Reaching the quantum limit with a gravitational wave telescope

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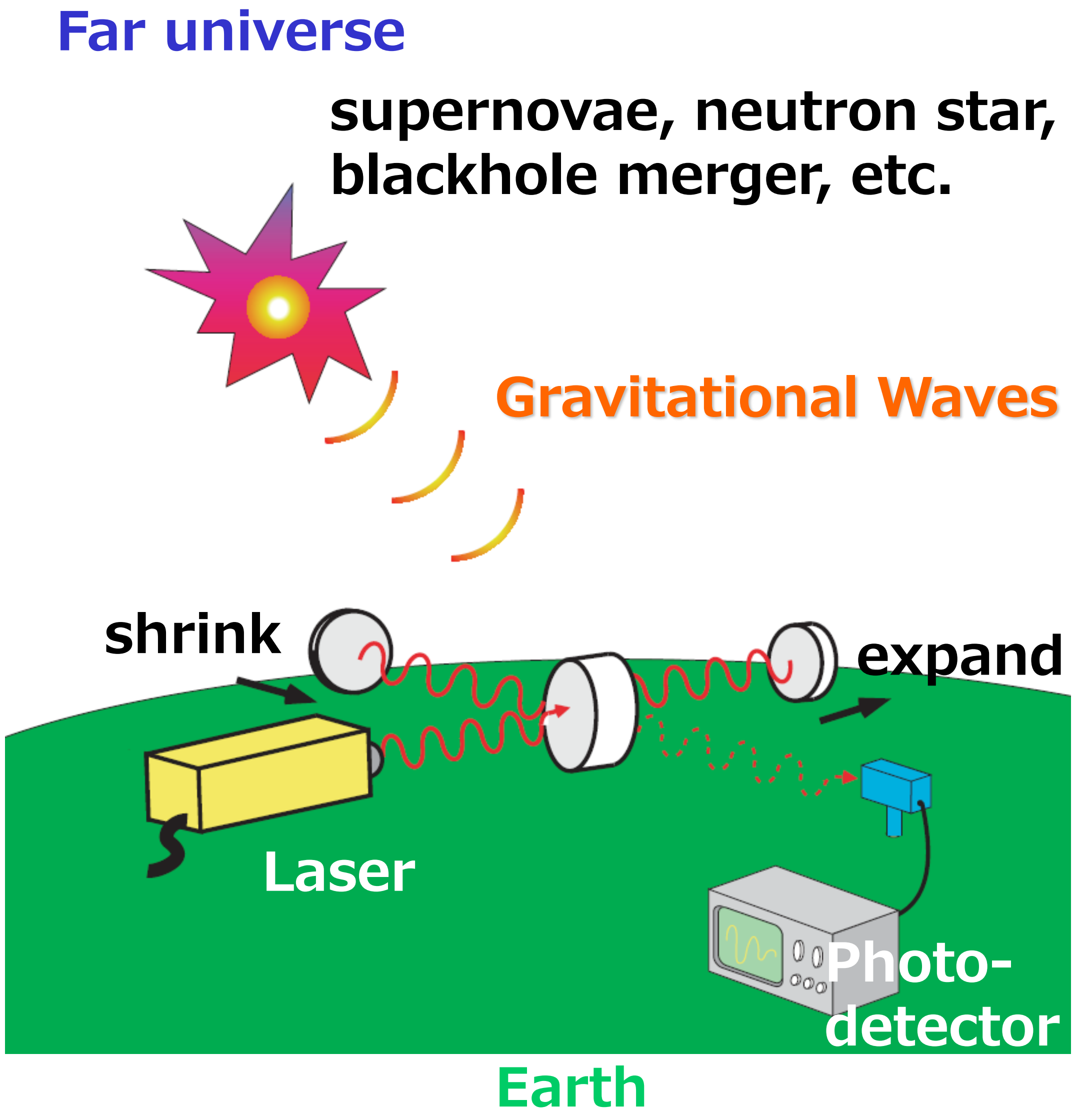
Laser interferometric GW detector



Massive astronomical events

⇩

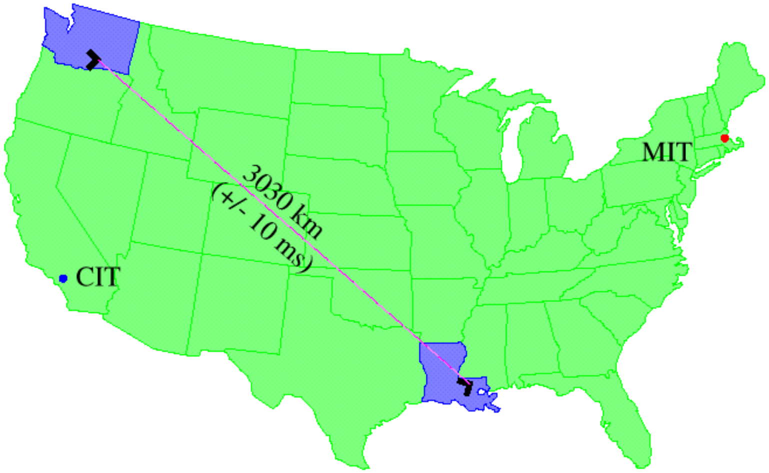
Optical path length changes

⇩

Can be observed by
a large interferometer

- LIGO in US [4km]
- Virgo in Italy [3km]
- GEO in Germany [600m]
- KAGRA in Japan [3km]

Advanced LIGO



- 4km interferometer x2
- 10-times better sensitivity than LIGO
- Started obs. in 2015

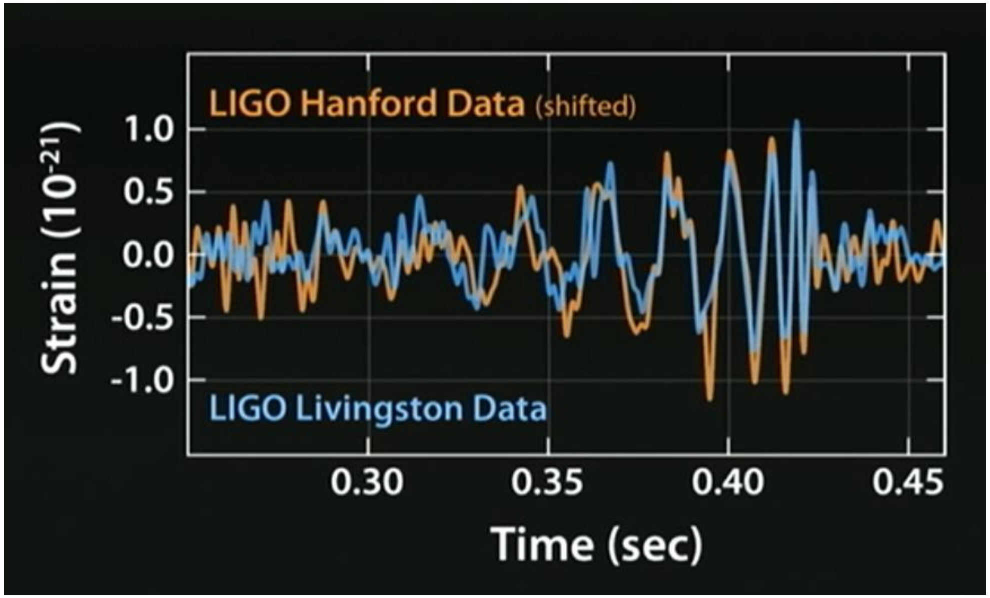
KAGRA

Kamioka, Gifu pref.

