

# Classical Pendulum Feels Quantum Back-Action

University of Tokyo  
Nobuyuki Matsumoto

GWADW2014

# Outline

- Introduction of our recent research

arXiv:1312.5031v1 [quant-ph] 18 Dec 2013

## Classical Pendulum Feels Quantum Back-Action

Nobuyuki Matsumoto<sup>1,\*</sup>, Yuta Michimura<sup>1</sup>, Gen Hayase<sup>2</sup>, Yoichi Aso<sup>1</sup>,  
and Kimio Tsubono<sup>1</sup>

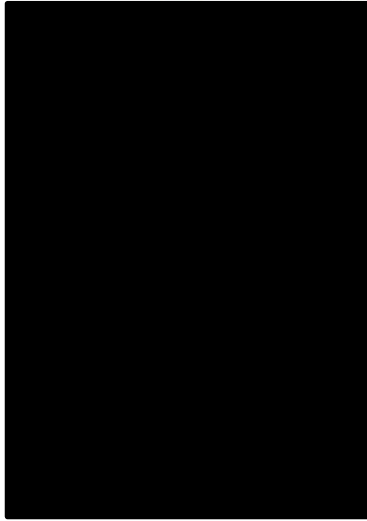
<sup>1</sup>*Department of Physics, Graduate School of Science,  
The University of Tokyo, Bunkyo, Tokyo 113-0033, Japan*

<sup>2</sup>*Department of Chemistry, Graduate School of Science,  
Kyoto University, Sakyo, Kyoto 606-8502, Japan*

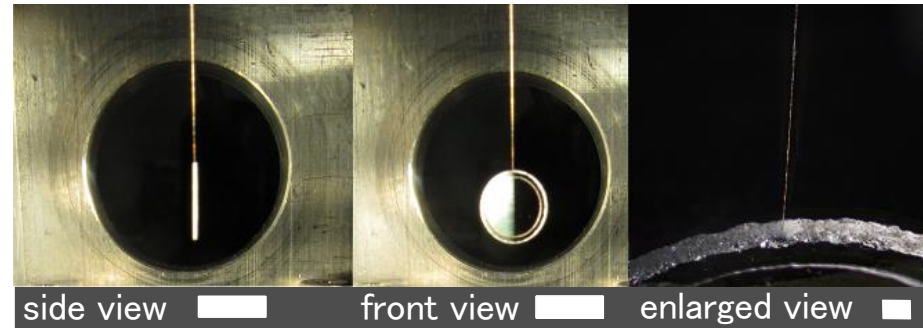
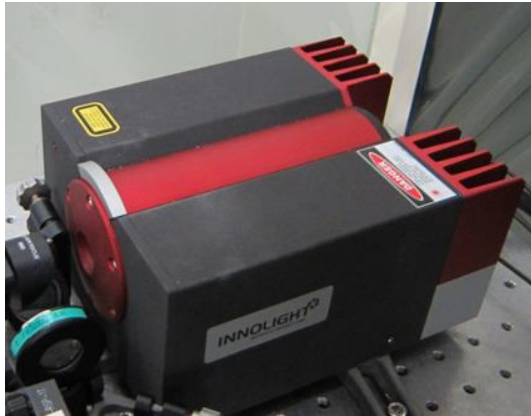
- Advantages of the triangular optical cavity  
(suppl. info. A.1)  $\Rightarrow$  [Opt. Express 22, 12915 \(2014\)](#)
- Observation of Quantum Back-Action  
(RPN: Radiation Pressure Noise)

# What is Quantum Back-Action?

Image  
(classical case)



Our case



Vacuum **fluctuation** of the coherent light sways the mirror

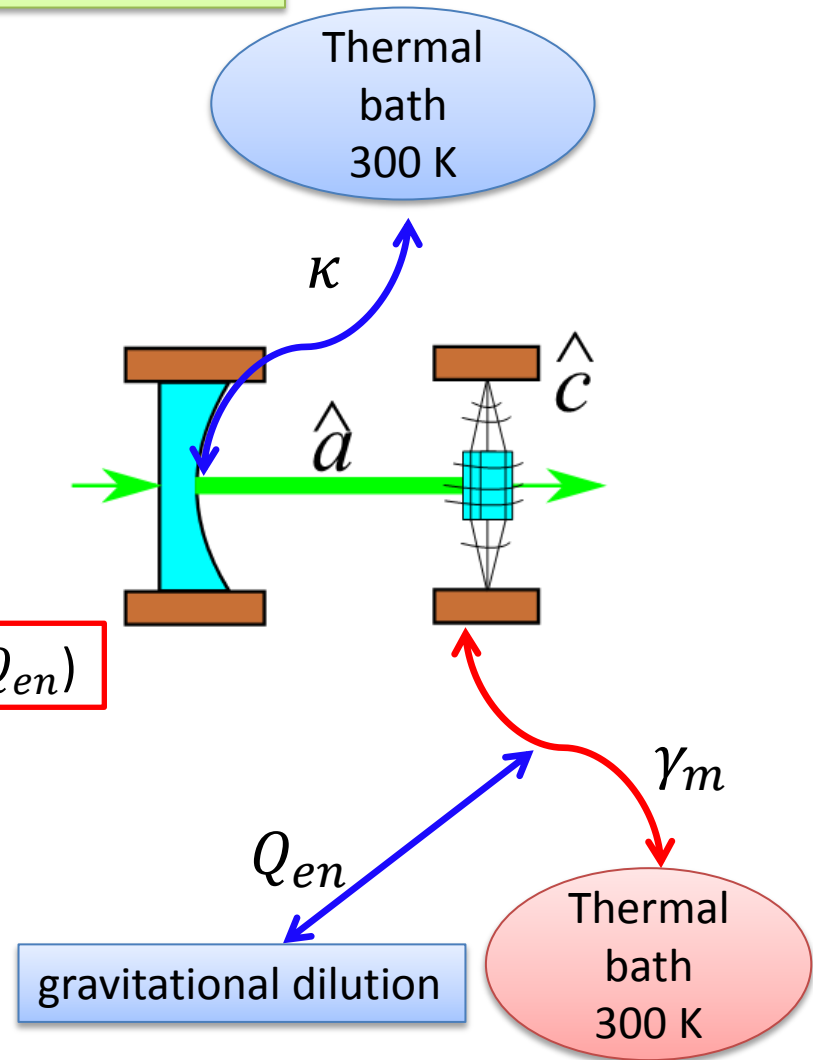
# Requirement for reaching the SQL

Quantum back-action/thermal fluctuating force > 1

$$R_s = \frac{S_{FF,q}^{(2)}}{S_{FF,th}^{(2)}} = \frac{|G_{opt}|^2}{(n_{th} \kappa \gamma_m)} > 1,$$

Increase of intra-cavity power

Decrease of mechanical decay ( $Q_{en}$ )



- $\hat{a}$ : photon annihilation operator
- $\hat{c}$ : phonon annihilation operator
- $\kappa$ : decay rate of the optical cavity
- $\gamma_m$ : decay rate of the pendulum
- $Q_{en}$ : Q enhancement factor due to the dilution

# Requirement for reaching the SQL

Quantum back-action/thermal fluctuating force > 1

$$R_s = \frac{S_{FF,q}^{(2)}}{S_{FF,th}^{(2)}} = \frac{|G_{opt}|^2}{(n_{th} \kappa \gamma_m)} > 1,$$

Increase of intra-cavity power

$$Q_{en} < \sqrt{\frac{G}{Y} 8lmg \frac{1}{\tau P_{circ}}} = \sqrt{\frac{2lmg}{(1+\sigma)} \frac{\kappa}{P_{in}}}$$

