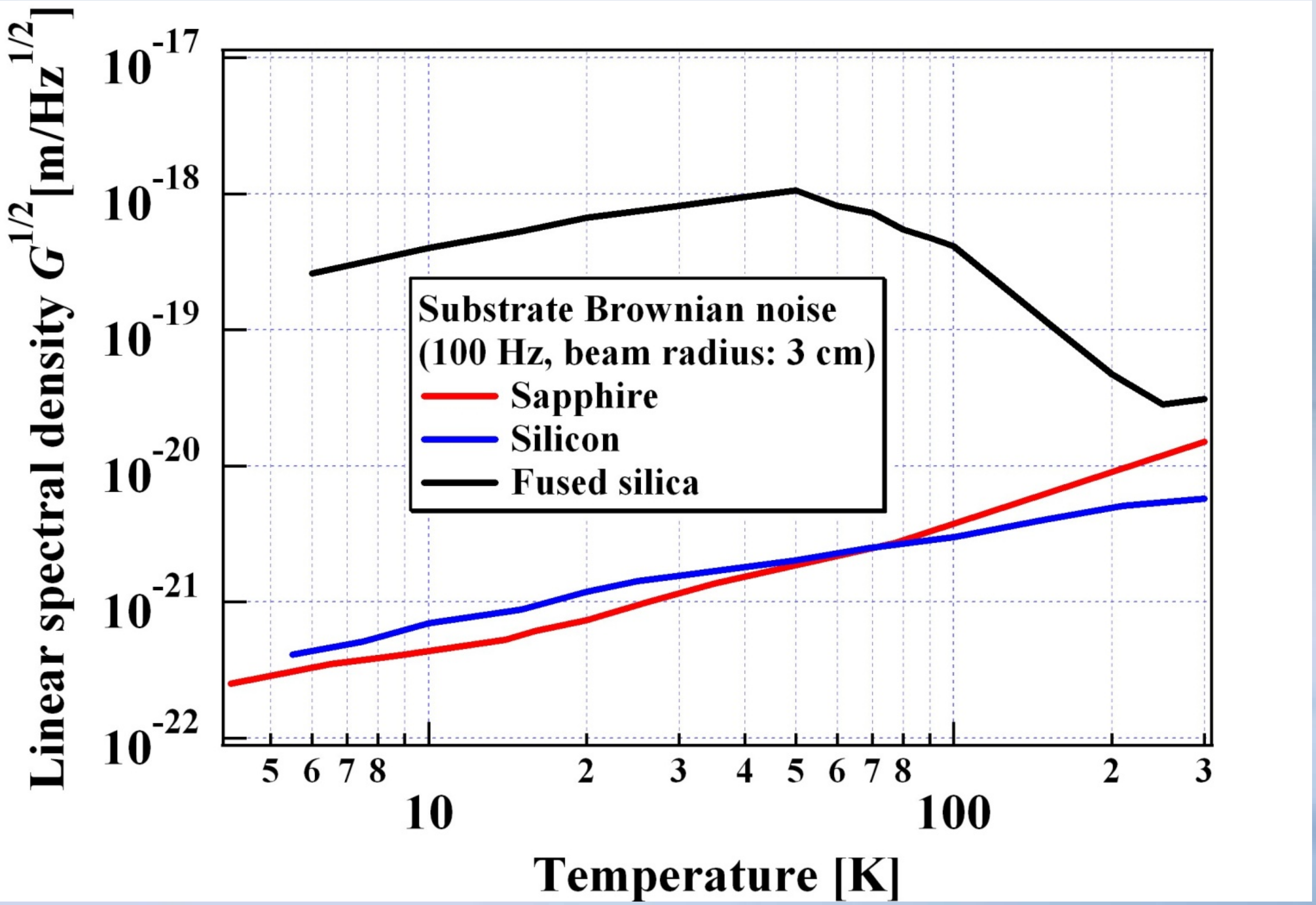


T. Uchiyama *et al*., Physics Letters A 261 (1999) 5-11.
R. Nawrodt *et al.*, Journal of Physics: Conference Series 122 (2008) 012008.
C. Schwarz *et al.*, 2009 Proceedings of ICEC22-ICMC2008.