- 3km interferometer
- Underground
- Cryogenic mirrors
- Joined the observation run in 2020

Credit: ICRR

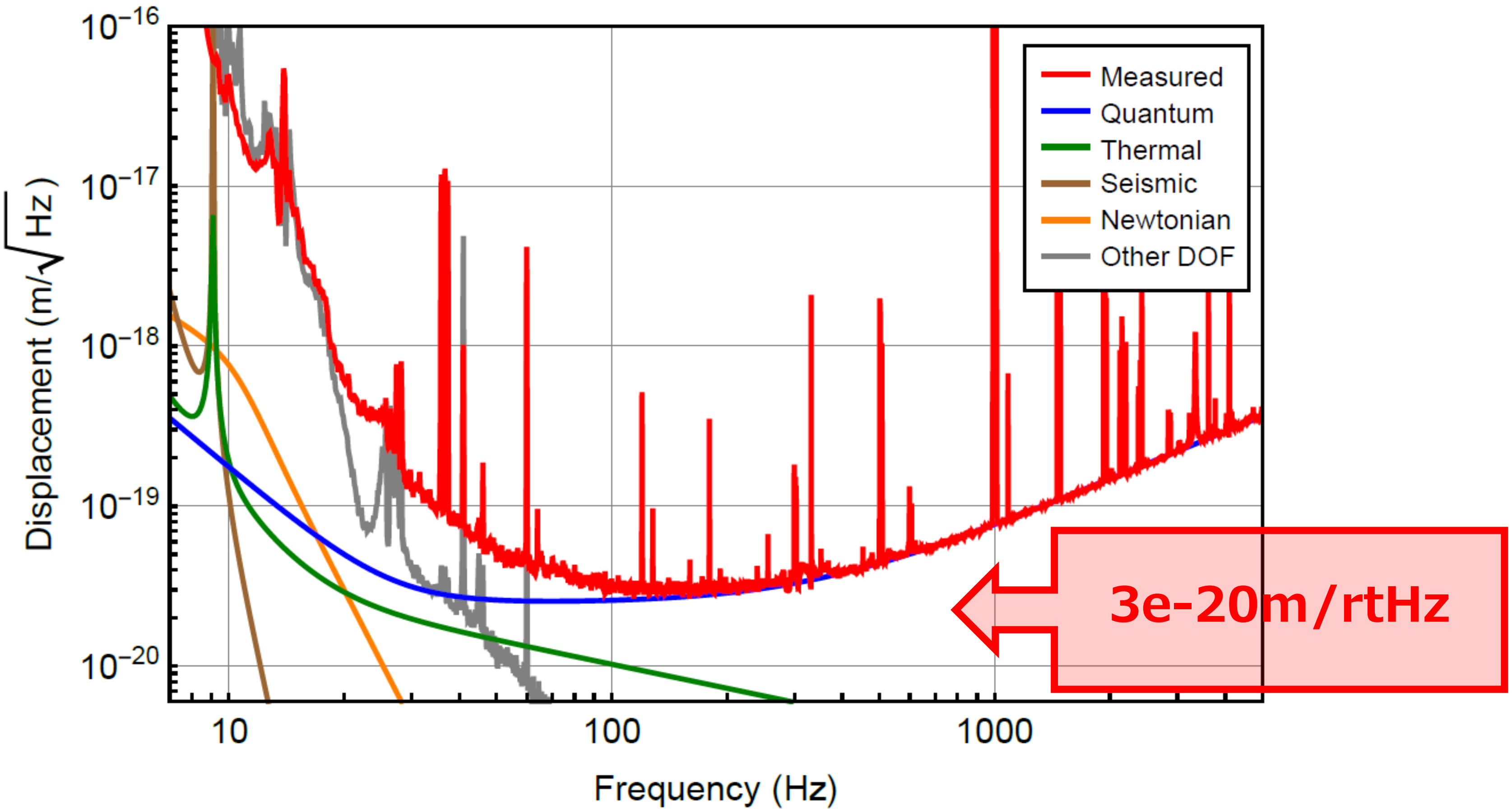
Advanced LIGO's first detection



- **Binary blackholes with 36 and 29 solar masses**
- **62 Ms BH was generated after the merger**
- **Two detectors observed it with a 10-ms time diff.**
- **Waveform matched to numerical relativity prediction**
- **The source is 1.3B light-years from Earth**
- **SNR was 24 and FAR was 1/200k-yrs or less**

(False Alarm Rate)

Sensitivity of Advanced LIGO in 2015



3e-20m/rtHz

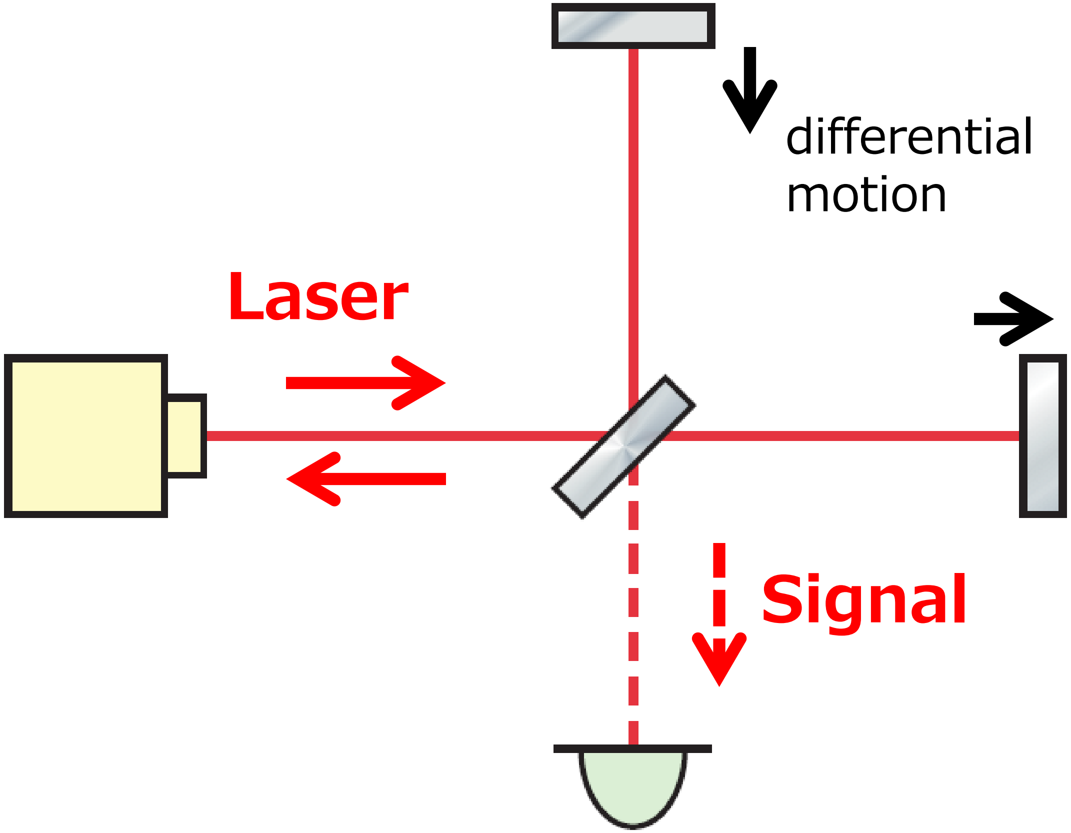
- **Mainly limited by quantum and control noises**
- **The sensitivity is x2 better now**

Contents of the talk

1. Gravitational waves
2. Quantum noise of light
3. Toward the standard quantum limit
4. Summary

Quantum noise

(IFO=Interferometer
GW=Gravitational Waves)



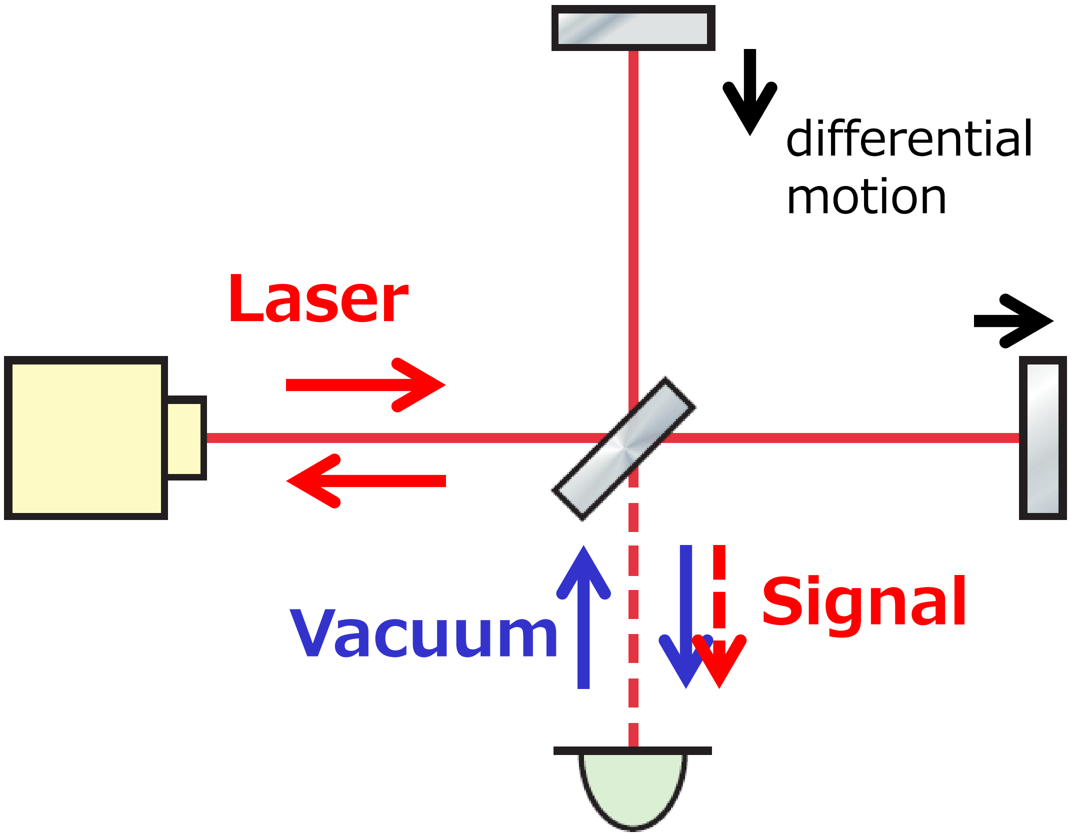
Operating Michelson IFO at dark fringe
(Light goes back to laser)

GW causes differential motion of the mirrors to send signal light to the dark port.

