Kerr instability in coatings

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Background

A mysterious scattering and reduction of power-recycling gain with increasing laser power were reported in LLO and LHO at the September LVK meeting. (Current circulating power in the arm is 370kW in LHO.)

→ Would the Kerr effect have anything to do with the mysteries?

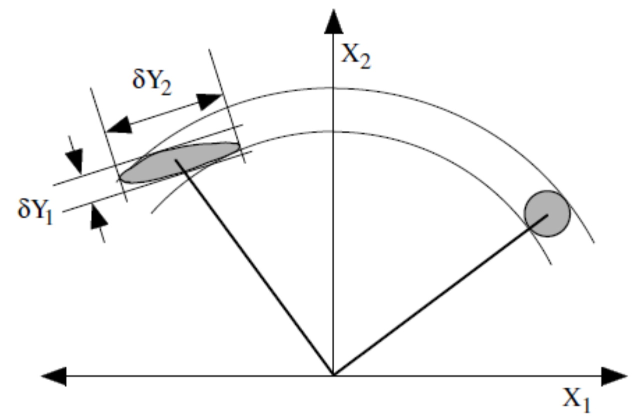
→ Should we consider the Kerr effect in the selection of new coating materials?

Optical Kerr effect

Polarization component of frequency ω induced in the medium can be written as

$$P_j(\omega) = \epsilon_0 \left[\chi^{(1)}_{jk}(\omega) + \frac{3}{4}\chi^{(3)}_{jklm}(\omega,\omega,-\omega)E_l E_m^*\right] E_k$$

where $\chi^{(3)}_{jklm}$ is the third-order nonlinear susceptibility.



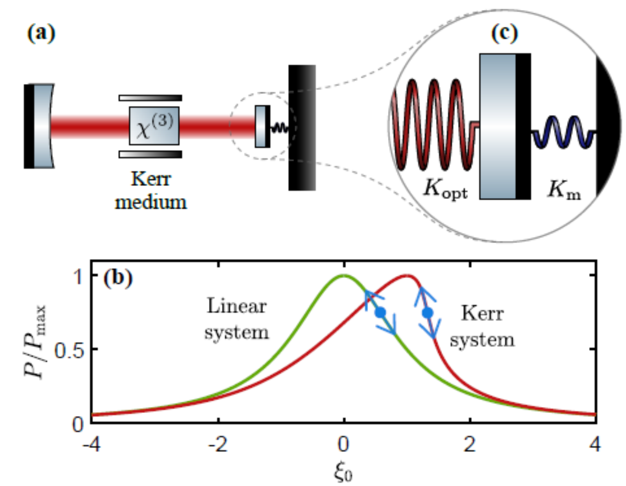
Kerr Squeezing [A.White 2000]

The real part of susceptibility contributes to the refractive index:

$$n = n_0 + n_2 I, \qquad n_2 = \frac{3Re[\chi^{(3)}_{1111}]}{4\epsilon_0 c n_0^2}$$

Light intensity

and this $\chi^{(3)}$ leads to Kerr squeezing, Kerr amplification, Kerr lensing, filamentation, etc.



Kerr-enhanced optical spring [S.Otabe 2023]

Kerr lensing (self-focusing)

[K.Kuroda, Nonlinear Optics]

The light propagation with a nonlinear effect can be described by the following *nonlinear Schroedinger equation (NSE)*:

$$\frac{\partial E}{\partial z} - \frac{i}{2k}\nabla_{\perp}^{2}E - i\gamma|E|^{2}E = 0, \qquad \gamma = \frac{\omega_0\epsilon_0 n_0 n_2}{2}.$$

Gaussian beam is described in the form:

$$F(x,y,z) = G(z)\exp\left[\frac{ik(x^2+y^2)}{2q(z)}\right]$$

where $G(z)$ is the amplitude of the beam at the center, $q(z)$ is the complex wave parameter. Plugging this in to NSE, and doing some math, we obtain the beam spot with the Kerr lensing:

$$\rho = \rho_0\sqrt{1 + \frac{z^2}{z_R^2}\left(1 - \frac{P}{P_c{}'}\right)}, \qquad P_c' = \frac{\lambda^2}{8\pi n_0 n_2}$$

→ The beam size starts to decrease if the laser power P is larger than the critical power $P_c{}'$, which is ~36kW for the tantala layer.