4. Interferometer after revolution

(5) Reduction of thermal noise

Structure damping in substrate

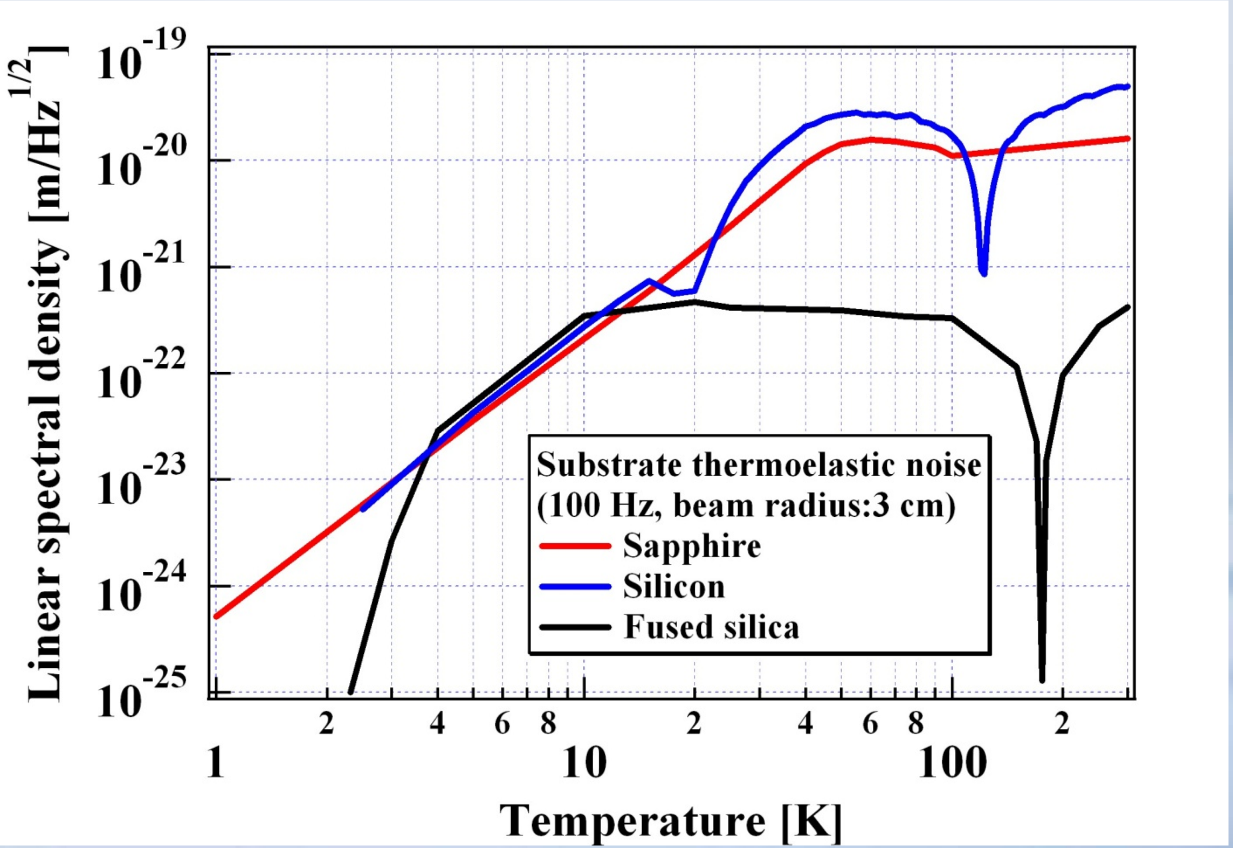


Fused silica can not be used !

4. Interferometer after revolution

(5) Reduction of thermal noise

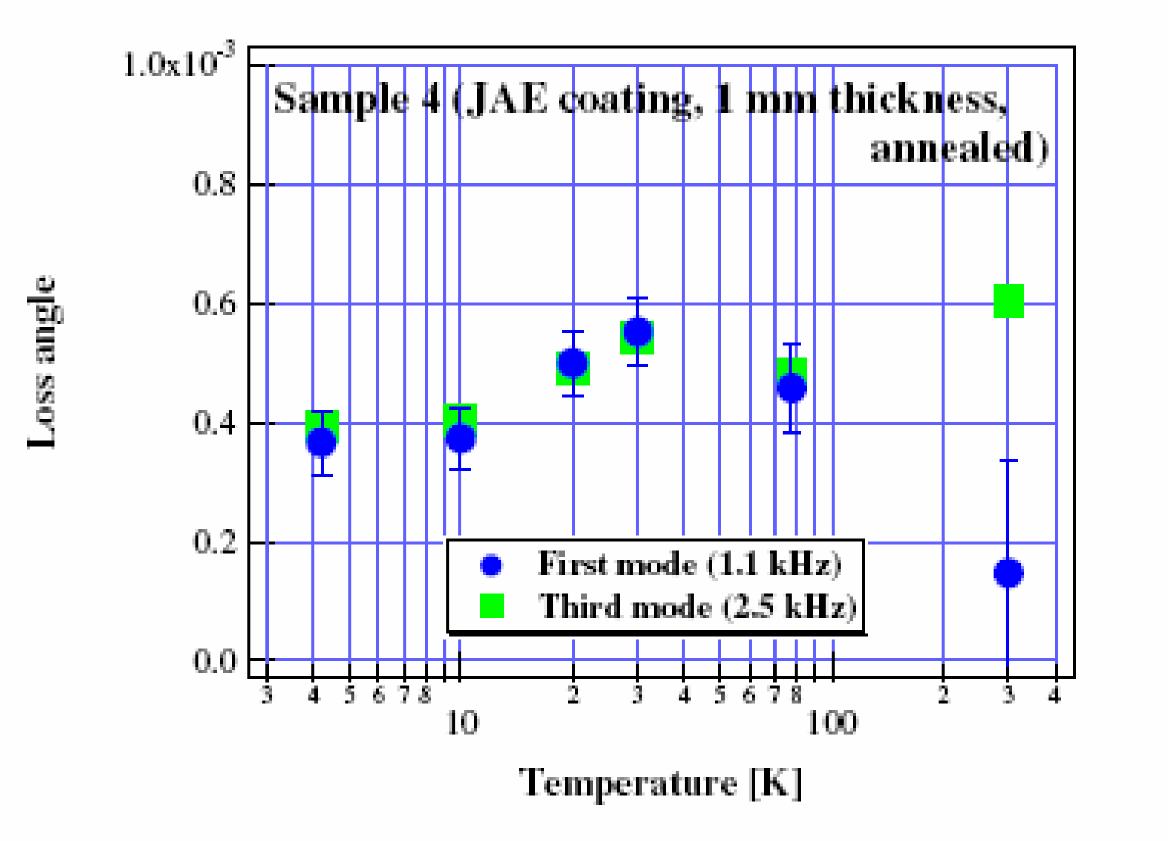
Thermoelastic damping in substrate



4. Interferometer after revolution

(5) Reduction of thermal noise

Structure damping in coating



Loss angle is almost independent of temperature.