Trade-off

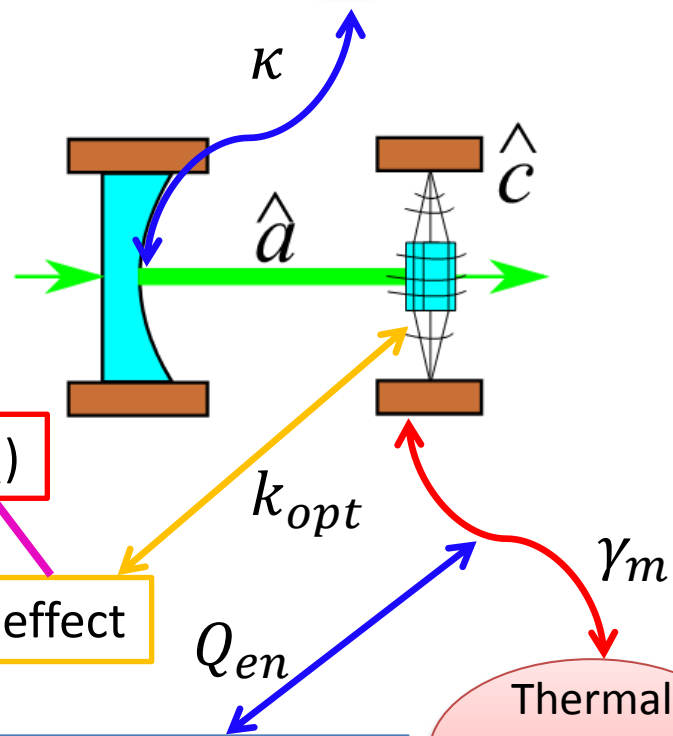
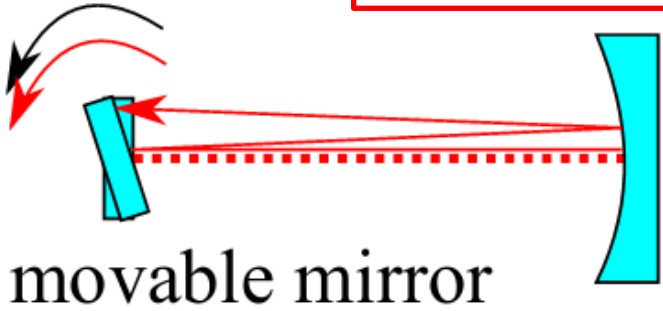
Decrease of mechanical decay ( $Q_{en}$ )

Anti torsional spring effect

gravitational dilution

Thermal bath 300 K

Thermal bath 300 K



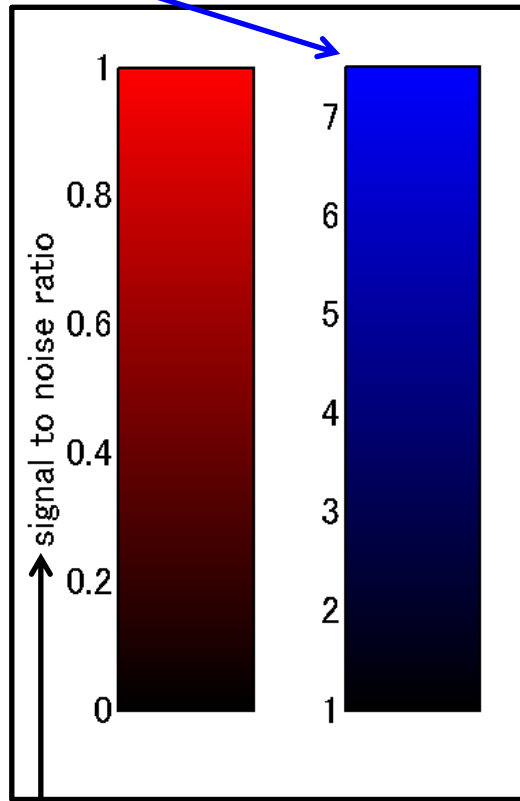
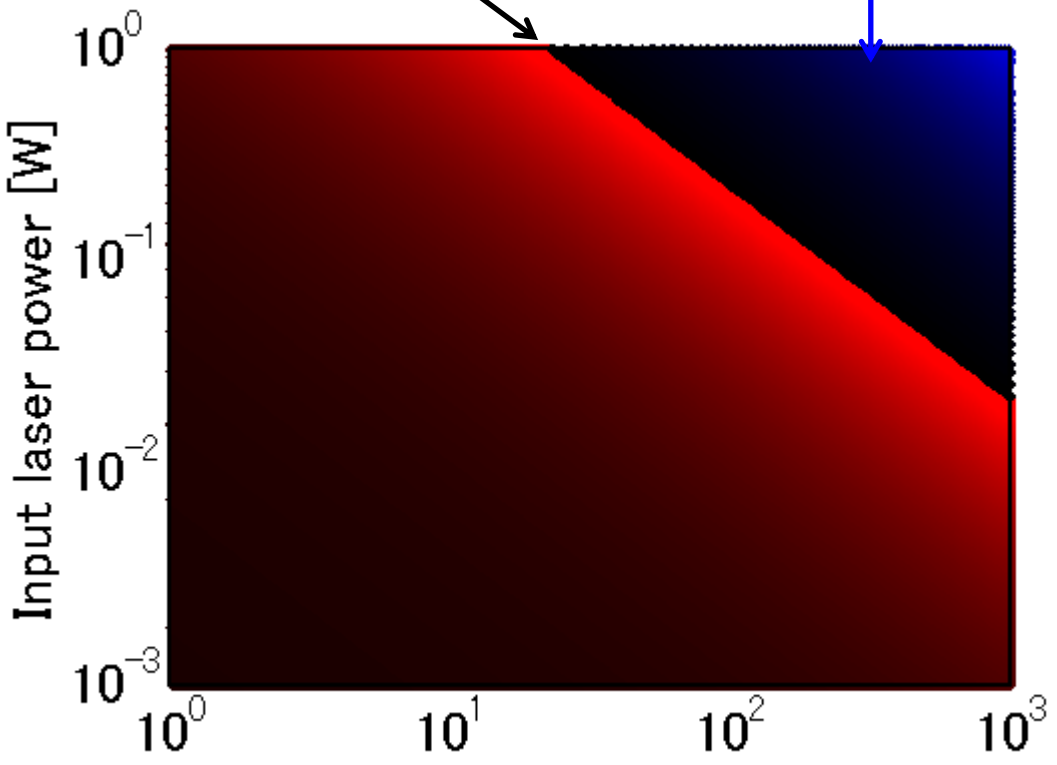
trade-off is **not** considered