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The beam size could become zero, but in reality the paraxial approximation breaks before that and light scatters into higher order modes (Kerr instability). In fact, Rayleigh range in LIGO is so long and the coating layer is so thin that it is quite far from the instability.

A numerical simulation is necessary to argue the influence in a quantitative manner.

Filamentation

[K.Kuroda, Nonlinear Optics]

Let us see another approach to study Kerr instability. A plane wave is a solution to *nonlinear Schroedinger equation (NSE).* Assuming the beam fluctuates to x direction at wavenumber α, and the propagation coefficient of the fluctuation in x direction is β, we have

$$E(x,y,z) = \left(E_0 + ae^{i(\alpha x+\beta z)} + be^{-i(\alpha x+\beta z)}\right)e^{i\left(k+\gamma|E_0|^2\right)z}.$$

Plugging this in to NSE, we finally obtain the following condition for β to have non-zero $a\,, b$:

$$\beta = \pm\frac{\alpha}{2k}\sqrt{\alpha^2 - 4k\gamma|E_0|^2}$$

This β will be imaginary if the spatial frequency α is lower than $\sqrt{2k\gamma}E_0$. The lowest spatial frequency of the beam is roughly $2\pi/d$. Thus, the stability condition is:

$$\frac{4\pi^2}{d^2} \geq \frac{P}{P_c}\frac{4\pi^2}{d^2}, \qquad P_c = \frac{\pi\lambda^2}{16n_0n_2}$$

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→ Low order modes will become unstable while higher order modes remain stable. The beam will be split to thinner beams called filaments. The critical power P_c is ~180kW for tantala.

This can be a new type of tilt instability. A numerical simulation is necessary to argue the influence in a quantitative manner.

Numerical simulation

A well-known scheme to deal with such a nonlinear equation is to use Finite-Difference Time-Domain (FDTD) simulation.

Solve Maxwell eq with $n^{(0)} = n_0$ and obtain $I^{(0)}$

↓

Solve Maxwell eq with $n^{(1)} = n_0 + n_2 I^{(0)}$ and obtain $I^{(1)}$

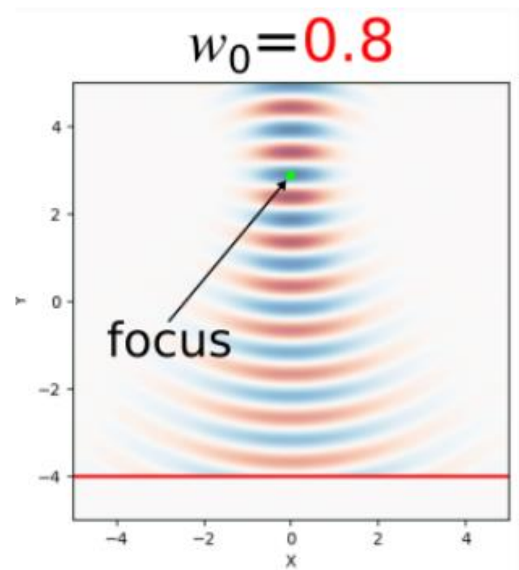
↓

Solve Maxwell eq with $n^{(2)} = n_0 + n_2 I^{(1)}$ and obtain $I^{(2)}$

⋮

Iterate the calculation until the intensity profile converges.