Laser fluctuation goes back to the laser.
⇒ What would be the noise source then?

Quantum noise

(QN=Quantum Noise
SNR=Signal-to-Noise Ratio)



Even without light, there exists vacuum

(zero mean, non-zero variance)

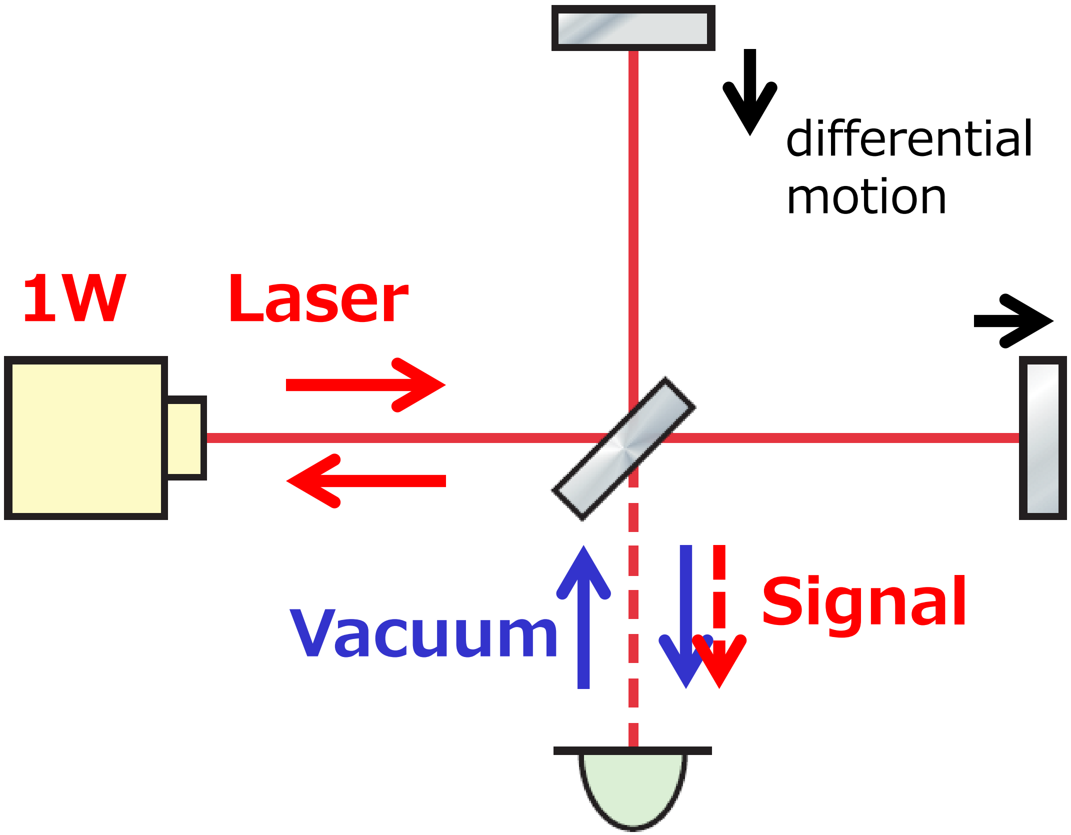
⇩

Vacuum field entering from dark port is the source of QN.

Vacuum fluctuation is equivalent to ½ photon
⇒ SNR is defined by the ratio to signal photons

Quantum noise

(IFO=Interferometer
GW=Gravitational Waves)



1W laser light contains N=5e18 photons/sec.

(λ is set to 1064nm)

⇩

As GW changes the path length by ΔL,

$$\Delta\mathrm{N} = \left(\sqrt{N}\,\frac{4\pi\Delta\mathrm{L}}{\lambda}\right)^2$$ **photons**

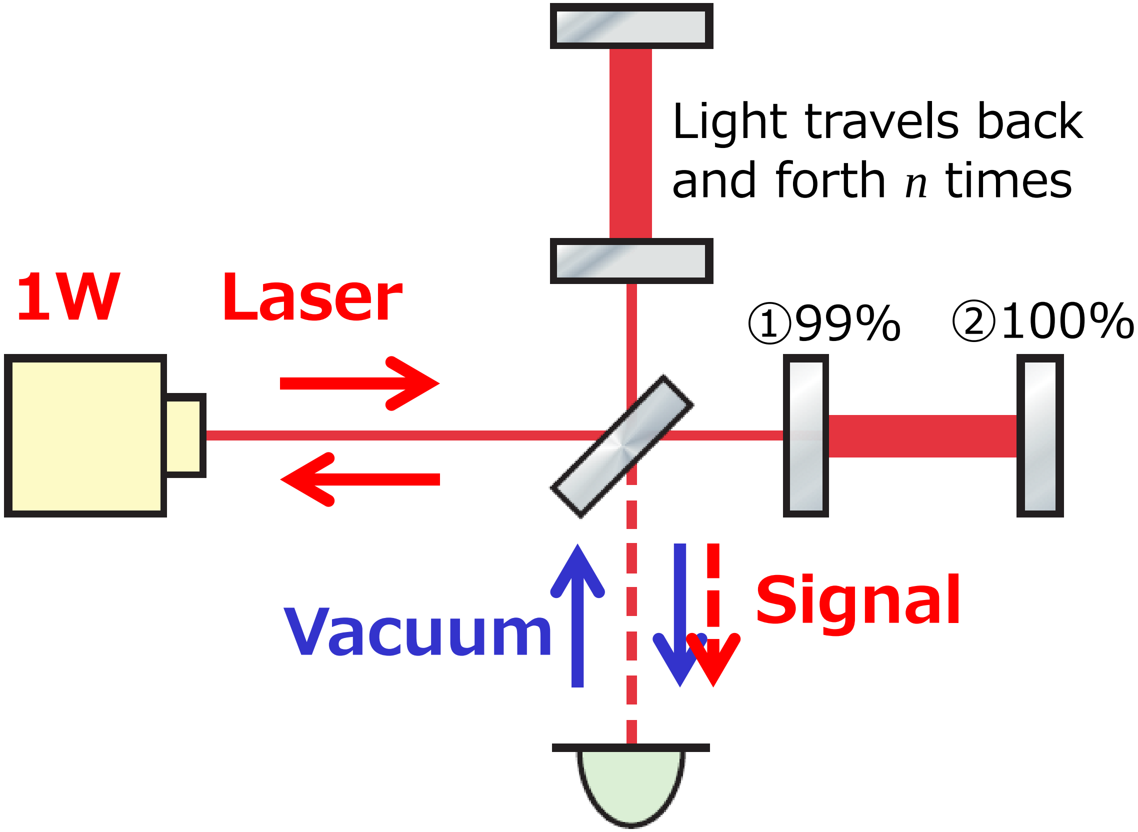
leak to the dark port

Sensitivity is given by solving $\Delta\mathrm{N} \sim 1/2$

⇒ For 1W IFO, it is ΔL=5e-17(m/rtHz)

Optical cavity

(IFO=Interferometer)



1W **Laser**

Light travels back and forth n times

①99% ②100%

Vacuum

Signal

With a cavity to circulate light 400 times, both $\sqrt{N}$ and ΔL increase by 400