K. Yamamoto *et al*., Physical Review D 74 (2006) 022002.

4. Interferometer after revolution

(5) Reduction of thermal noise

Structure damping in coating

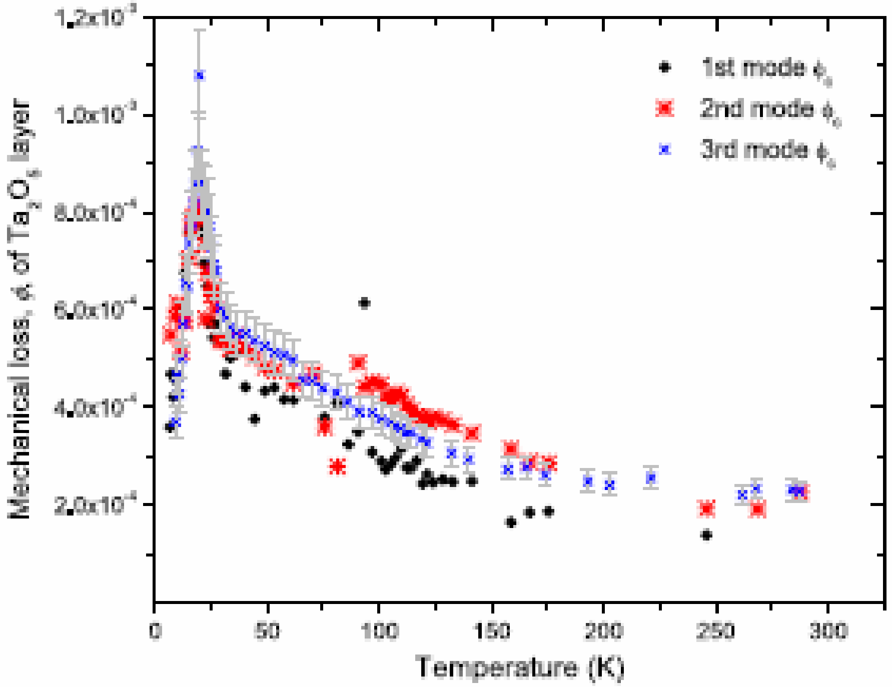


Figure 4. Temperature dependence of the loss of the doped Ta_2O_5 coating.

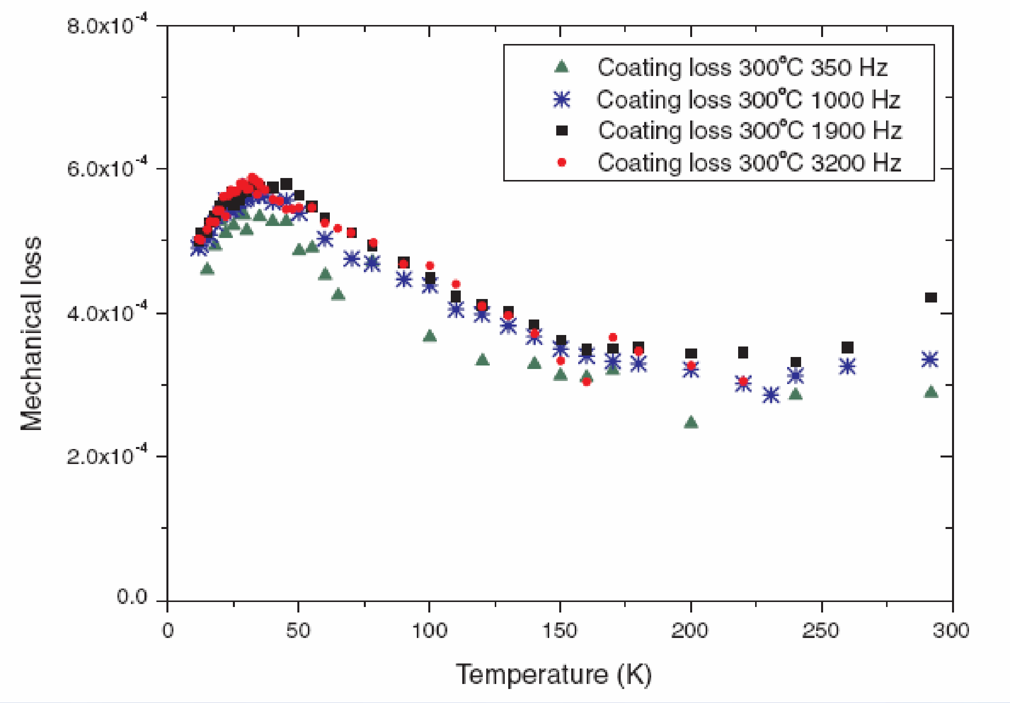
Peak at 20 K ?

I. Martin *et al.*, Classical and Quantum Gravity 25(2008)055005.

4. Interferometer after revolution

(5) Reduction of thermal noise

Structure damping in coating



Peak at 20 K ?

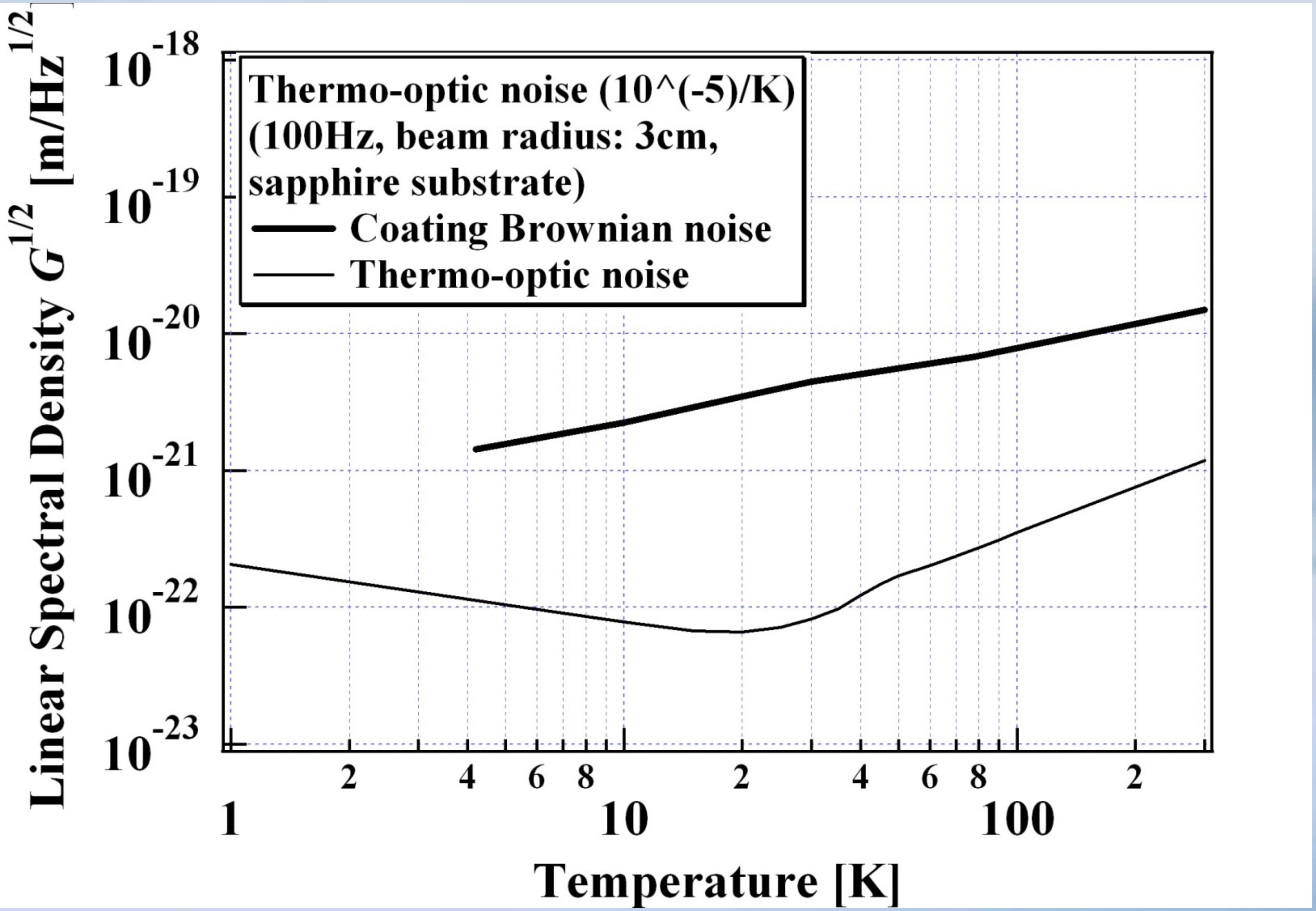
Annealing suppresses peak (not perfectly).