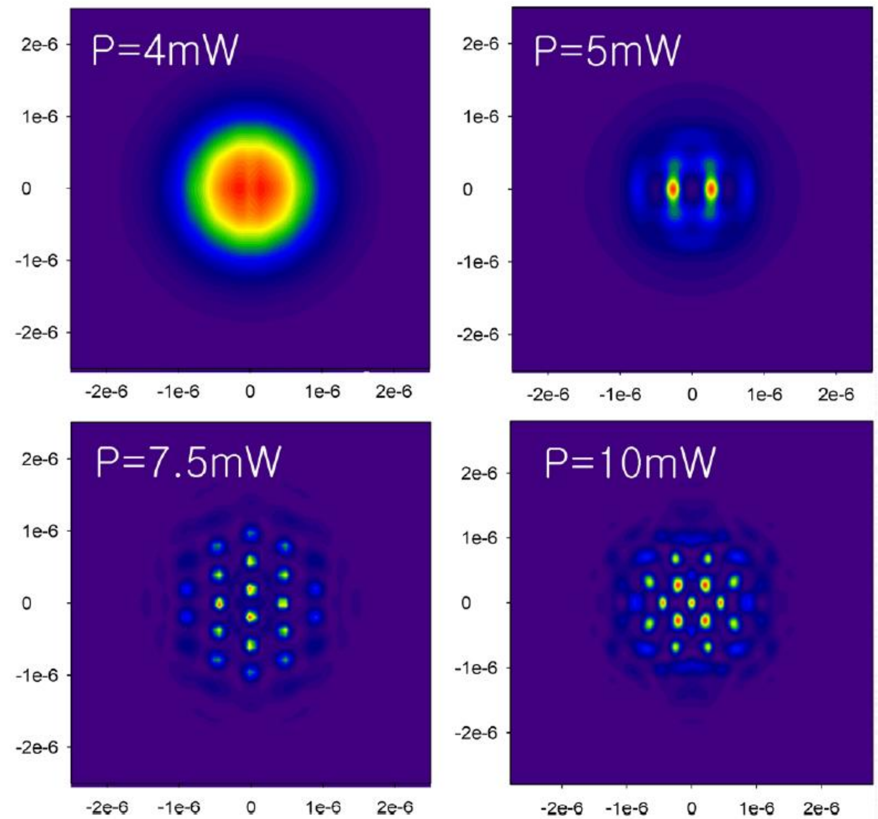
https://meep.readthedocs.io/en/latest/



Numerical simulation

Shown in right is a plot in a paper using FDTD to calculate the Kerr filamentation. The nonlinear susceptibility is extremely high. The filamentation can be clearly seen at P>5mW.

$$n_2 = 4 \times 10^{-10}\ [\mathrm{m}^2/\mathrm{W}]$$
$$w = 1 \times 10^{-6}\ [\mathrm{m}]$$
$$\ell = 1.7 \times 10^{-6}\ [\mathrm{m}]$$



[H-H.Lee et al., Opt. Express, 2004]

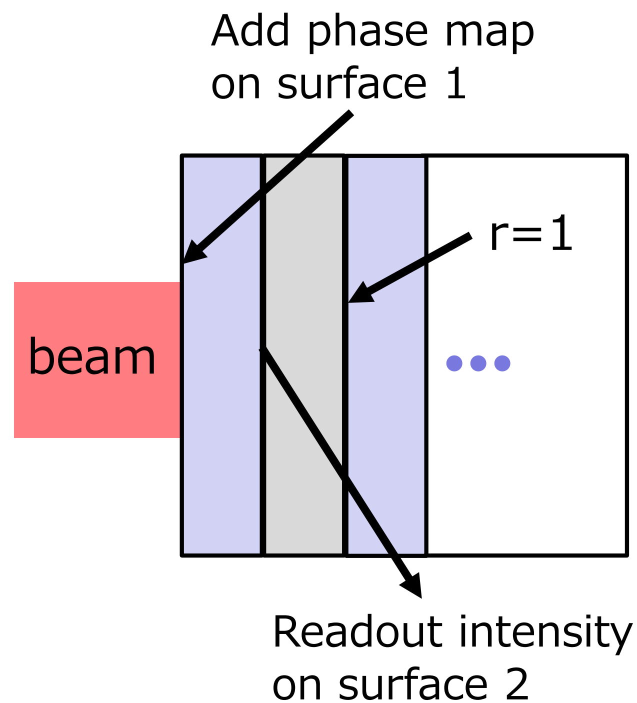
FDTD with FINESSE3

A similar process can be performed with FINESSE.

- Run code with no map and get transmission profile $I^{(0)}$
- ↓ Run code with a map $n_2 I^{(0)}$ and get profile $I^{(1)}$
- ↓ Run code with a map $n_2 I^{(1)}$ and get profile $I^{(2)}$
- ⋮