⇓

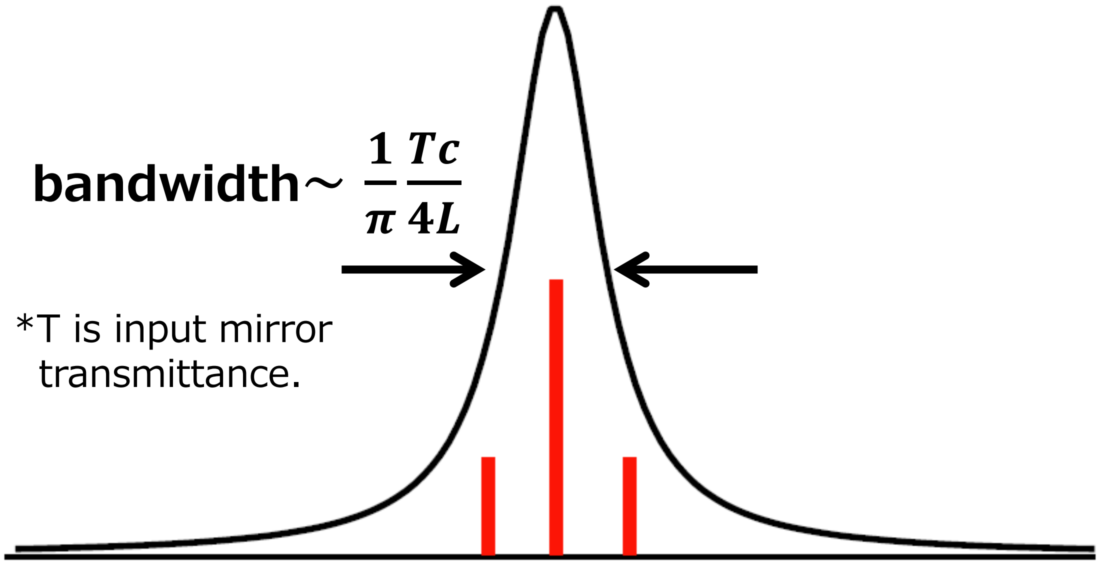
$$\Delta N = \left(400^2 \times \sqrt{N}\,\frac{4\pi\Delta L}{\lambda}\right)^2$$

Sensitivity is given by solving $\Delta N \sim 1/2$

⇒ For 1W IFO, it is ΔL=1e-19(m/rtHz)

Optical cavity

(BW=Bandwidth)



bandwidth$\sim \frac{1}{\pi}\frac{Tc}{4L}$

*T is input mirror transmittance.

Signal outside bandwidth will not increase in cavity.

⇩

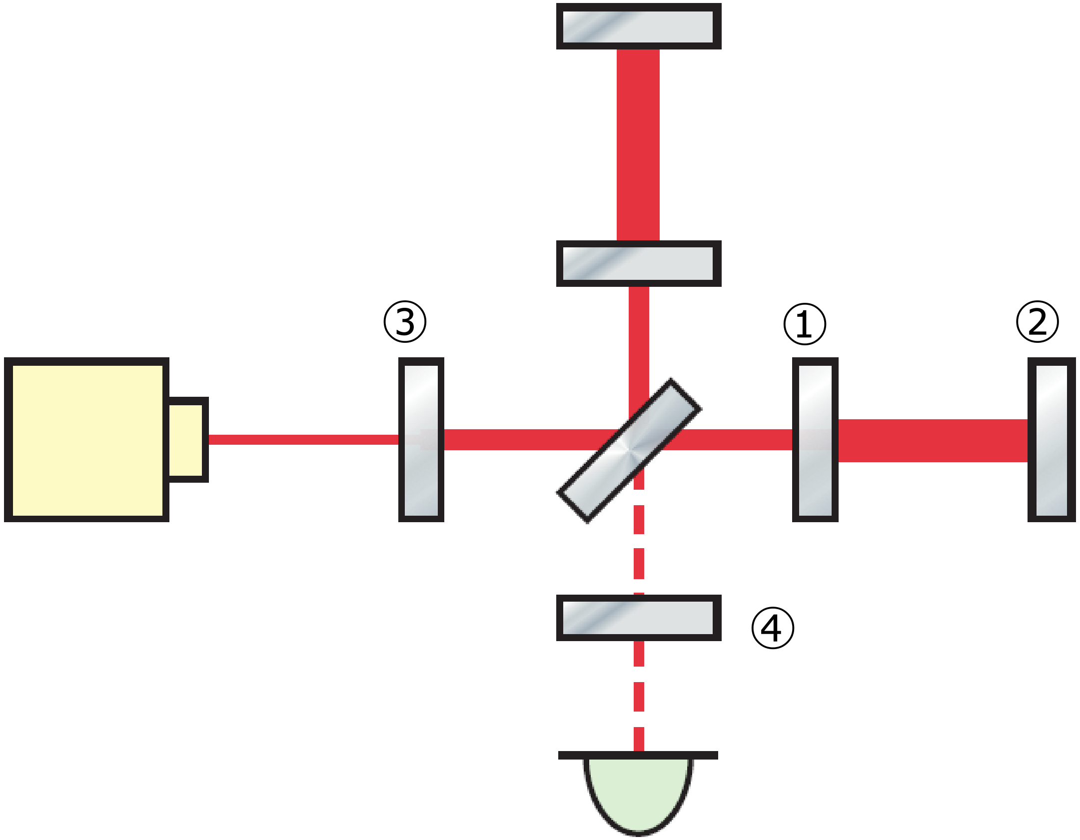
With 99% input mirror and L=4km, BW is~30Hz.

We like to have more light in the cavity but we do not want to decrease the BW

⇒ A coupled cavity

Coupled cavity

(BW=Bandwidth
BS=Beam Splitter)



**Coupled cavity w/①②③ determines the power.
Coupled cavity w/①②④ determines the BW.**

**"Power-recycled
Resonant-sideband-extraction"**

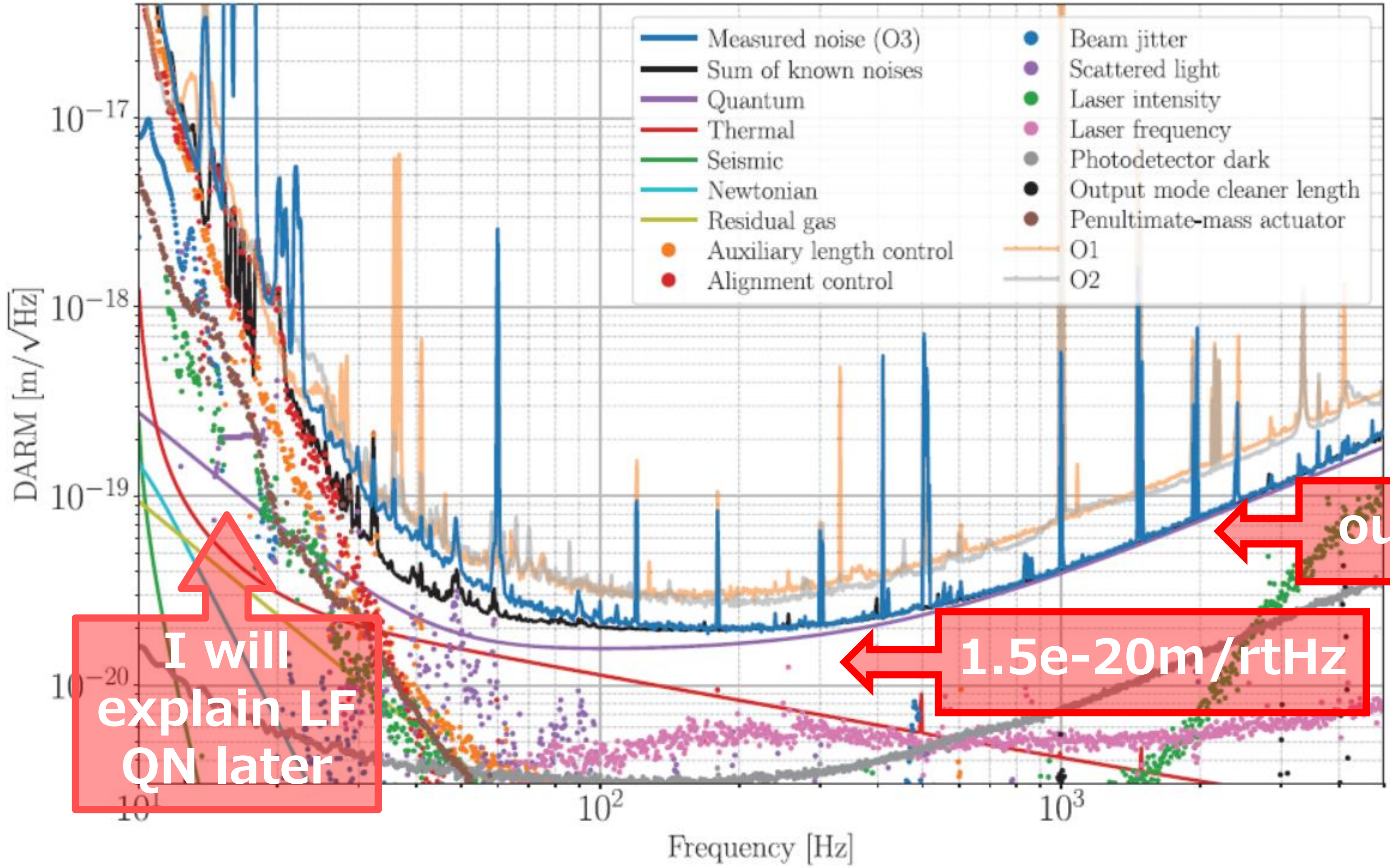
Both Advanced LIGO & KAGRA use this system.