It is assumed that loss is constant.

I. Martin *et al.*, Classical and Quantum Gravity 27(2010)225020.

4. Interferometer after revolution

(5) Reduction of thermal noise



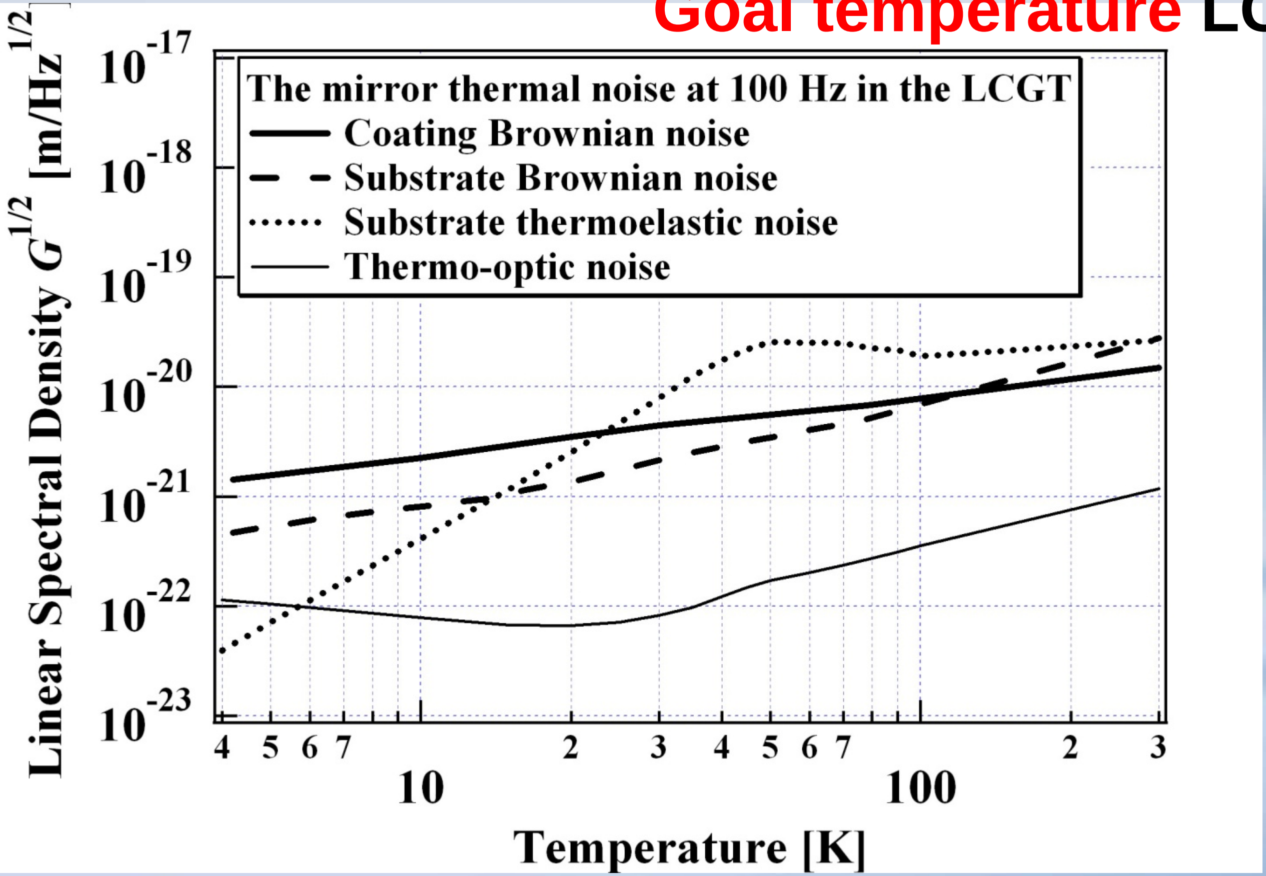
Thermo-optic noise is **less serious**.