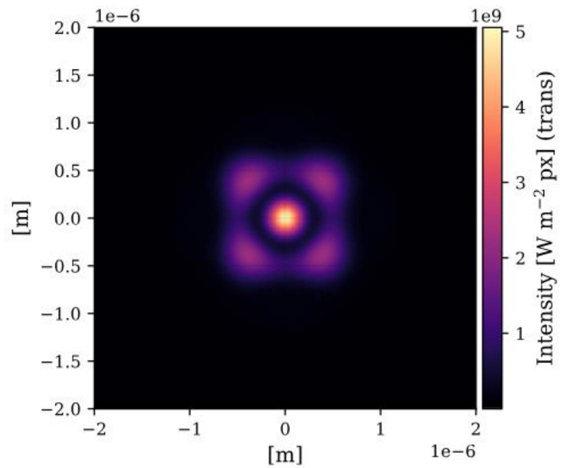
Iterate the calculation until the intensity profile converges.

https://finesse.ifosim.org/docs/latest/index.html

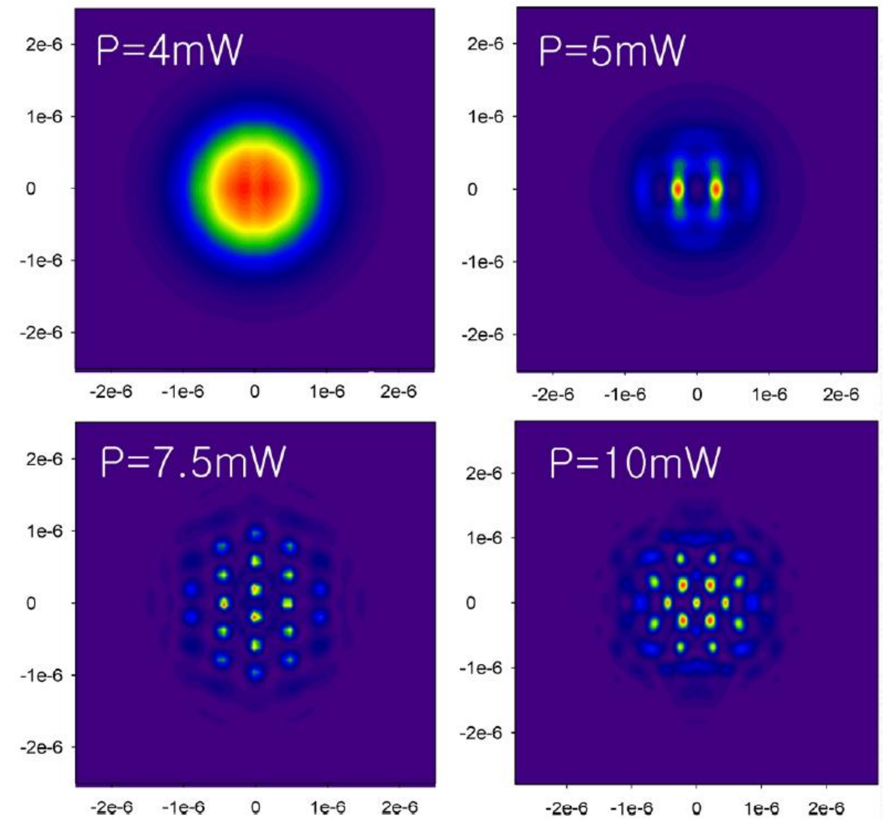


FDTD with FINESSE3

We ran FINESSE3 with the same parameter as [Lee2004]. The calculation did not converge. The plot below is the intensity profile after 10 iterations.



Injected power was 5mW and we calculated up to n+m=30 modes. We removed the circular aperture and decrease the map size to 2um.



[H-H.Lee et al., Opt. Express, 2004]