Currently, Advanced LIGO uses ~1.5kW at BS and sensitivity reaches ΔL=2e-20(m/rtHz).

Quantum noise of LIGO in 2020

(BW=Bandwidth
LF=Low Frequencty
QN=Quantum Noise)

[LIGO, PRD 2020]



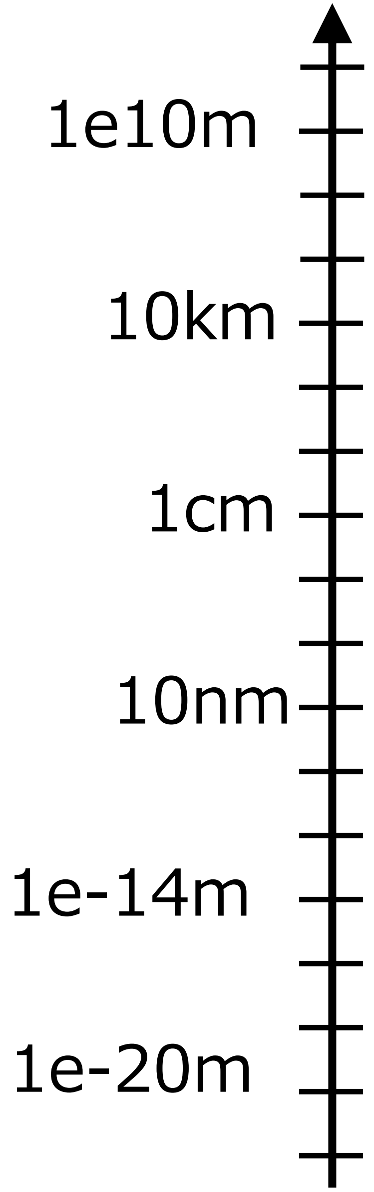
outside BW

1.5e-20m/rtHz

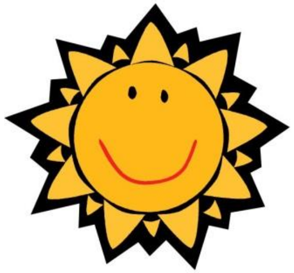
I will explain LF QN later

A 3dB squeezing was injected to effectively double the arm power (to be explained later).

How small are we going to measure?



Sun-Earth distance
(150,000,000km)



ratio
= 10^-21

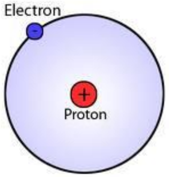
GW detector baseline (3~4km)

human

(2.09m)

strain
= 10^-24

Atom (1A)

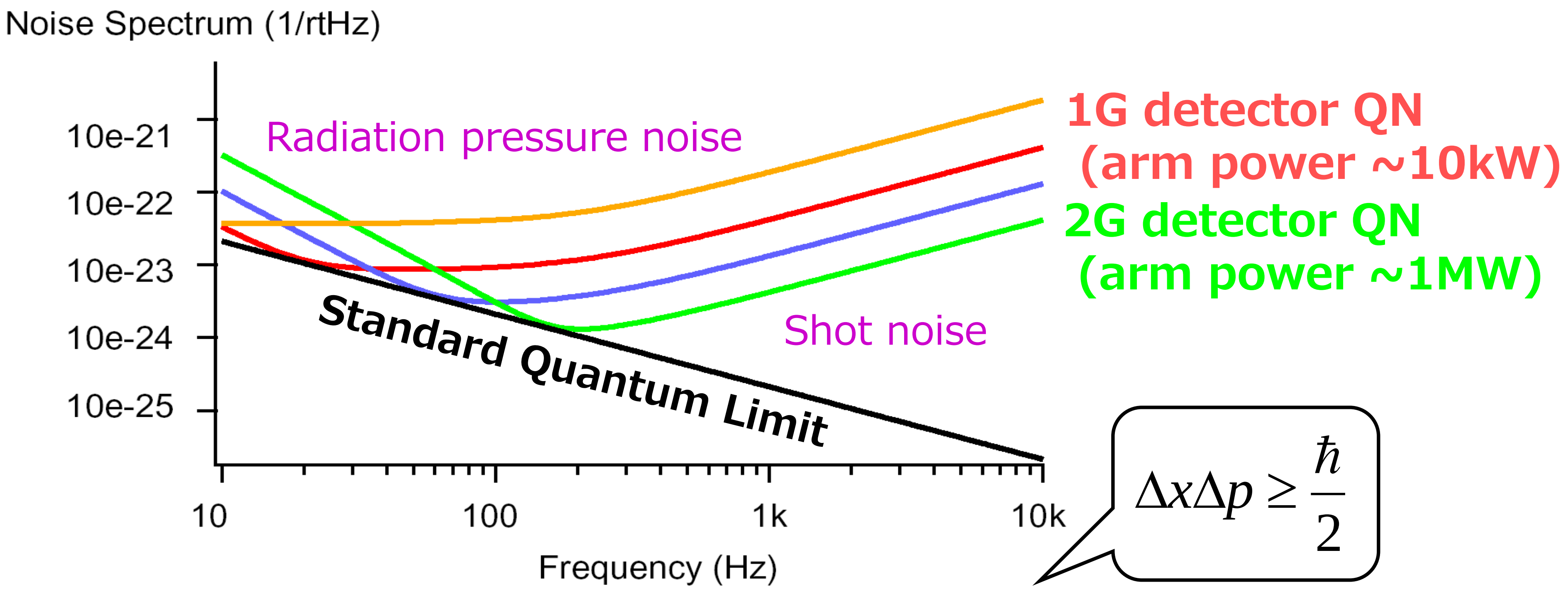


1G sensitivity (1e-19m)

Quantum limit (100Hz;40kg)

2G sensitivity goal

Quantum noise in GW detector



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Uncertainty Principle

High precision → **Back action**

||

Shot noise reduction w/high power laser

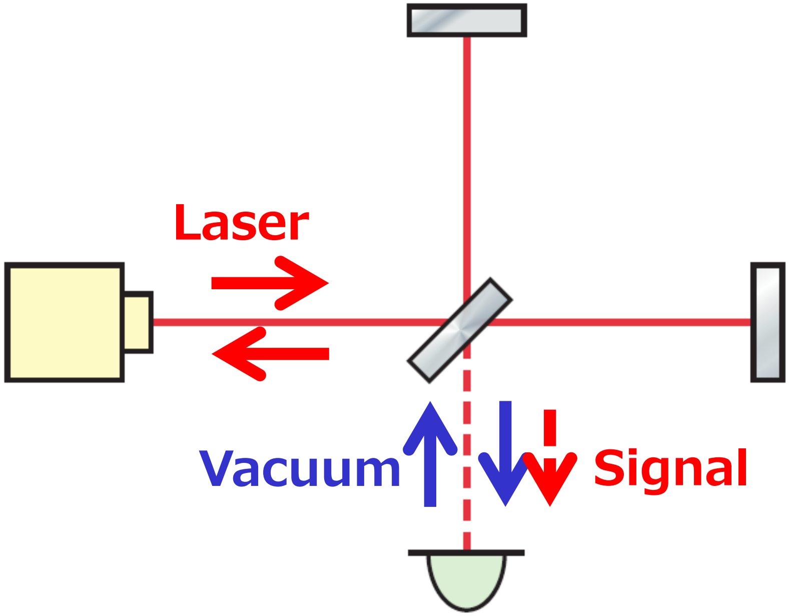
||

Radiation pressure

There is a limit that cannot be exceeded by simply increasing laser power.

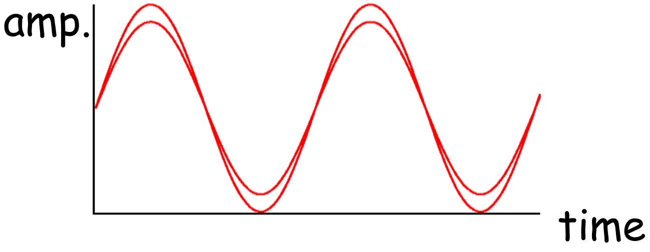
Source of quantum noise

(SQL=Standard
Quantum Limit)



Vacuum field consists of amplitude and phase fluctuations.

