Suspension thermoelastic noise in vertical direction

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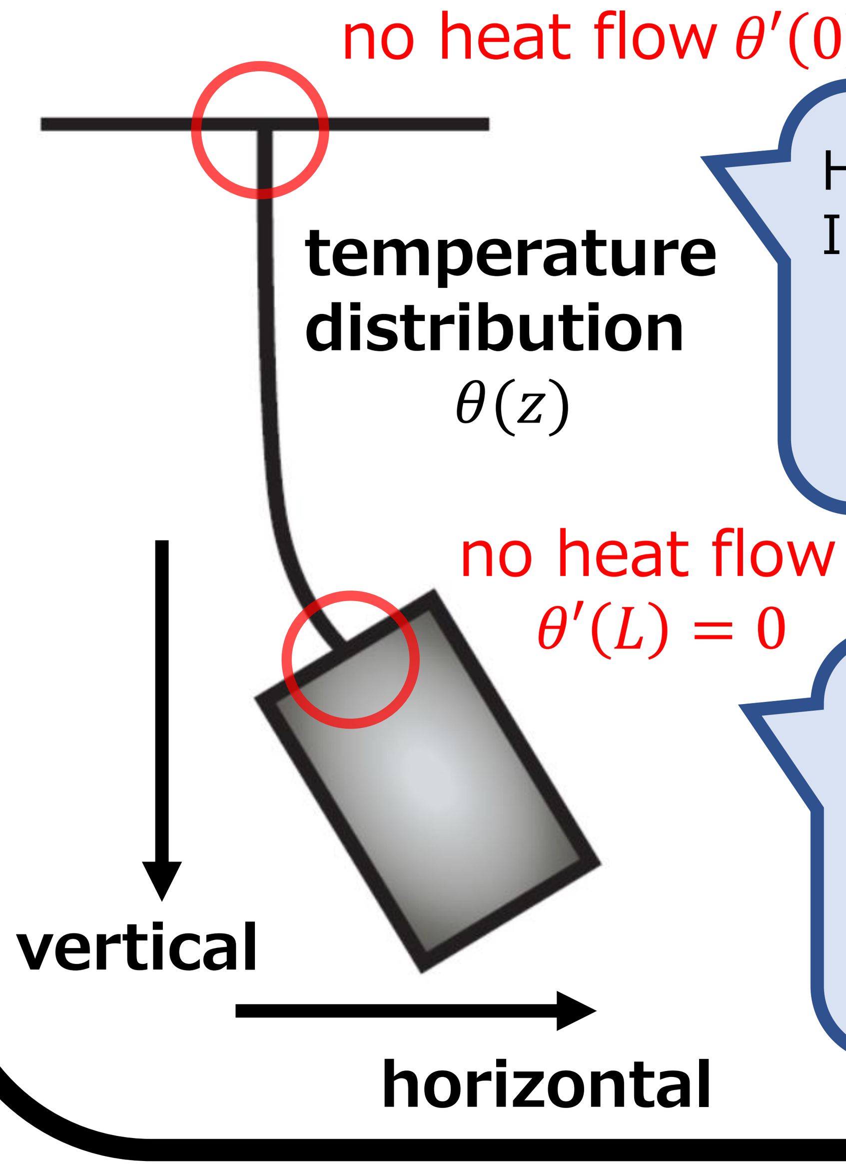
Basics

- Brownian thermal noise is a mechanical displacement of a mirror surface or a suspension fiber.
- Thermoelastic noise is temperature fluctuation that couples to displacement via thermal expansion.

Motivation



Heat flow BC varies with IFO configuration.
LV: no insulator
NEMO,Voyager: no flow
KAGRA,ET: heat flow

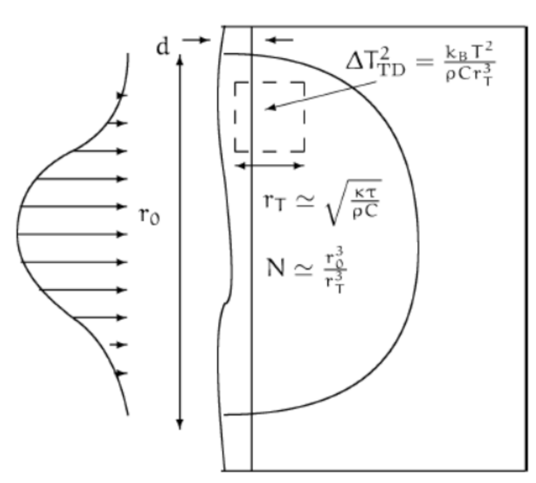
No literature found that shows derivation of vertical suspension thermoelastic noise (VSTN).