4. Interferometer after revolution

(5) Reduction of thermal noise

Structure damping in coating

Goal temperature LCGT : 20 K



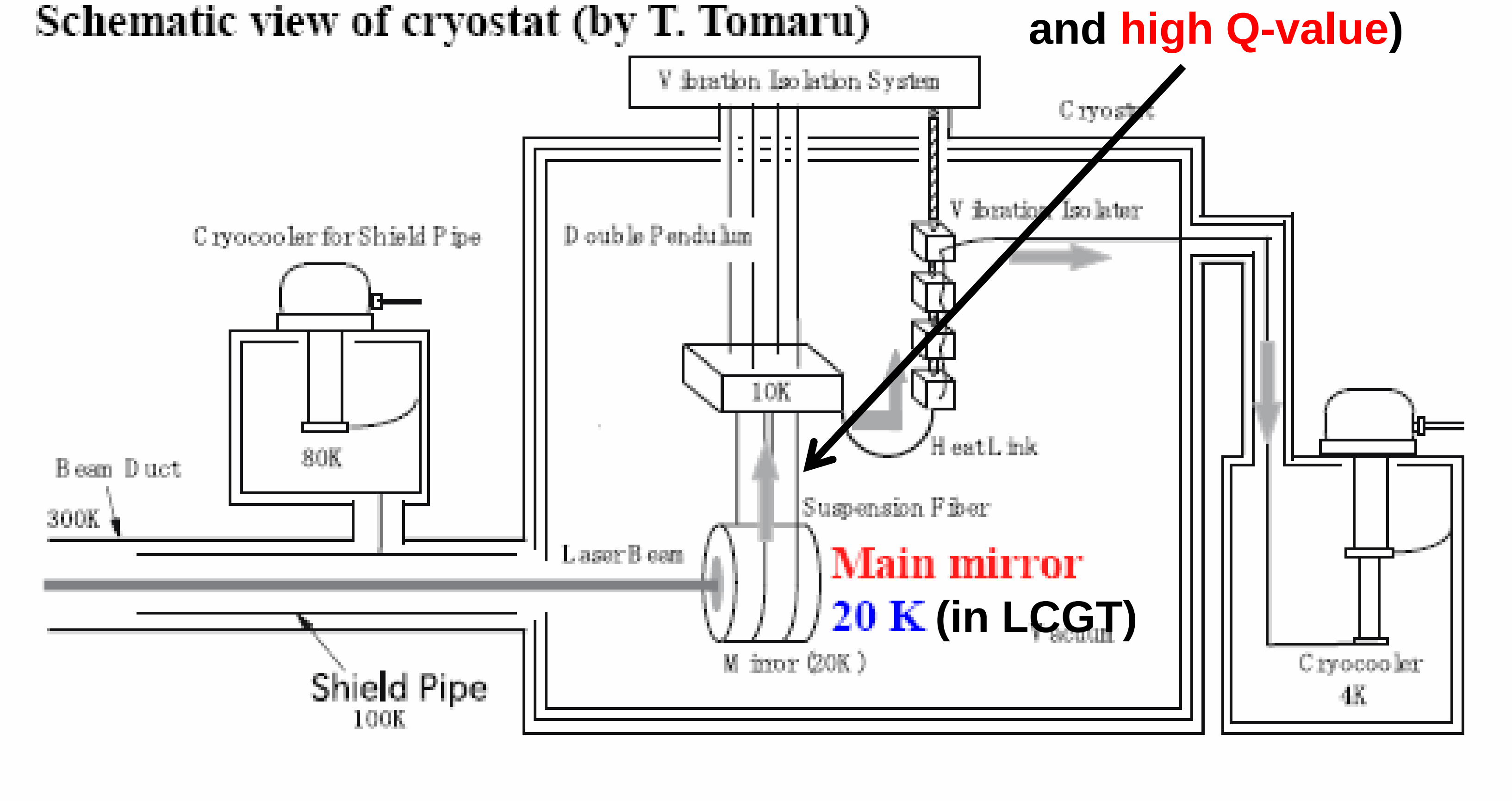
Coating thermal noise is the most serious problem !

4. Interferometer after revolution

(5) Reduction of thermal noise

How can we cool mirrors ?

Sapphire or silicon fiber (high thermal conductivity and high Q-value)



Schematic view of cryostat (by T. Tomaru)

Main mirror
20 K (in LCGT)

4. Interferometer after revolution

(5) Reduction of thermal noise **100 m, Kamioka (Japan)**

Cryogenic Laser Interferometer Observatory (CLIO,Japan)