# signal to noise ratio of RPN to TN

Back-action=Thermal noise

SQL can be reached

Increasing Back-action force



Finesse~1000  
Mass~5 mg  
Intrinsic Q~1000

Decreasing Thermal fluctuating force

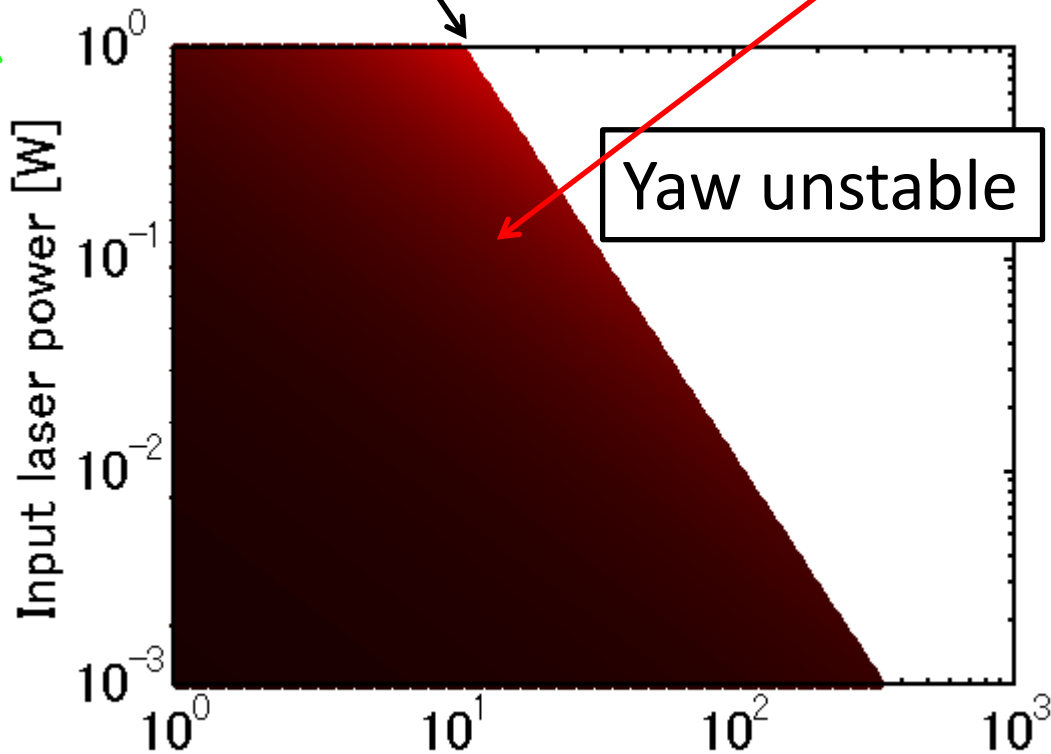
Back-action/thermal noise

trade-off is considered

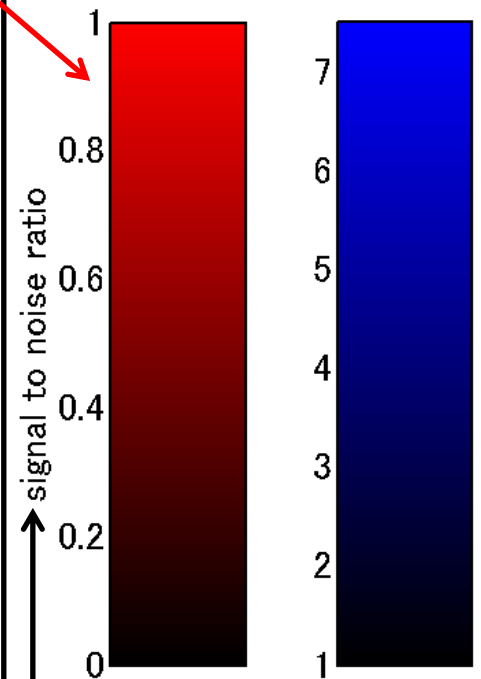
$$Q_{\text{en}} < \sqrt{\frac{G}{Y} 8lmg \frac{1}{\tau P_{\text{circ}}}} = \sqrt{\frac{2lmg}{(1+\sigma)} \frac{\kappa}{P_{\text{in}}}}$$

$k_{\text{wire}} = k_{\text{opt}}$ , for yaw motion

SQL can **not** be reached



Increasing  
Back-action  
force



Finesse~1000  
Mass~5 mg  
Intrinsic Q~1000

Q enhancement factor

Decreasing  
Thermal fluctuating force

Back-action/thermal noise

# Optically trapped mirror for reaching the standard quantum limit

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and Kimio Tsubono**

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[Opt. Express 22, 12915 \(2014\)](#)

- Our strategy for reaching the **SQL**

Key feature:

Optical **positive** torsional spring effect  
(triangular optical cavity)

⇒ Large gravitational dilution under high circulating power

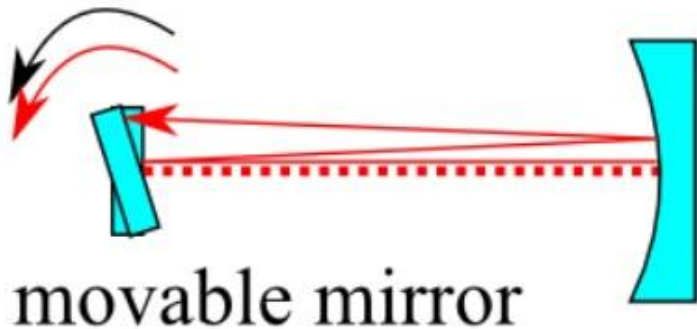


# The difference between linear and triangular cavities

Anti-restoring force

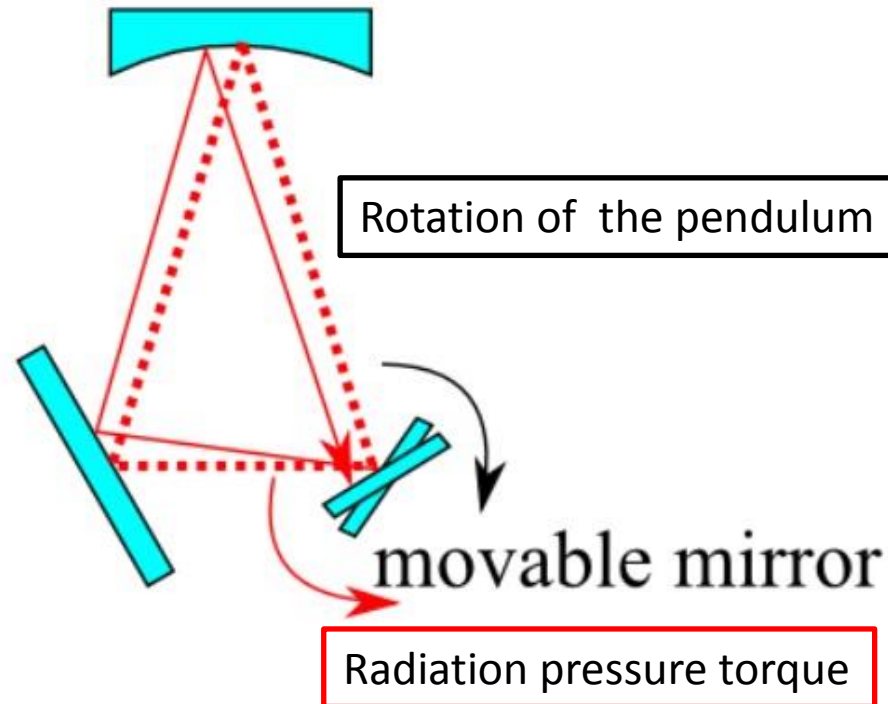
Rotation of the pendulum

Radiation pressure torque



Restoring force?