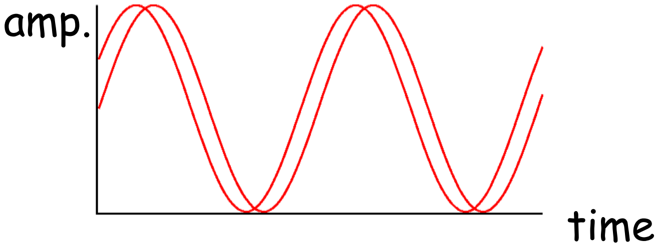
SQL derives from a commutator of these two components.



photon number fluctuation

↓

radiation pressure noise



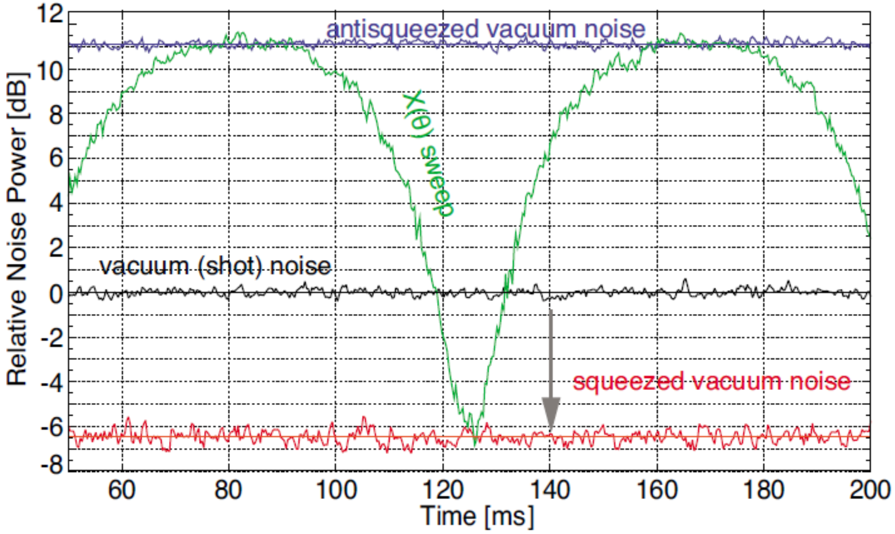
phase fluctuation

↓

shot noise

Optical squeezing

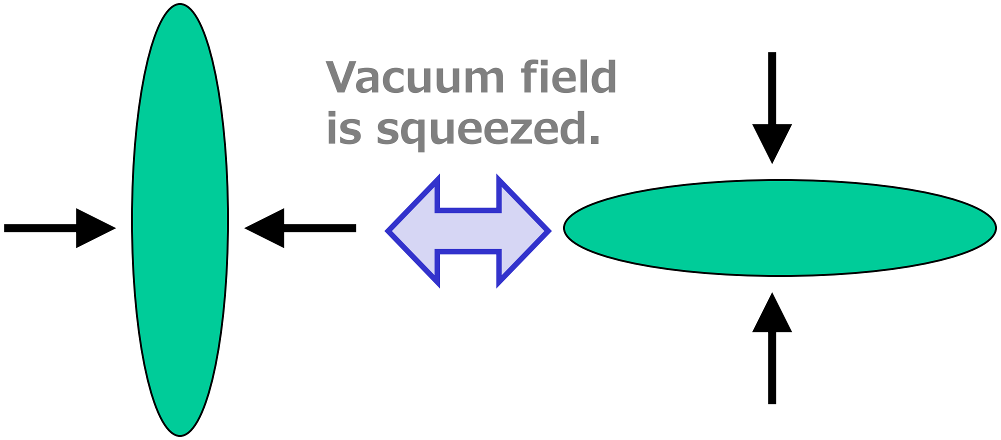
(RP=Radiation Pressure
LF=Low Frequency
HF=High Frequency)



[Mehmet, Opt.Lett. (2010)]

7dB squeezing

Optical parametric amplification process creates a correlation in upper and lower sidebands.



Vacuum field is squeezed.

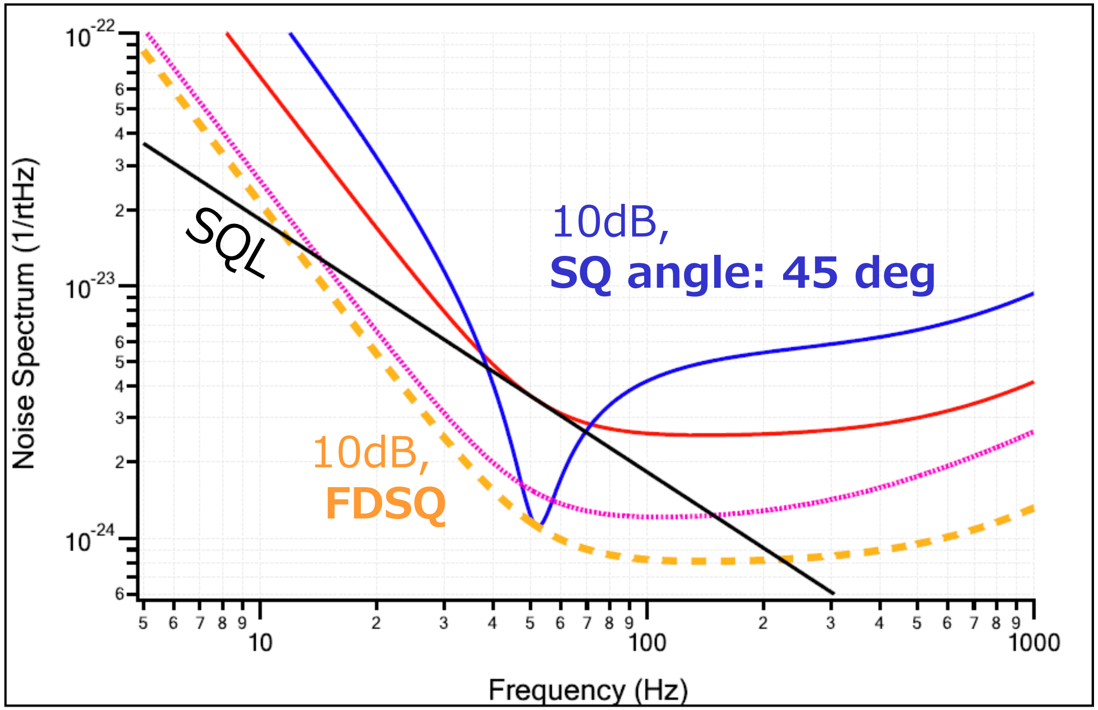
shot noise is low

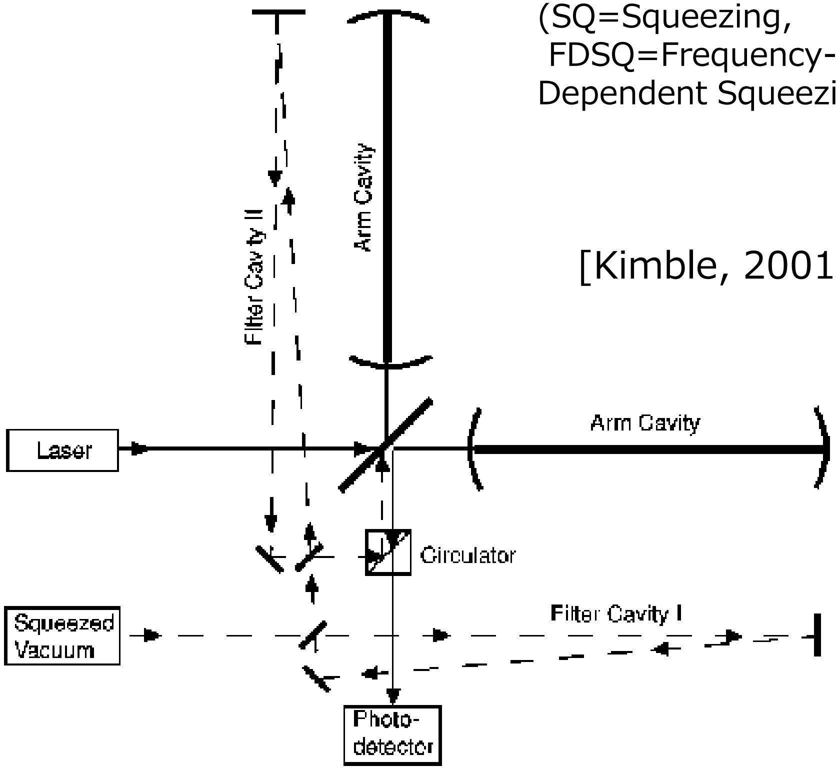
RP noise is low

Sensitivity improves either at LF or HF according to the choise of the squeezing angle.