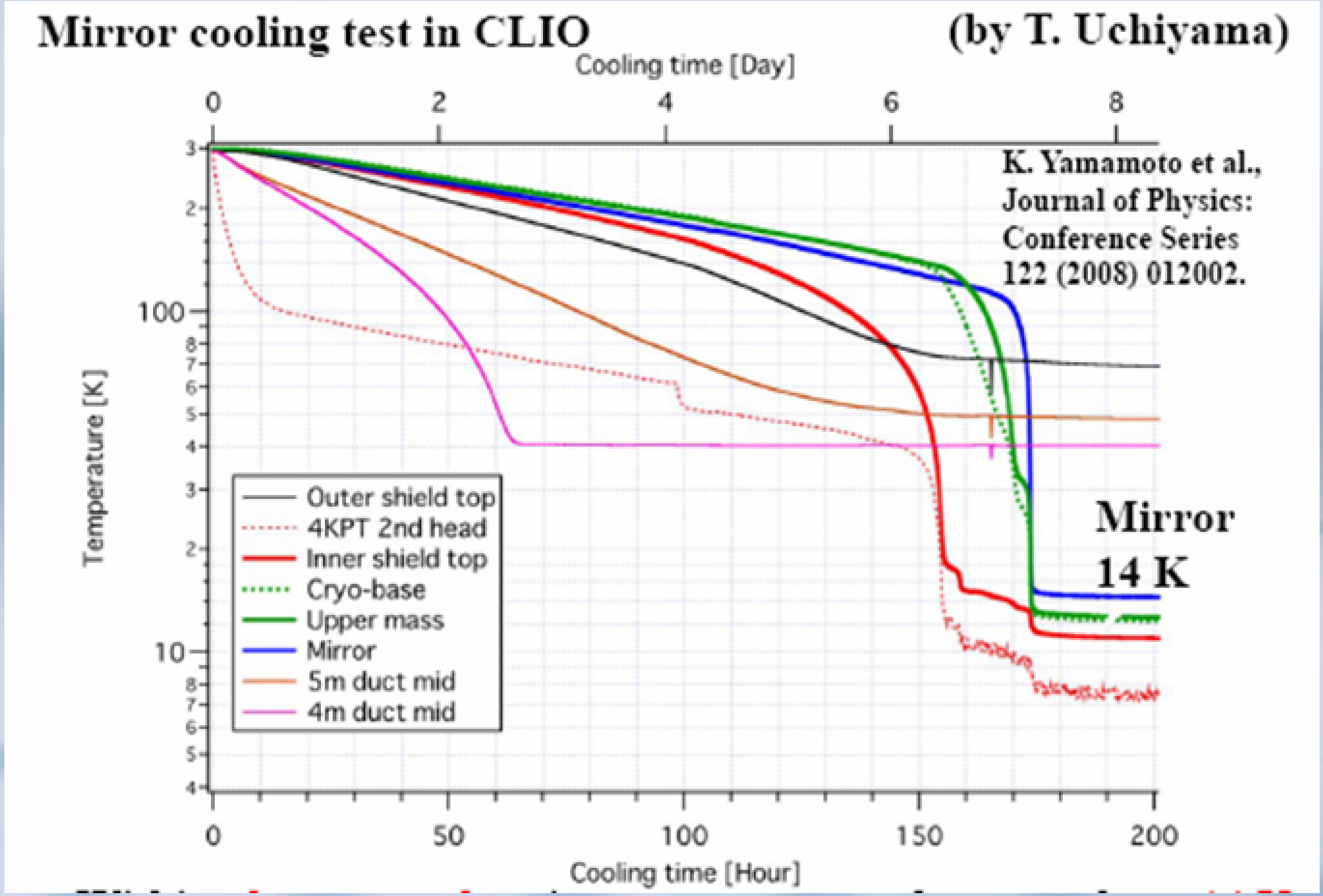
Result of CLIO for cryogenic techniques are introduced.

CLIO cryostat have already been installed
(different scale from those of LCGT and ET).

S. Kawamura, Classical and Quantum Gravity 27 (2010) 084001.

4. Interferometer after revolution

(5) Reduction of thermal noise


Mirror cooling test in CLIO (by T. Uchiyama)

K. Yamamoto et al., Journal of Physics: Conference Series 122 (2008) 012002.

Within about a week, mirror temperature became about 14 K (mirror temperature must be below 20 K).

4. Interferometer after revolution

(5) Reduction of thermal noise

Cryocooler

Why ?

Usual case : Liquid nitrogen and helium

Safety and maintenance in mine

↓

Cryocooler

Usual cryocooler : Gifford-McMahon cryocooler

Large vibration

↓

Pulse-tube cryocooler (without solid piston)

But, vibration of commercial one is not enough small.

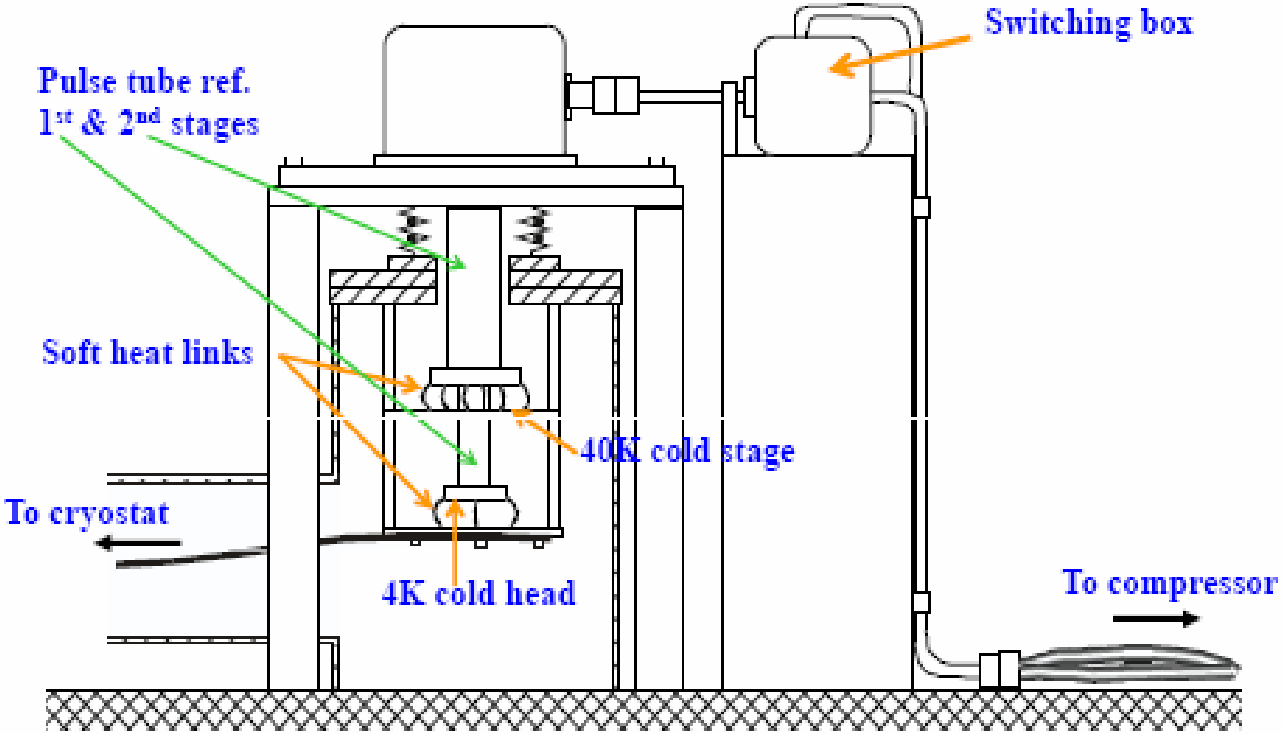
4. Interferometer after revolution

(5) Reduction of thermal noise

Schematic view of silent pulse-tube cryocooler

Quiet refrigerator was developed (design in 2003)

T. Tomaru *et al.*, Classical Quantum Gravity 21 (2004) S1005.
T. Tomaru *et al.*, Cryocoolers 13 (2005) 695.
R. Li. *et al.*, Cryocoolers 13 (2005) 703.
patent: Pa-3 Tomaru et al., 2003; Suzuki et al., 2003.