Rotation of the pendulum



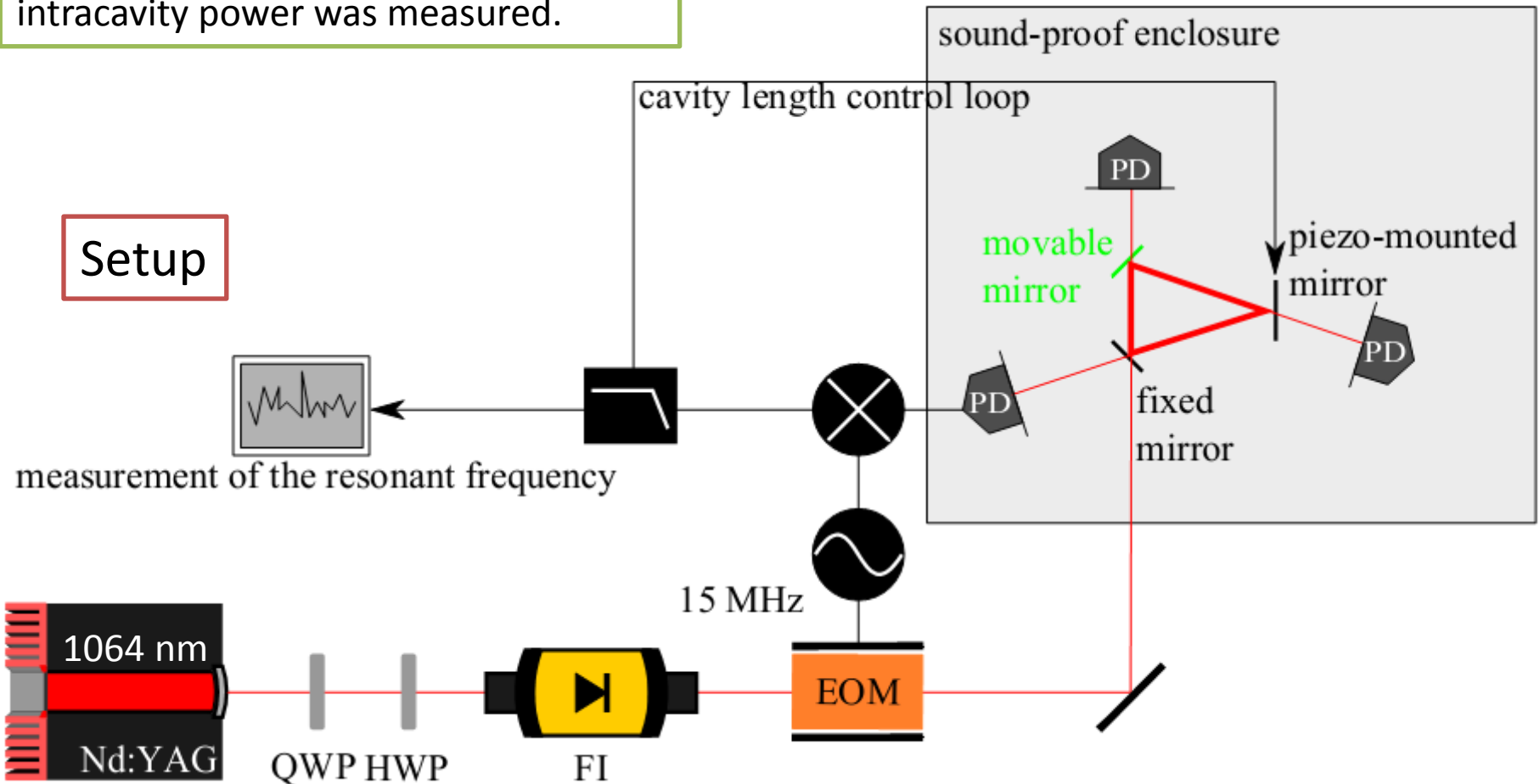
..... aligned

— misaligned

# Measurement of an optical torsional spring in a triangular cavity

The change of the resonant frequency of the yaw motion with increasing the intracavity power was measured.

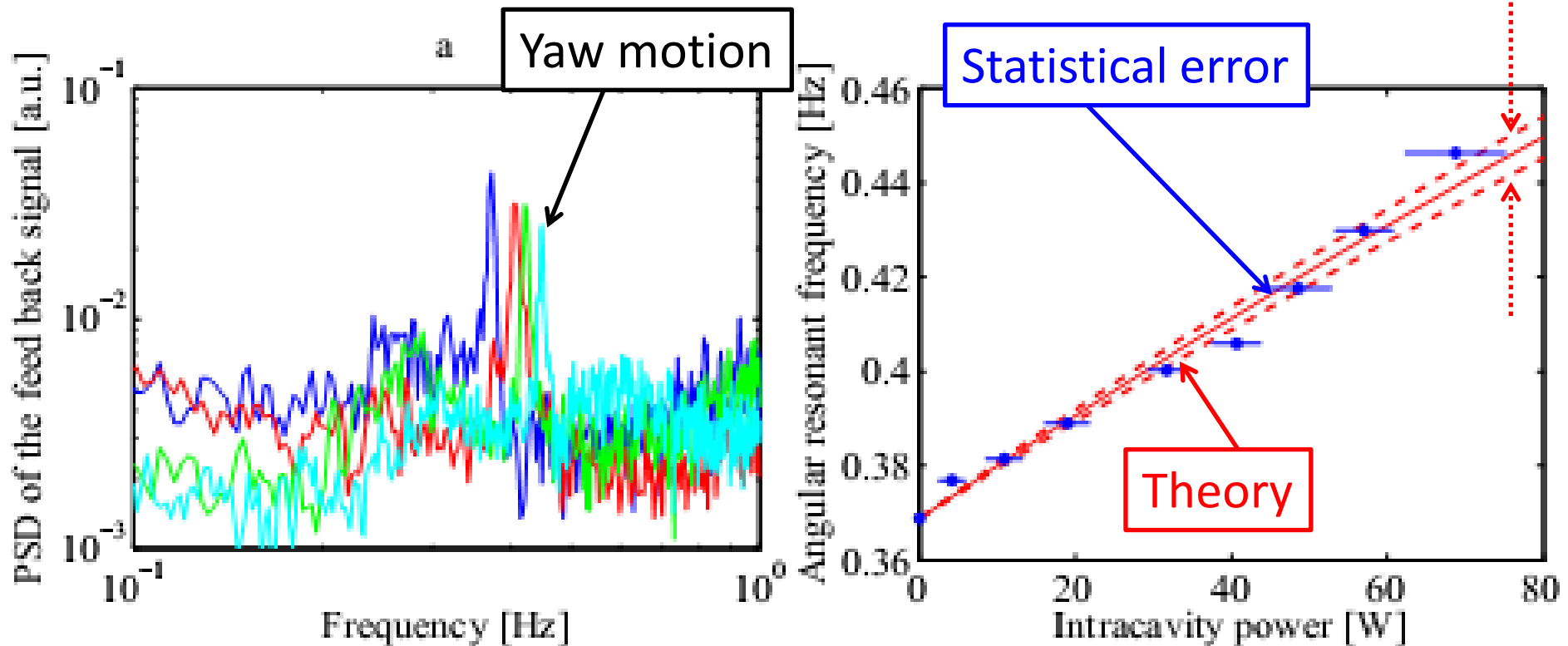
Setup



# Measurement of an optical torsional spring in a triangular cavity

- The change of the resonant frequency (yaw motion) with increasing the intracavity power