$$n_2 = 4 \times 10^{-10}\ [\mathrm{m}^2/\mathrm{W}]$$
$$w = 1 \times 10^{-6}\ [\mathrm{m}]$$
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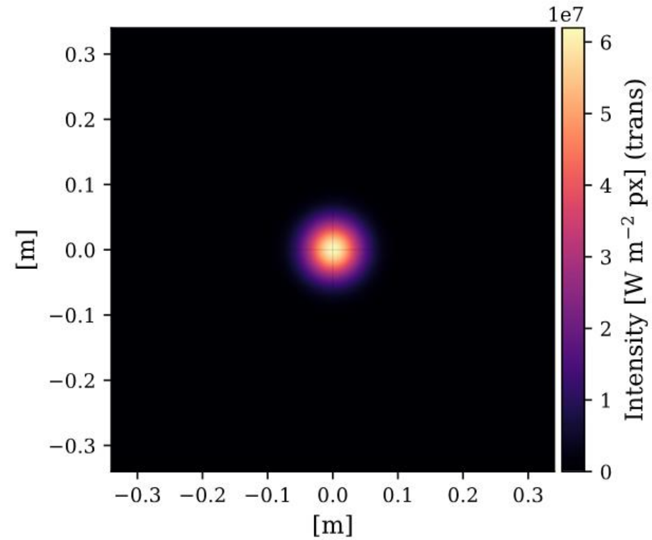
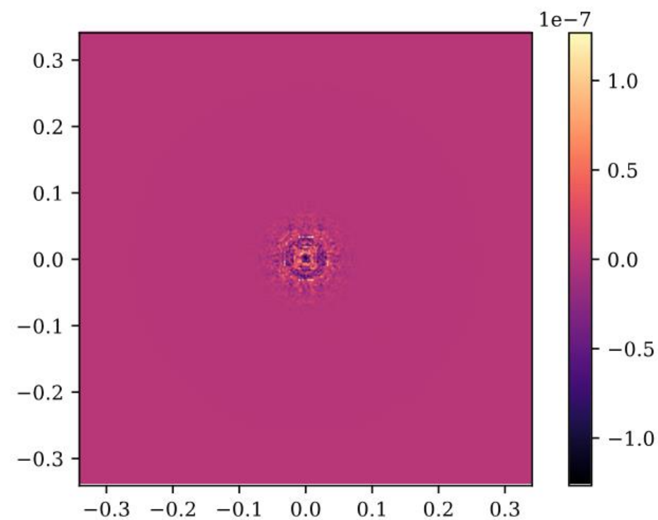
Simulation with LIGO parameters

We ran the code for the first tantala layer of aLIGO. The laser power in the anti-resonant cavity with silica/tantala interfaces is given as $370\mathrm{kW} \times n_s^2/n_t$ ($\simeq 370\mathrm{kW}$).

The nonlinear susceptibility of tantala is $6.0 \times 10^{-19}[\mathrm{m}^2/\mathrm{W}]$.

The intensity profile converged very quickly, only after 2 cycles. The scattering of the fundamental mode is so small that the image of the difference (bottom) is faint.