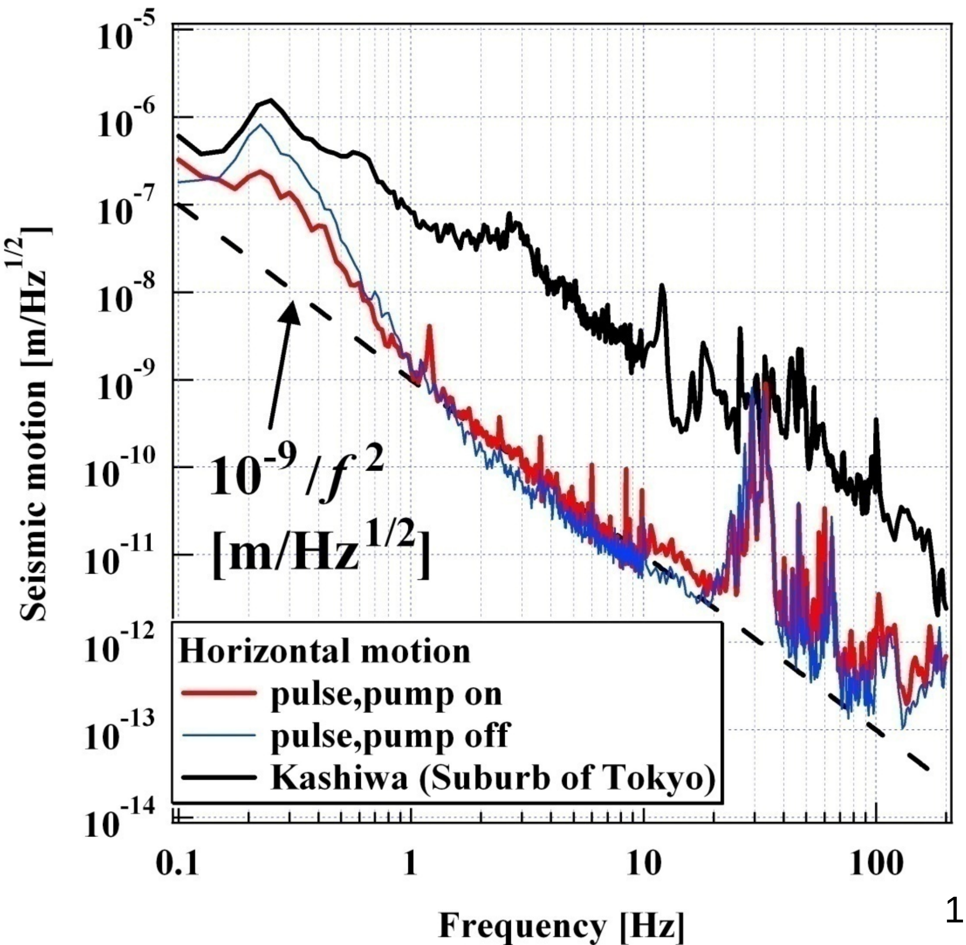


4. Interferometer after revolution

(5) Reduction of thermal noise

Measurement of vibration in cryostat with silent pulse-tube cryocooler at silent site

Cryocoolers do not increase vibration even if they are put on site with extremely small seismic motion.

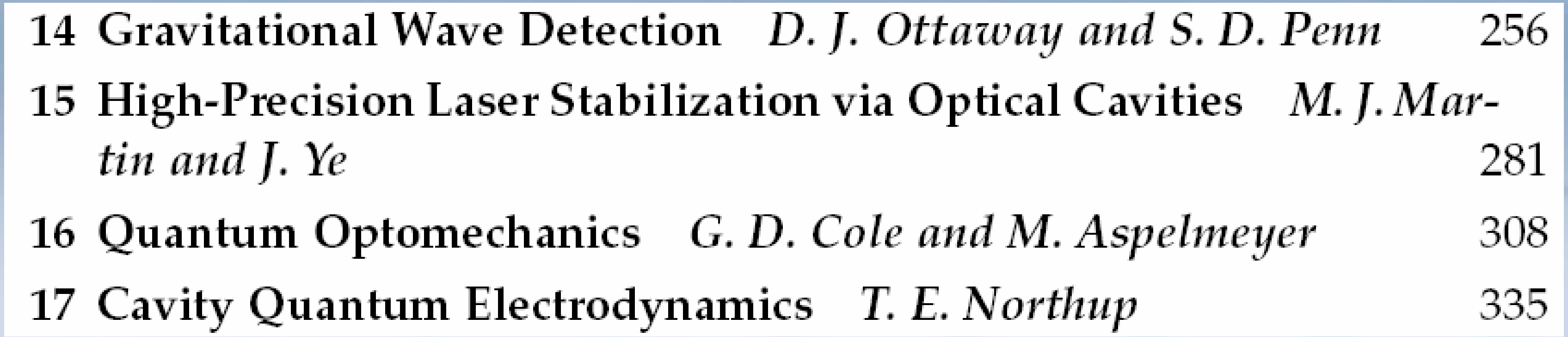


K. Yamamoto *et al*., Journal of Physics: Conference Series 32 (2006) 418.

4. Interferometer after revolution

(6) Impact on other fields

Contents of G.M. Harry, T. Bodiya, and R. DeSalvo (Editors)
Optical Coatings and Thermal Noise in Precision Measurements
Cambridge University Press, Cambridge (in press)



14 Gravitational Wave Detection *D. J. Ottaway and S. D. Penn* 256

15 High-Precision Laser Stabilization via Optical Cavities *M. J. Martin and J. Ye* 281

16 Quantum Optomechanics *G. D. Cole and M. Aspelmeyer* 308

17 Cavity Quantum Electrodynamics *T. E. Northup* 335

Thermal noise is also sensitivity limit
on the other fields in precision measurement.