⇒ **Positive** torsional spring effect



# The second half

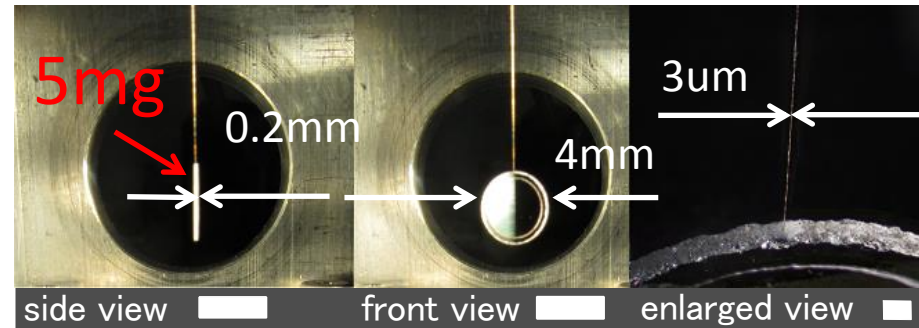
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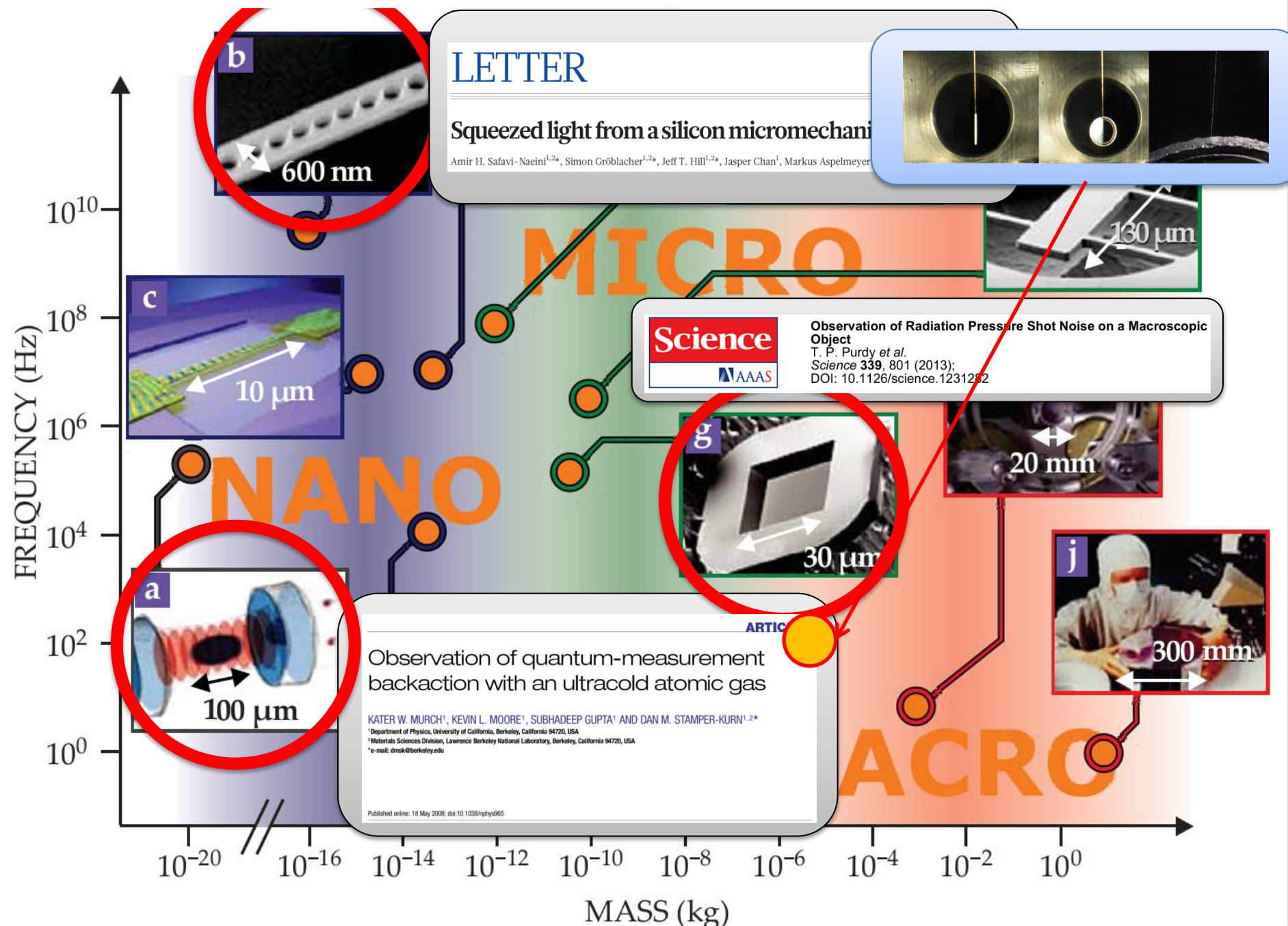
<sup>2</sup>*Department of Chemistry, Graduate School of Science,  
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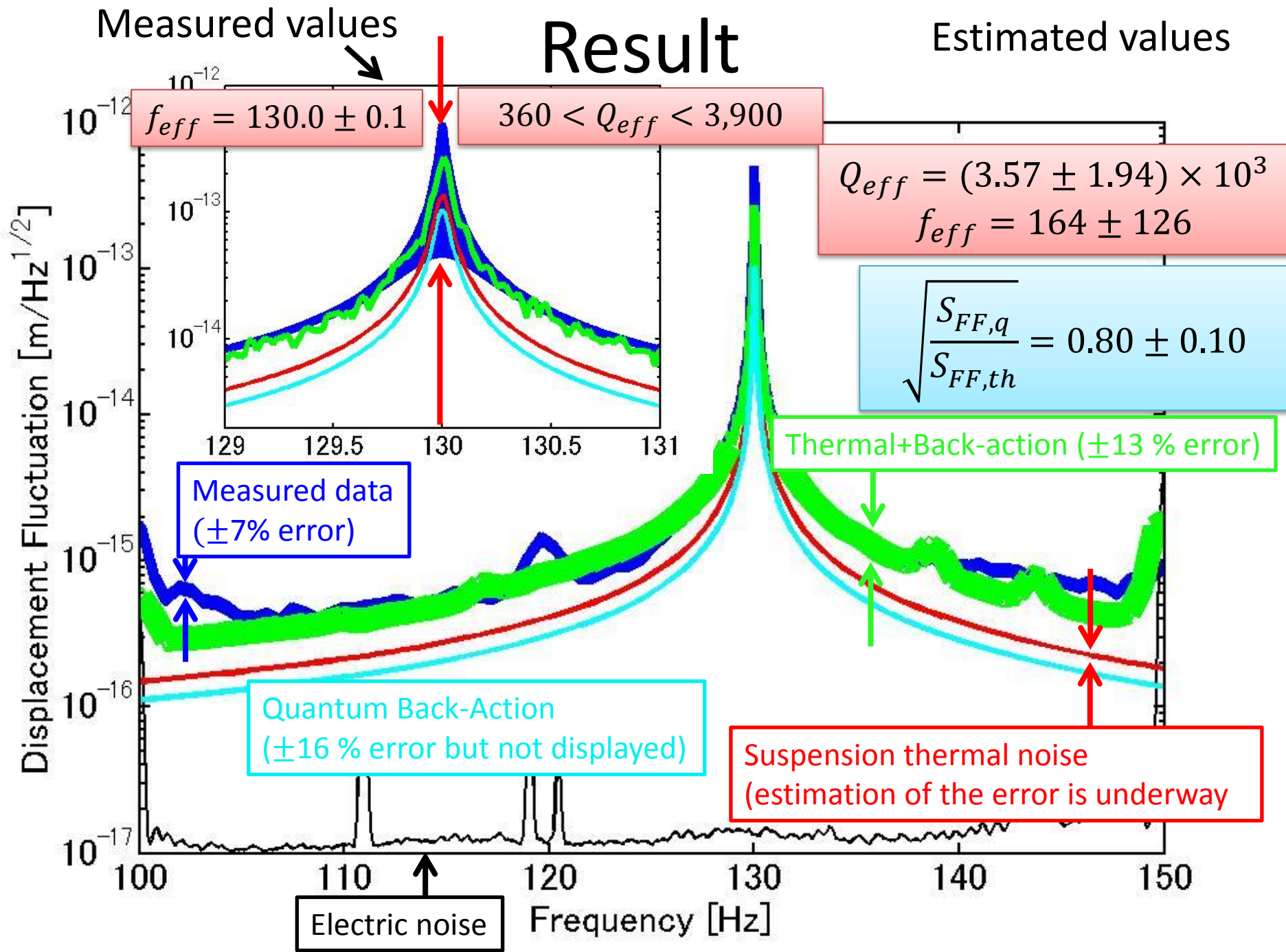


- Parts of the measurement is underway to show validity of our result. (error evaluation of the mechanical Q-factor, frequency noise)
- Our strategy for reaching the **RPN**

Key features:

- Optical positive torsional spring effect
- Gravitational dilution
- **Double optical spring** (Optical dilution)