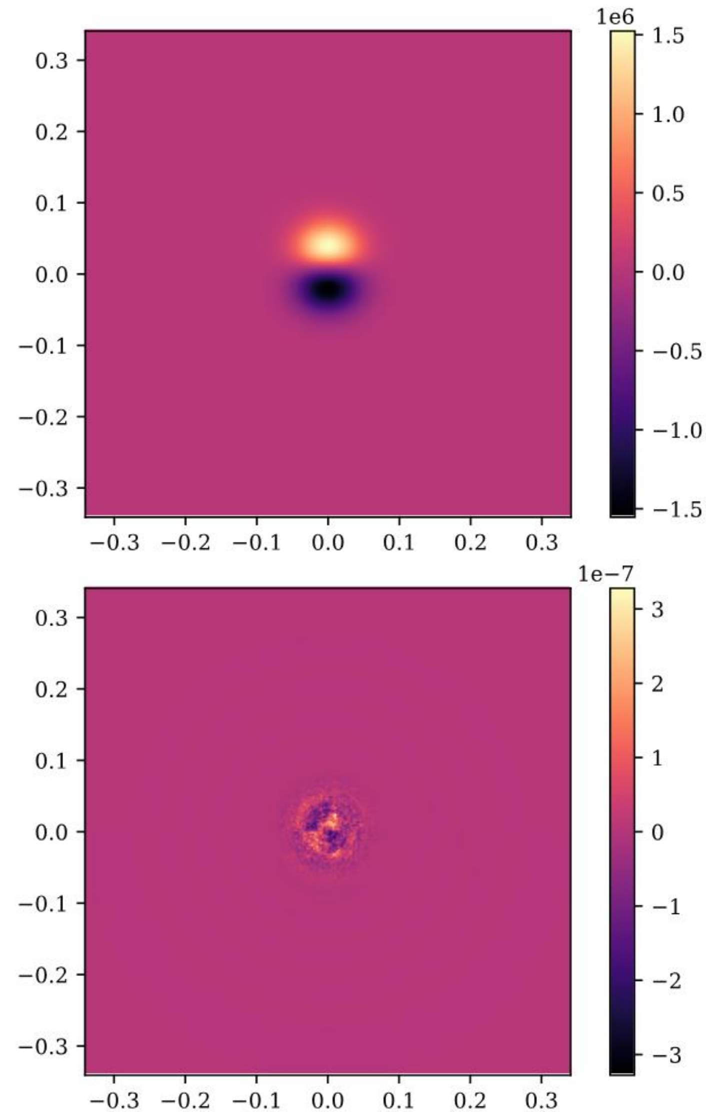




$n_s = 1.45, \quad n_t = 2.07$
$w_{ETM} = 62\mathrm{mm}$ ($w_{ITM} = 53\mathrm{mm}$)

Tilt instability

Next, we introduced 1% TEM10.
The increase of TEM10 is not clear,
so let us increase the power by x10.

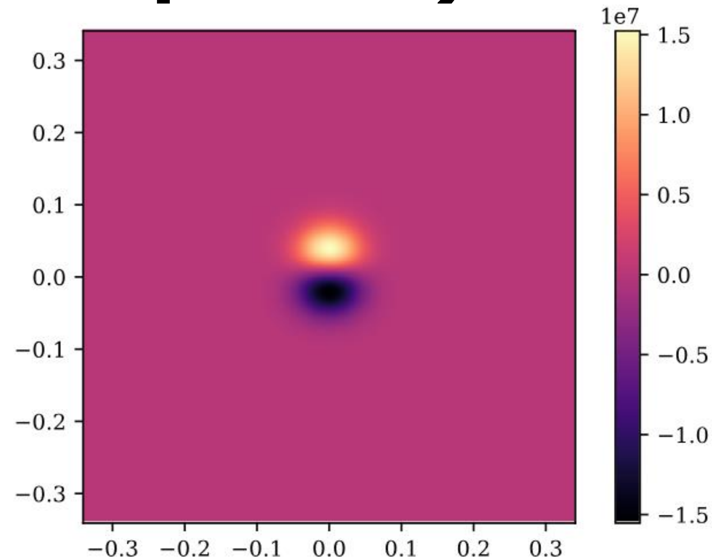


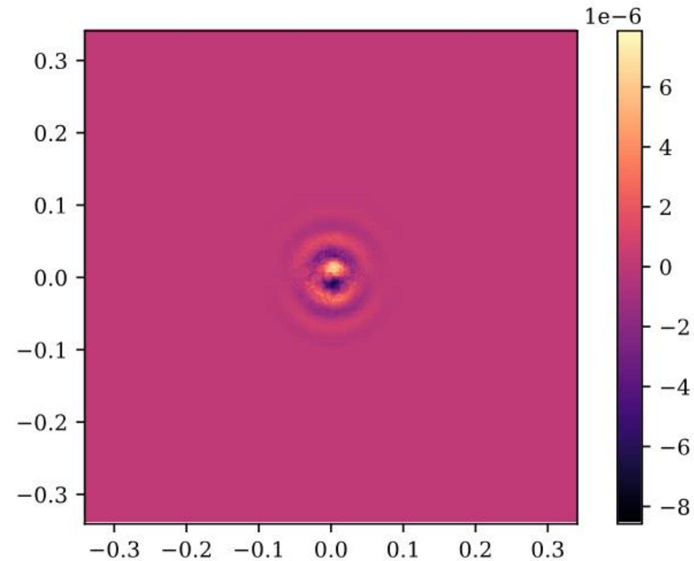
$n_s = 1.45, \quad n_t = 2.07$
$w_{ETM} = 62\text{mm}$ ($w_{ITM} = 53\text{mm}$)

Tilt instability (x10 power)

With x10 power, the increase of TEM10 with Kerr lensing can be clearly seen but the increase is at tiny amount (~4e-13).

The increment can be ignored as it is much smaller than the roundtrip optical loss.





$n_s = 1.45, \quad n_t = 2.07$
$w_{ETM} = 62\text{mm}\ (w_{ITM} = 53\text{mm})$

Summary

- We found that current circulating power in LIGO exceeds the critical power of Kerr instability.
- However, the influence of the instability seems small due to the large Rayleigh range.
- We performed FDTD simulation using FINESSE3 for quantitative argument of the Kerr instability, and the influence was indeed very small.
- We do not think the Kerr instability could affect the selection of the new coating materials.