Maybe very small?

- Elastic equation at LF reads: $\boldsymbol{Eu'' - E\alpha\theta' = 0}$. This with $\boldsymbol{\theta'(L) = 0}$ may seem to yield $\boldsymbol{u(L) = 0}$ as $\boldsymbol{\theta}$ can be given as $\boldsymbol{\theta = \sum \theta_n \cos\frac{n\pi x}{L}}$ and then $\boldsymbol{u}$ can be given as $\boldsymbol{u = \sum u_n \sin\frac{n\pi x}{L}}$.
- If you pull down the mass very slowly, fiber temperature increases uniformly and there will be no thermal dissipation (i.e. no TE noise).

Previous studies

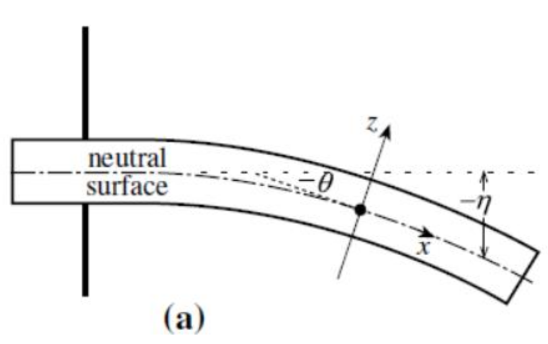
Langevin approach
Braginsky used this method to calculate mirror TE noise. Langevin force is applied and the surface fluctuation is calculated. By taking the ensemble average of the fluctuation, the noise spectrum is derived.



[V.Braginsky (2003)]

Levin's direct method Levin introduced this method to calculate mirror TE noise. Virtual entropy is applied to the surface and the temperature distribution is calculated. By integrating the temperature gradient over the volume, the noise spectrum is derived.

Single mode Q This is a typical method used by many people to calculate thermoelastic noise of a blade spring or a suspension in the horizontal direction. The real and imaginary parts of a single mode oscillator are calculated. From the ratio, i.e. Q, the noise spectrum is derived.



[K.Thorne, lecture note]

Heat eq. and Elastic eq.

Exact form (one-dimensional)

$$\boldsymbol{i\omega\theta - \chi\theta'' = \underline{-i\omega SE\chi u'}} \quad \text{(Heat eq.)}$$

$$\boldsymbol{\rho\omega^2 u + Eu'' \underline{- E\alpha\theta'} = 0} \quad \text{(Elastic eq.)}$$

In Langevin approach:

Heat source is ignored. Langevin term added.

$$\boldsymbol{i\omega\theta - \chi\theta'' = F} \quad \text{(Heat eq.)}$$

$$\boldsymbol{\rho\omega^2 u + Eu'' - E\alpha\theta' = 0} \quad \text{(Elastic eq.)}$$

solve & plug in

In Levin's method:

Expansion is ignored. Virtual force added.

$$\boldsymbol{\rho\omega^2 u + Eu'' = 0;\ u'(L) = f} \quad \text{(El eq.)}$$

$$\boldsymbol{i\omega\theta - \chi\theta'' = -i\omega SE\chi u'} \quad \text{(Heat eq.)}$$

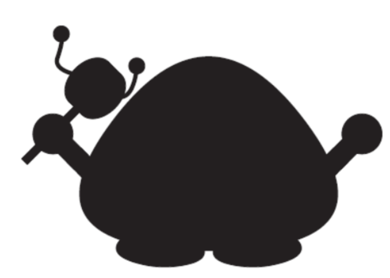
solve & plug in

In either method, the interaction term is ignored in one of the two equations where we added a virtual term, and the simultaneous diff equations can be solved in sequence.