# Double optical spring

① Triangular cavity

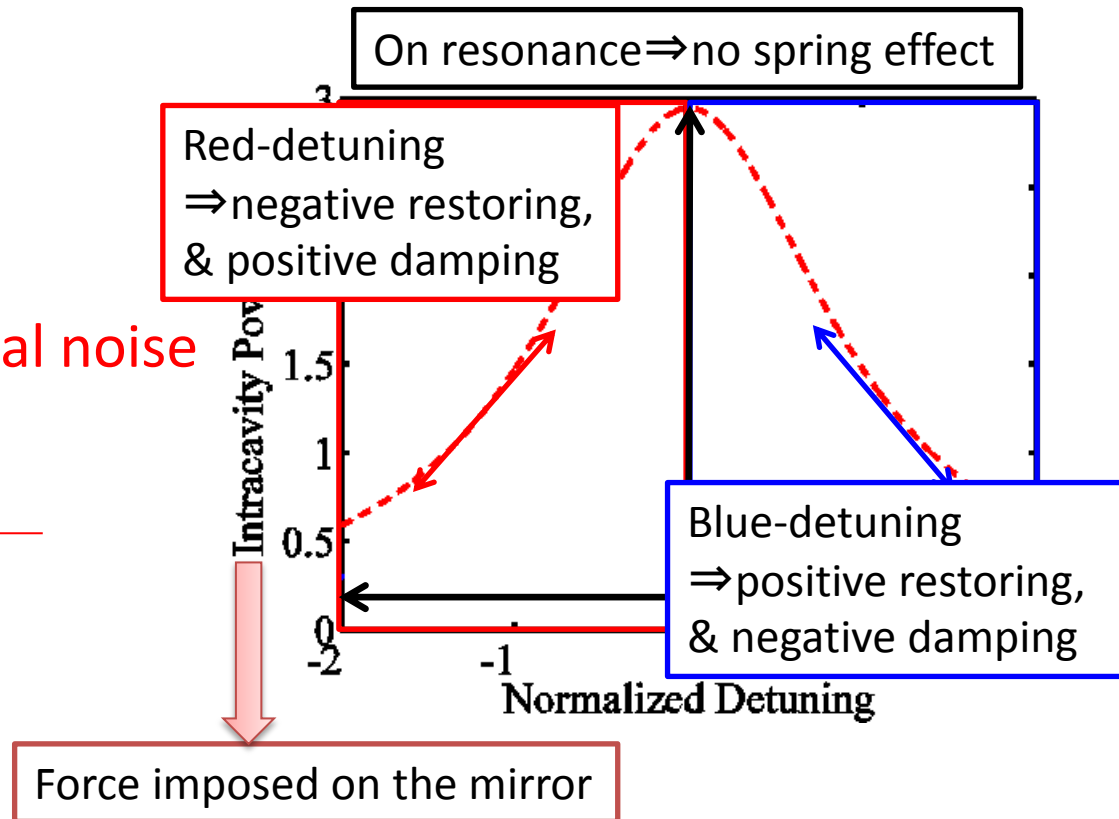
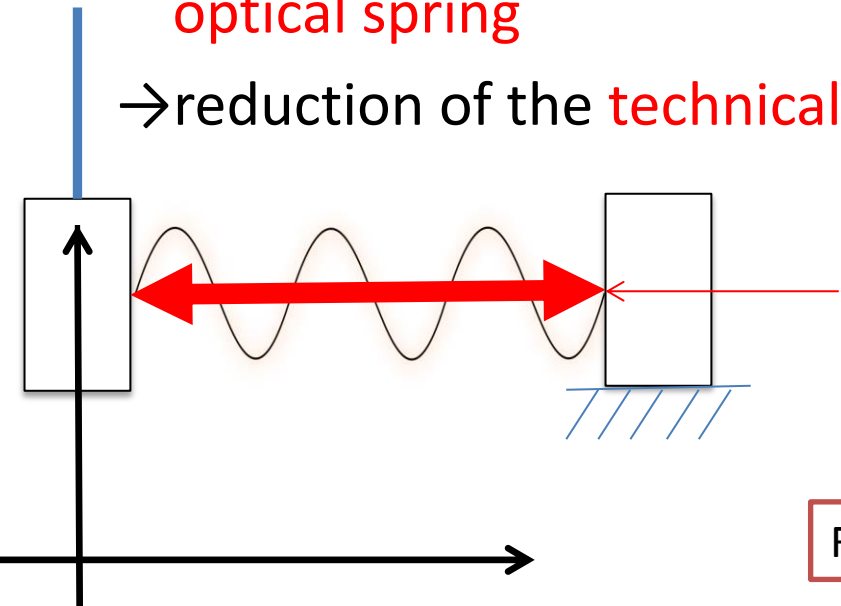
→ stability & low dissipation

② Stability

→ high intracavity power

→ use of ( **translational** )  
**optical spring**

→ reduction of the **technical noise**



Intra-cavity power

Linear range  
(PDH method)

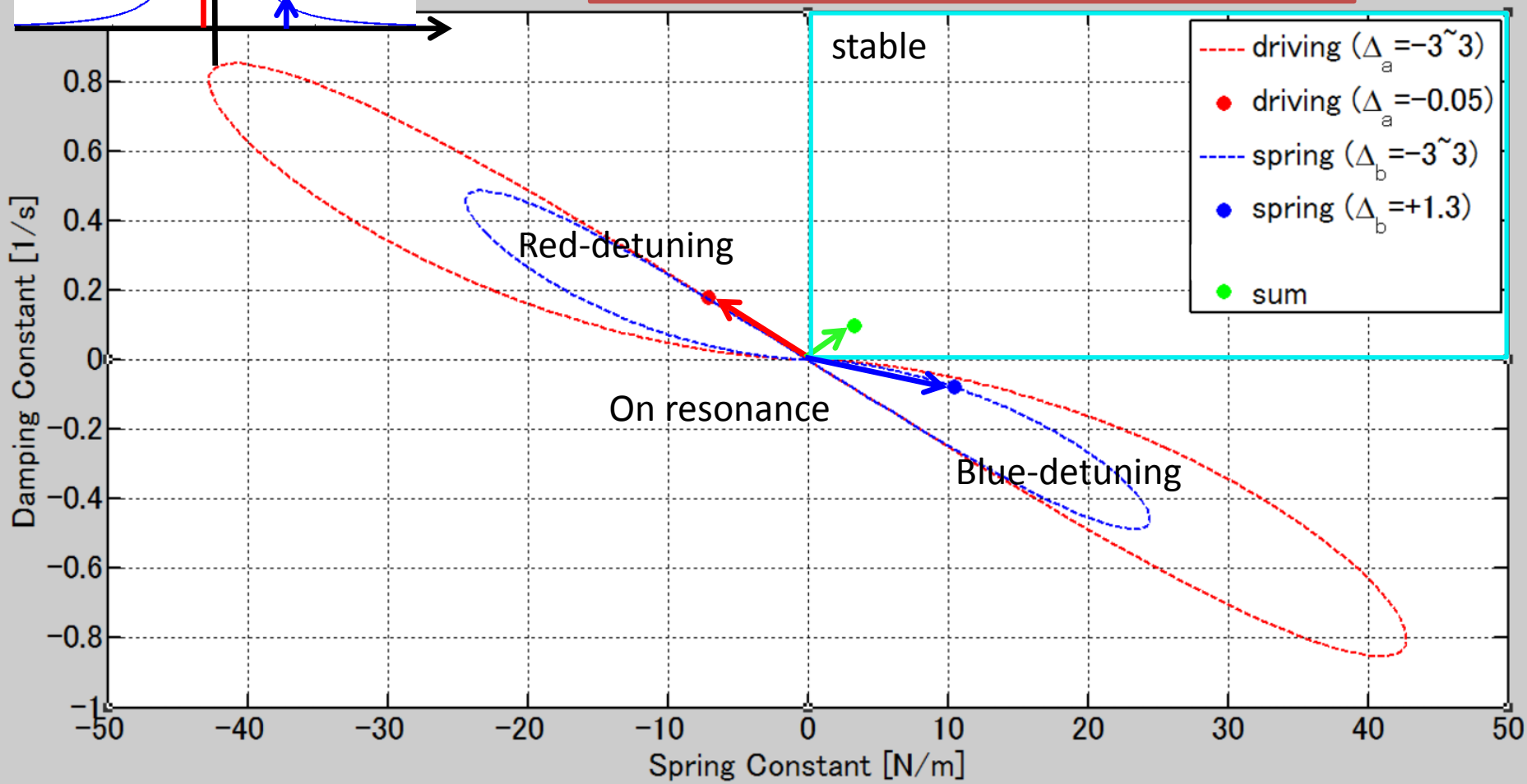
Resonant freq.  $\sim 2\text{Hz}$   
Q-value  $\sim 47e4$