Frequency-dependent squeezing

orage: lossless, pink: with loss





(SQ=Squeezing, FDSQ=Frequency-Dependent Squeezing)

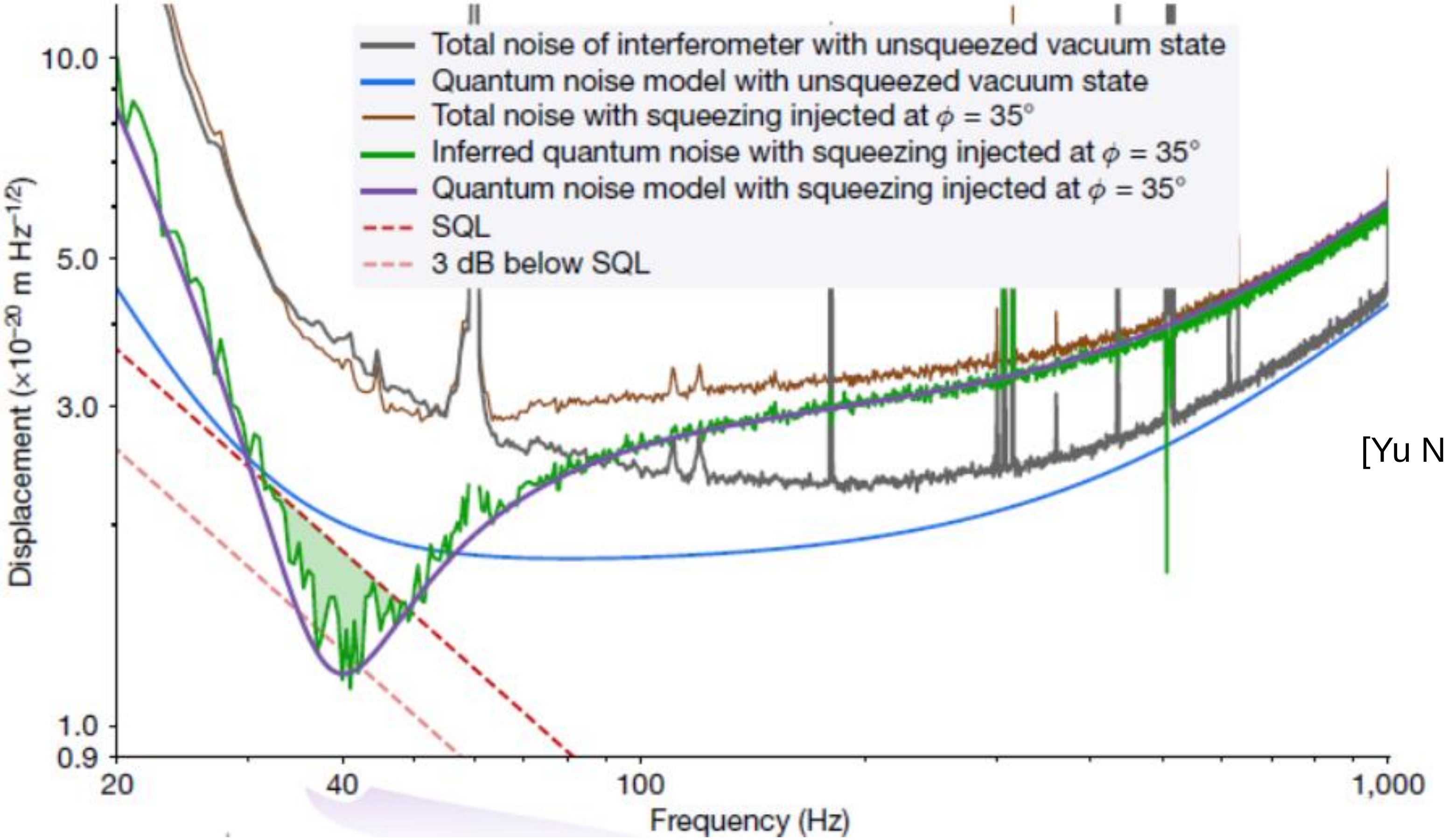
[Kimble, 2001]

- **SQ angle is rotated in filter cavities**
- **Rotation angle depends on the frequency**

This technique has been installed in LIGO & Virgo.

Toward the SQL

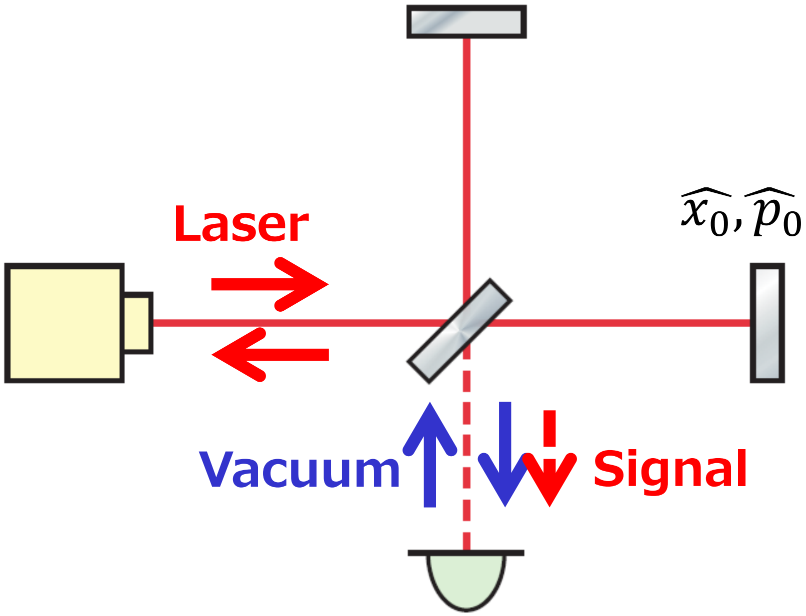
(QN=Quantum Noise
SQL=Standard Quantum Limit)



[Yu Nature 2020]

LIGO demonstrated their QN exceeds the SQL by 3dB with a post-processing removal of classical noise.

How come the sensitivity can go beyond the SQL?



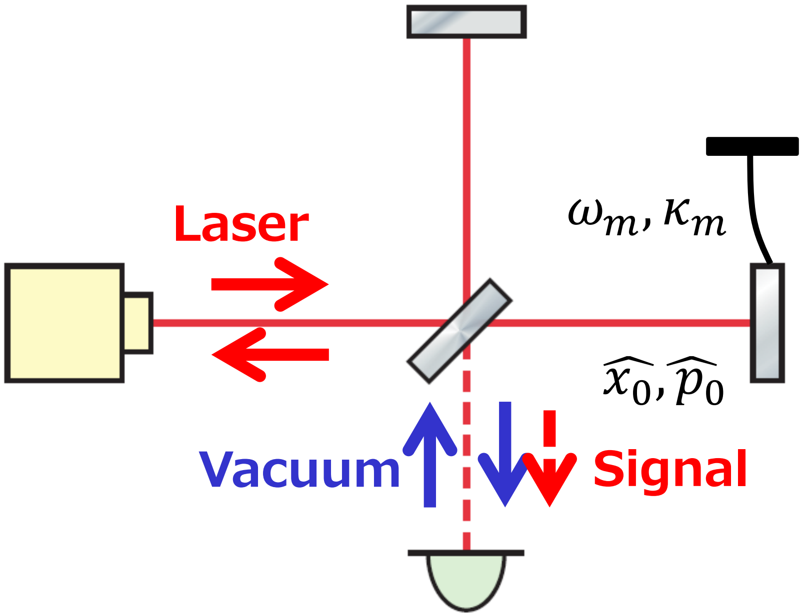
Brangisky's explanation in his paper, [PRD 67, 082001 (2003)] **is as follows:**

"GWD measures the external force on the mirror. Its initial position $\widehat{x_0}$ is not measured and remains quantum."

The mirror fluctuates with back action noise but one can measure the external force without seeing the fluctuation of the test mass.

In other words, the output field y(t) commutes at different times: $[y(t), y(t')] = 0$.

How come the sensitivity can go beyond the SQL?