4. Interferometer after revolution

(6) Impact on other fields

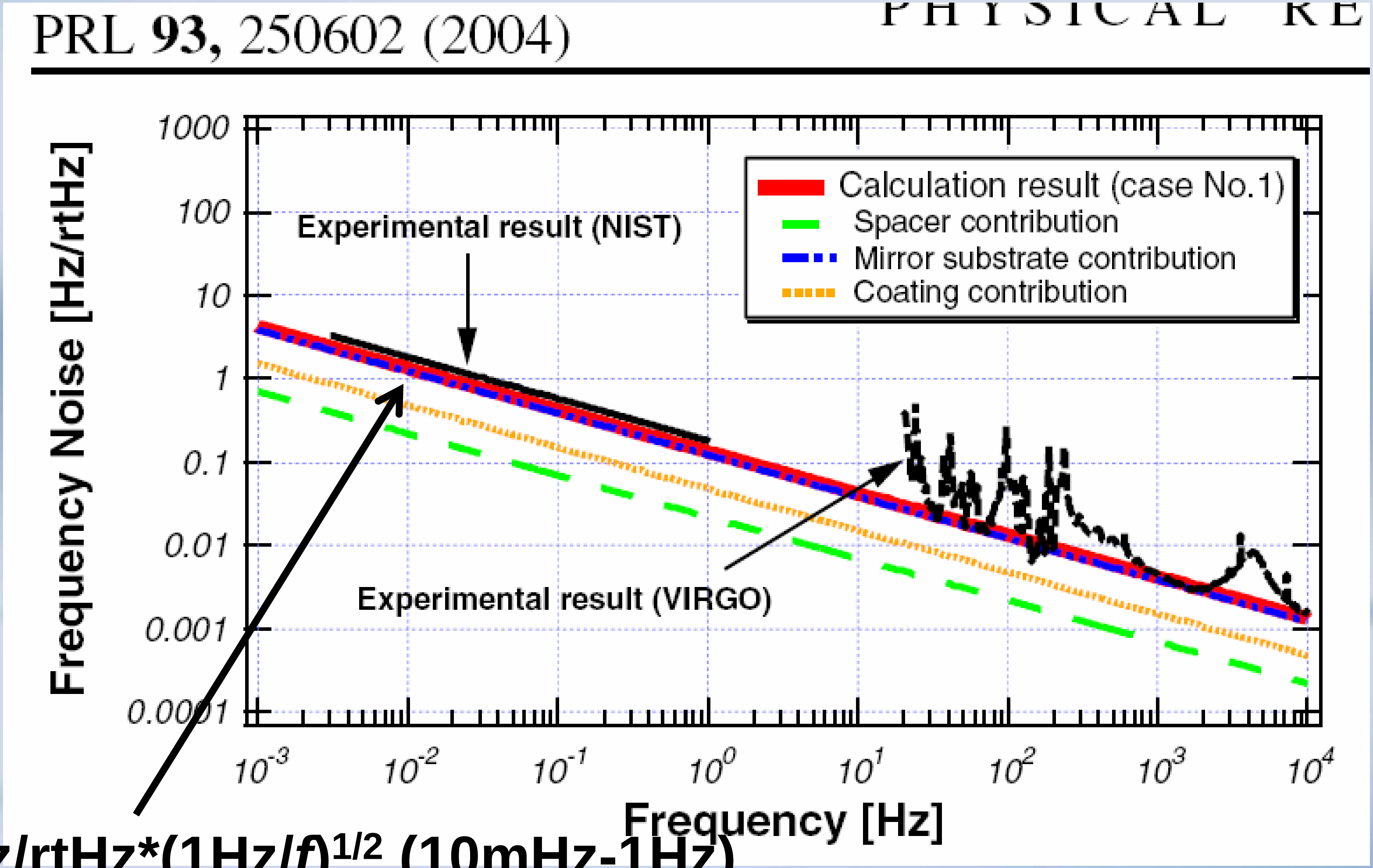
For example …
Cavity as reference for laser frequency stabilization

Current best laser frequency stabilization
with rigid cavity at room temperature
is limited by thermal noise of mirrors.

K. Numata *et al*., Physical Review Letters 93 (2004) 250602.

4. Interferometer after revolution

(6) Impact on other fields



PRL **93,** 250602 (2004)

PHYSICAL RE

0.1Hz/rtHz*(1Hz/f)$^{1/2}$ (10mHz-1Hz)

Allan deviation :4*10^{-16} (10mHz-1Hz)

4. Interferometer after revolution

(6) Impact on other fields **Hot paper (ISI Web of Knowledge)**

Thermal-noise limit in the frequency stabilization of lasers with rigid cavities

著者名: Numata K, Kemery A, Camp J

ジャーナル名: PHYSICAL REVIEW LETTERS 巻: 93 号: 25 記事番号: 250602 発行: DEC 17 2004

被引用数: 106 引用文献: 24

1 paper every 3 weeks ! (until 18 June 2011)

5. Summary

(1)Long history of research of thermal noise (200 years !)

General theorem for thermal noise, Fluctuation-Dissipation Theorem, appears only 60 years ago.

(2)Resonant detector

Cooling : Liquid helium or dilution refrigerator

Low mechanical loss material : Al5056

(3)Interferometric detector

It is essential to measure Q-values.

Pendulum : Rigid and heavy support system

Mirror : Nodal support system

5. Summary

(4)Interferometric detector

Drastic progress of research on the end of 20th century

New kinds of thermal noise

Thermoelastic noise and coating thermal noise

Direct measurement of thermal noise and dissipation

Reduction of thermal noise

(larger beam and cryogenic techniques)

Impact on other fields

There are open questions and this field will be hot in future.

Vielen Dank fuer Ihre Aufmerksamkeit !

Thank you for your attention !