Resonant freq.  $\sim 130\text{Hz}$   
Q-value  $\sim 4000$

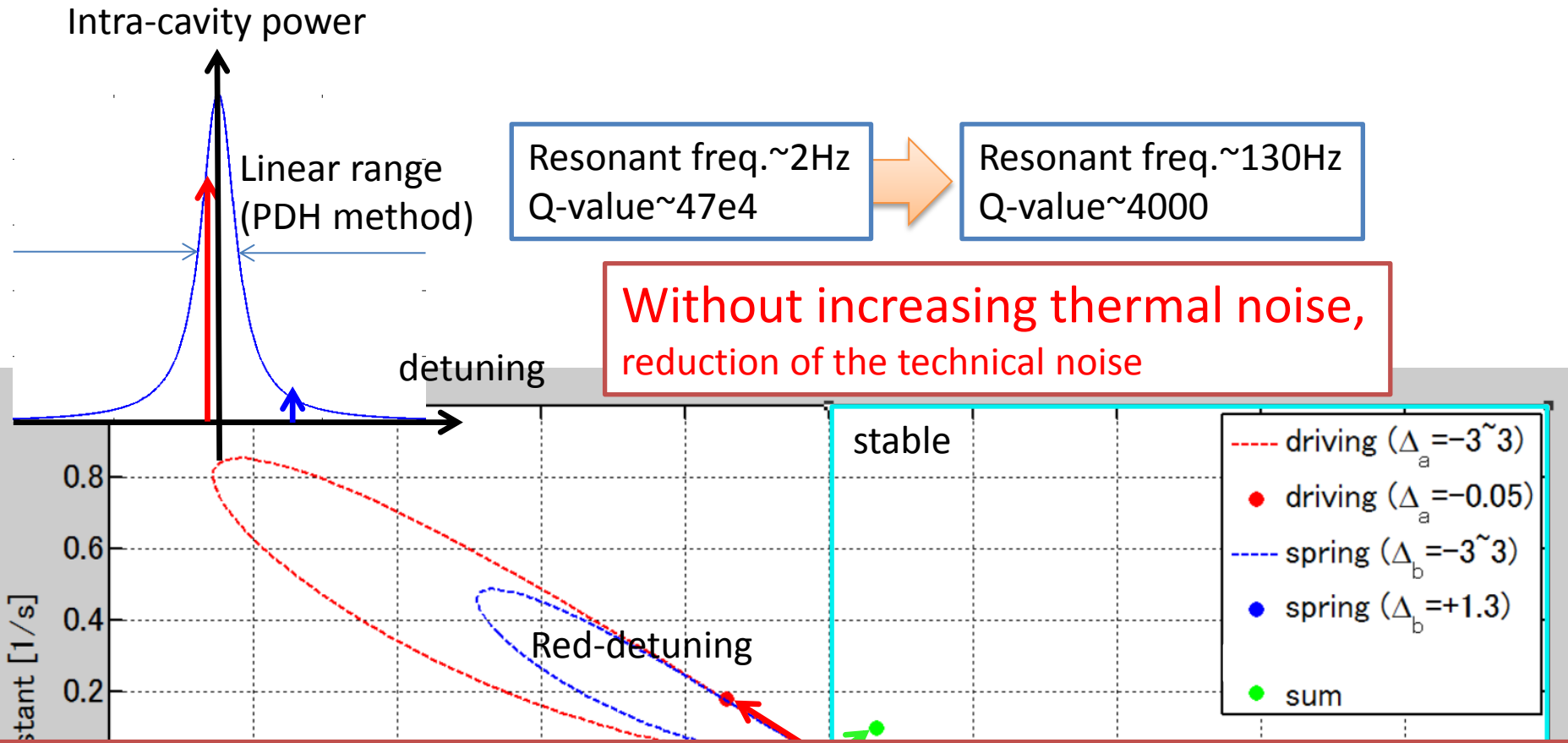
Without increasing thermal noise,  
reduction of the technical noise

detuning

stable

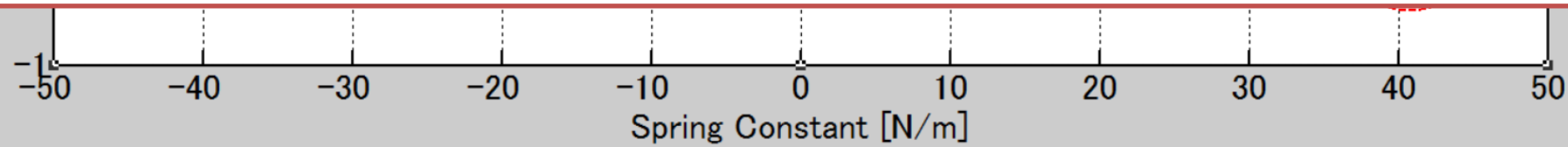






## Triangular cavity

⇒ increasing the signal,  
reduction of the thermal,  
& powerful double optical spring, under the **stable** system

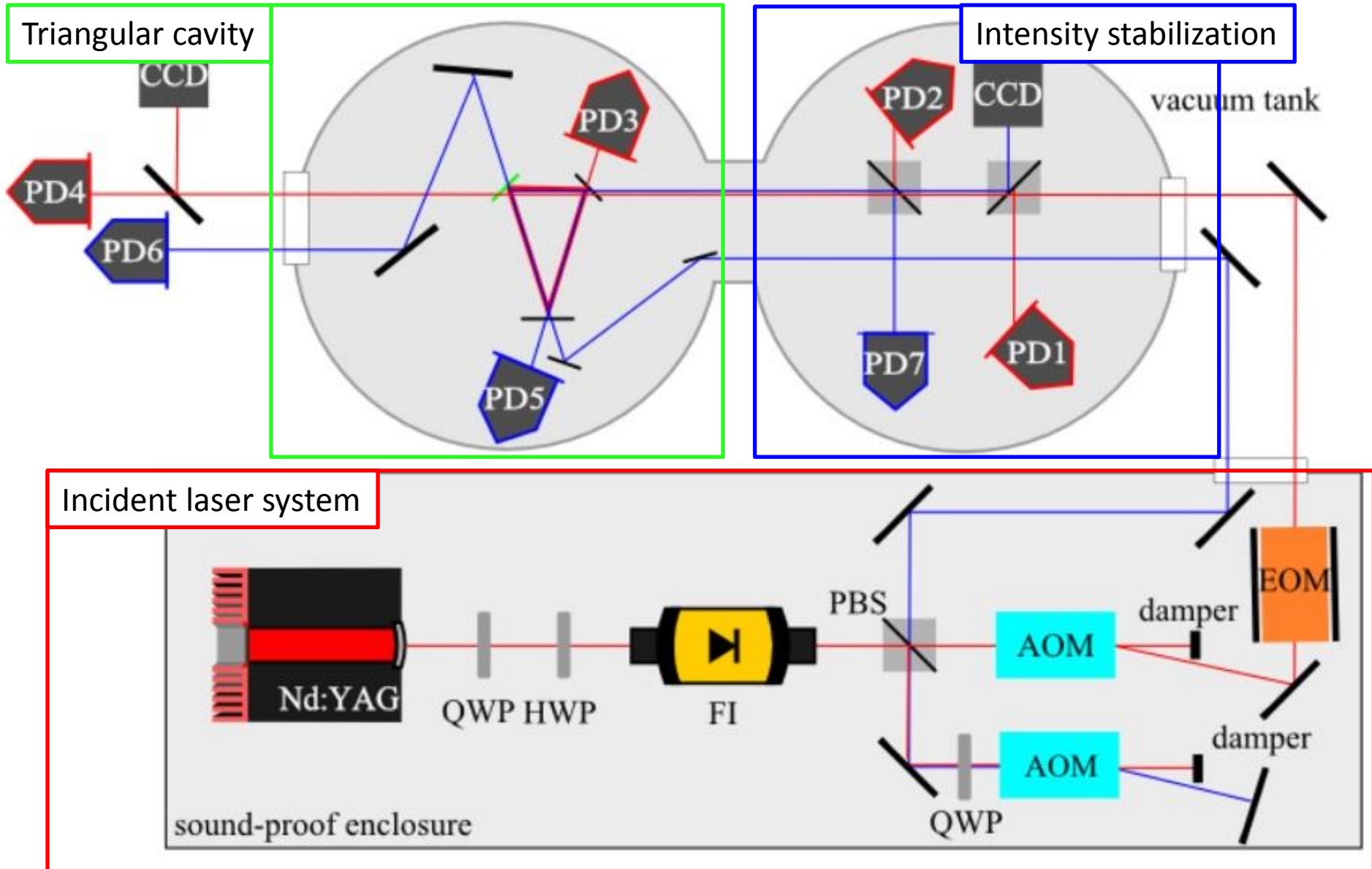


# Details of the experiment

## Main experiment

- i. Setup
- ii. Measurement of the cavity linewidth
- iii. Measurement of an optical spring
- iv. Quality factor of the pendulum
- v. Intensity stabilization of the laser
- vi. Calibration