$\chi = \kappa/\rho C$: thermal diffusivity, ρ: linear density, S: cross section , E: Young's modulus, α: thermal expansion, κ: thermal conductivity, C: heat capacity

Diffusive length

Diffusive length r_T is given by $r_T = \sqrt{2\chi/\omega}$.
For KAGRA at 20K, r_T =6.0m @ 100Hz
For KAGRA at 300K, r_T =140um @ 100Hz
For LIGO (300K), r_T =37um @ 100Hz.



The one-dimensional Langevin approach would work properly only if r_T is larger than the fiber diameter.

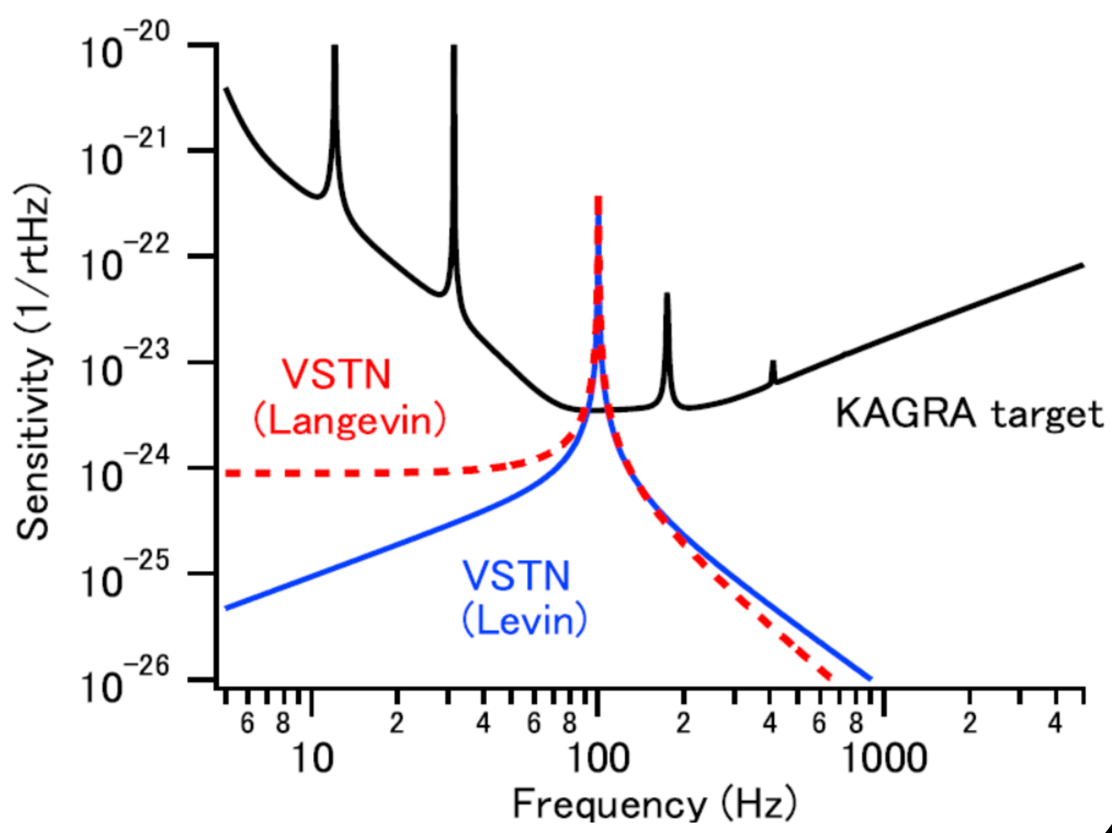
Results with Levin's method

In Levin's method, the VSTN power spectrum is given by

$$S_x = \frac{8k_B T^2\alpha^2 L^3}{\pi^2\kappa}\left(\frac{\pi^2}{6} - \frac{\pi^2}{4\beta}\frac{\sinh 2\beta - \sin 2\beta}{\cosh 2\beta - \cos 2\beta}\right)$$

Here $\beta = L/r_T$. The second term in the bracket approaches to $\pi^2/6$ with $\beta \to 0$.

In Langevin approach, the first term in the equation disappears and the result is a little different.
We also tried the single mode approach but the result has not matched.



Non-zero heat flow

Now we are almost ready to introduce a non-zero heat flow in our boundary condition. This will be the examination of **Maxwell's Daemon** in the thermal noise calculation.