Khalili's updated theory in his paper, [PRA 86, 033840 (2012)] **is as follows:**

"Oscillator's initial fluctuation $\hat{x}_0\cos\omega_m t$ dissipates in time and a thermal field enters. If T is low, the zero-point fluctuation of the thermal field takes over the initial quantum fluctuation."

$$\hat{x}_q(t) = e^{-\frac{\kappa_m t}{2}}\left[\hat{x}_0\cos\omega_m t + \frac{\hat{p}_0}{m\omega_m}\sin\omega_m t\right] + \int_{-\infty}^{t} e^{-\frac{\kappa_m(t-t')}{2}}\frac{\sin\omega_m(t-t')}{m\omega_m}F(t')\,dt'$$

$$\hat{y}(t) = \underbrace{\hat{z}(t)}_{\text{shot noise}} + \underbrace{\hat{x}_{BA}(t)}_{\text{RPN}} + \underbrace{\hat{x}_q(t)}_{\text{initial position}}$$

Either $z + x_{BA}$ or x_q does not commute at different times, but commutators cancel and y does commute at different times.

Zero-point fluctuation

[Gardiner and Zoller]

Thermal noise of a resister with resistance R:

$$\boldsymbol{S(\omega) = \frac{1}{\pi} k_B T R}$$

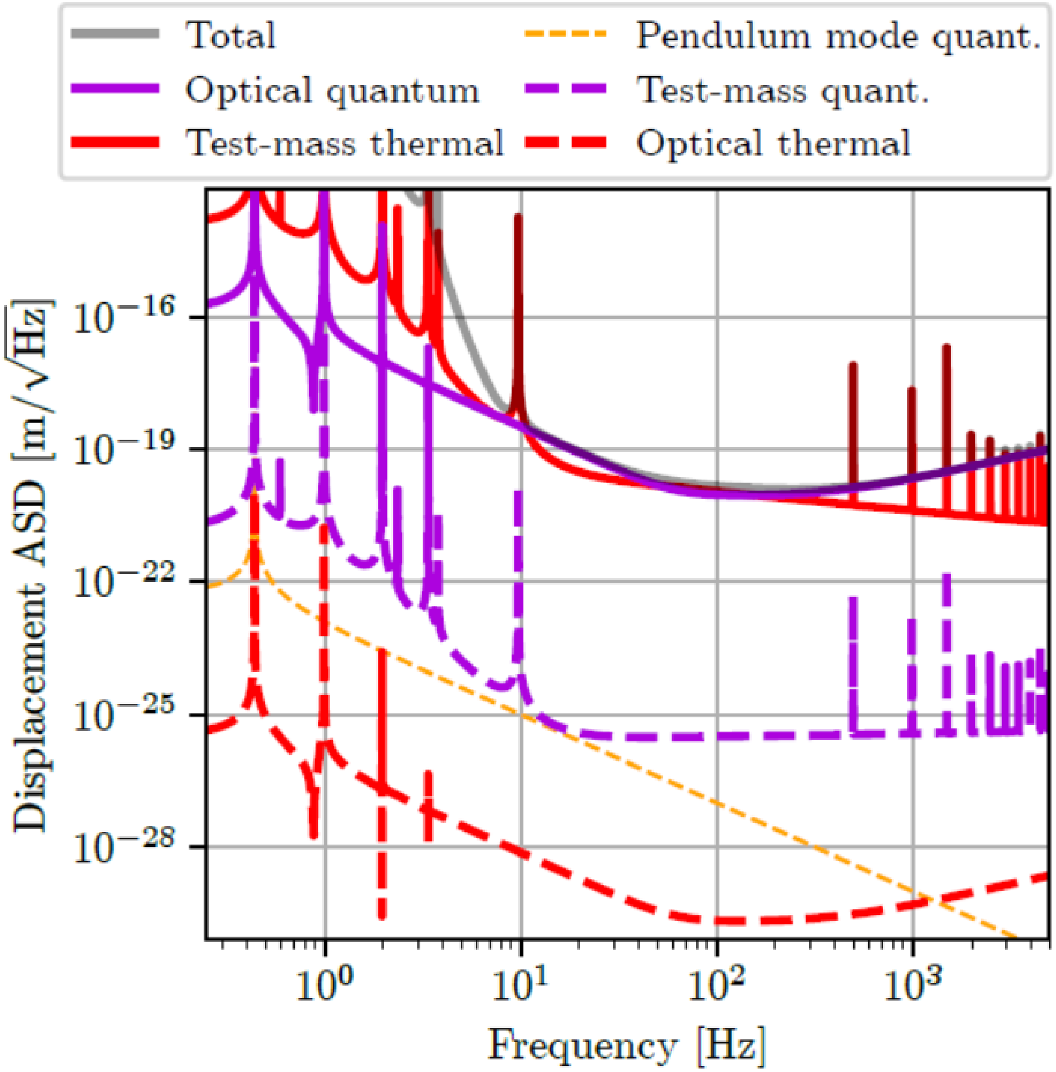
Rewrite it with Planck’s spectrum:

$$\boldsymbol{S(\omega) = \frac{1}{\pi} \hbar \omega R \frac{1}{exp[\hbar\omega / k_B T] - 1}}$$

Adding the zero-point fluctuation:

$$\boldsymbol{S(\omega) = \frac{R}{\pi} \left(\frac{1}{2} \hbar \omega + \frac{\hbar \omega}{exp[\hbar\omega / k_B T] - 1} \right)}$$

Zero-point fluctuation in GWD

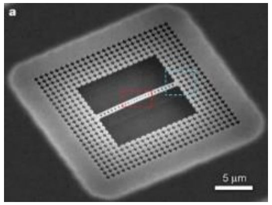


[Whittle, arXiv 2023]

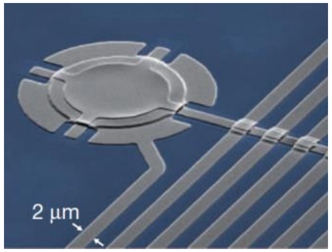
Whittle et al. demonstrated to calculate test mass quantum fluctuation in Advanced LIGO, which is very low compared with quantum noise of light.

(dashed curves)
orange: pendulum mode only
purple: all mechanical modes
red: thermal fluctuation of photons

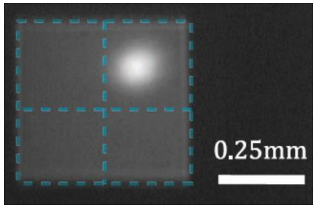
Macroscopic QM on various mass scales



331fg nanobeam



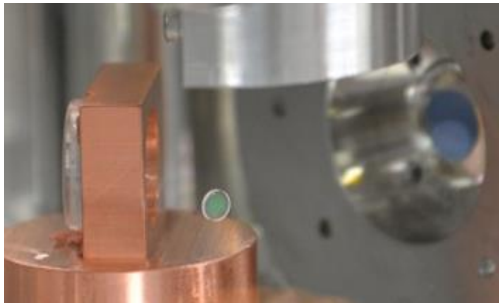
48pg membrane



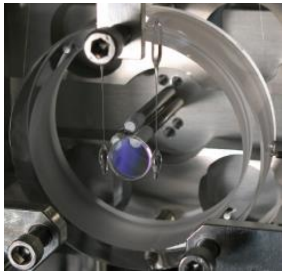
7ng membrane

50ng cantilever

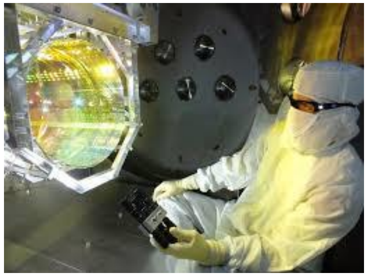
Planck mass
(~22µg)



7mg pendulum



1g pendulum



LIGO 40kg mass

Some microresonators have reached the SQL (not the zero-point fluctuation). None above the Planck mass has reached the SQL.

Reference

331fg nanobeam [Chan, Nature (2011)], 48pg membrane [Teufel, Nature (2011)], 7ng membrane [Peterson, PRL (2016)], 50ng cantilever [Cripe, Nature (2019)], 7mg pendulum [Matsumoto, PRL (2019)], 1g pendulum [Neben, NJP (2012)]

Summary

(SQL=Standard Quantum Limit
GW=Gravitational Wave
QM=Quantum Measurement)

- **Quantum noise in GW detector consists of quantum fluctuation of light.**
- **SQL can be surpassed with squeezing.**
- **Sensitivity of GW detector is quite close to exceed the SQL.**
- **Test mass quantum noise is a few orders below the quantum noise of light.**
- **Macroscopic QM experiments are going on in various mass scale.**