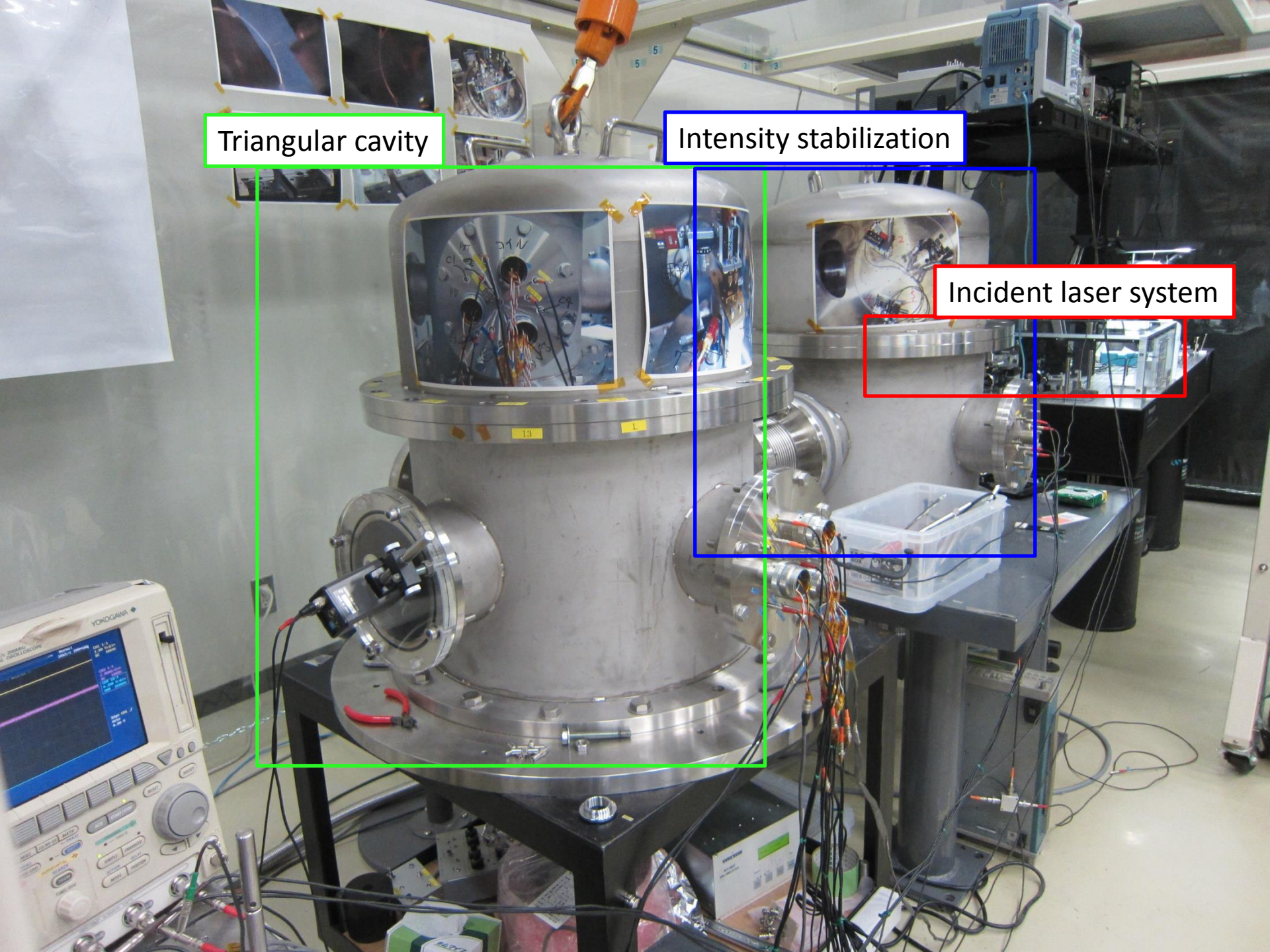
# Setup



Triangular cavity

Intensity stabilization

Incident laser system



# Setup

## Triangular cavity

Round trip length  $\sim 9\text{cm}$   
Finesse  $\sim 1100$   
Intracavity power  $\sim 3.6\text{W}$

$100 \pm 10 \text{ mW}$

Linear range  
(PDH method)

Intra-cavity power

$3.00 \text{ W}$

$0.61 \text{ W}$

detuning

50 mm

20 mm

Controlled mirror  
 $\sim 100\text{g}$

Fixed (half inch) mirror

$5 \pm 0.5 \text{ mW}$

Movable mirror  
 $\sim 5\text{mg}$

Stabilized laser

Nd:YAG laser (Innolight Mephisto)

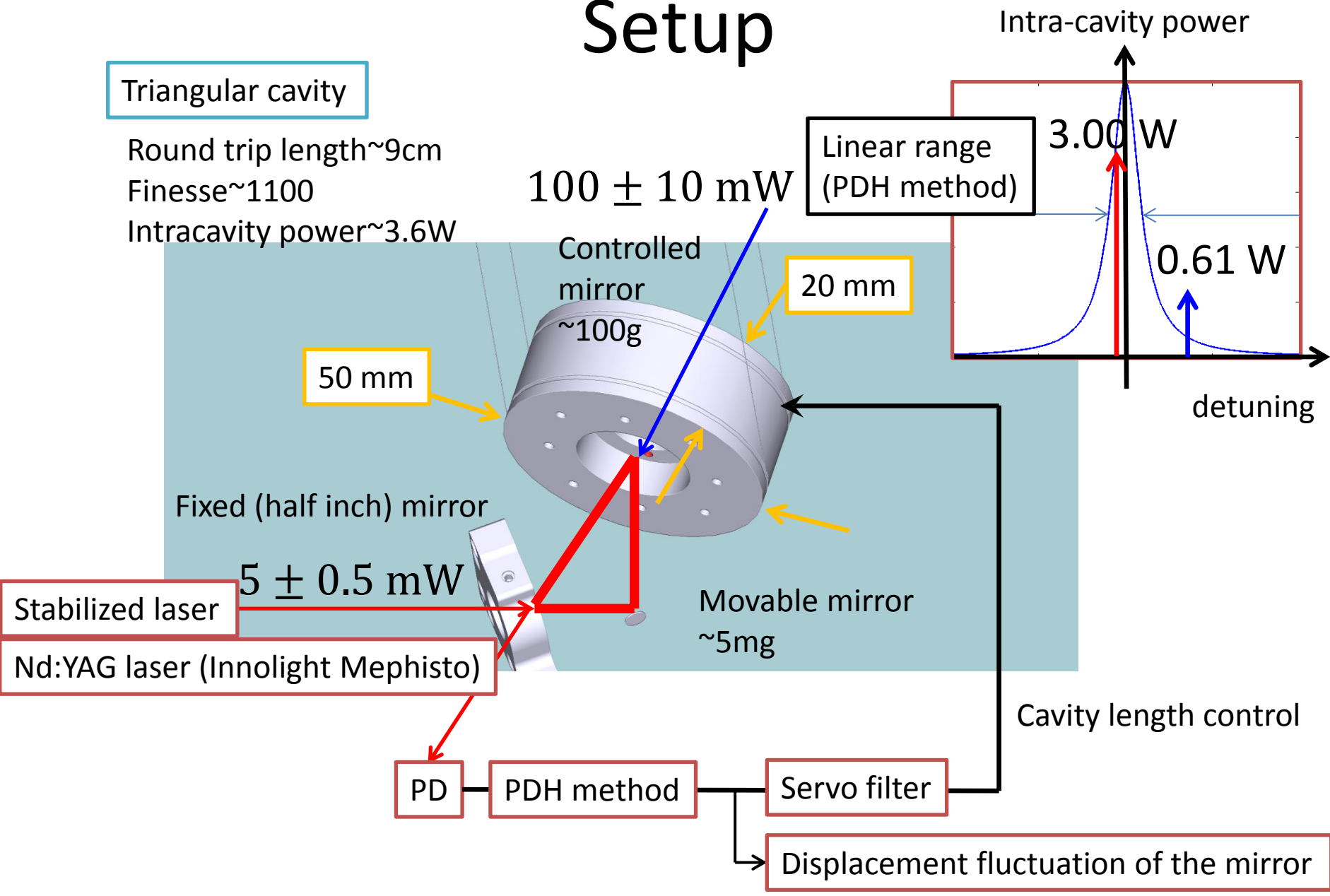
Cavity length control

PD

PDH method

Servo filter

Displacement fluctuation of the mirror



# Setup

Coil-springs

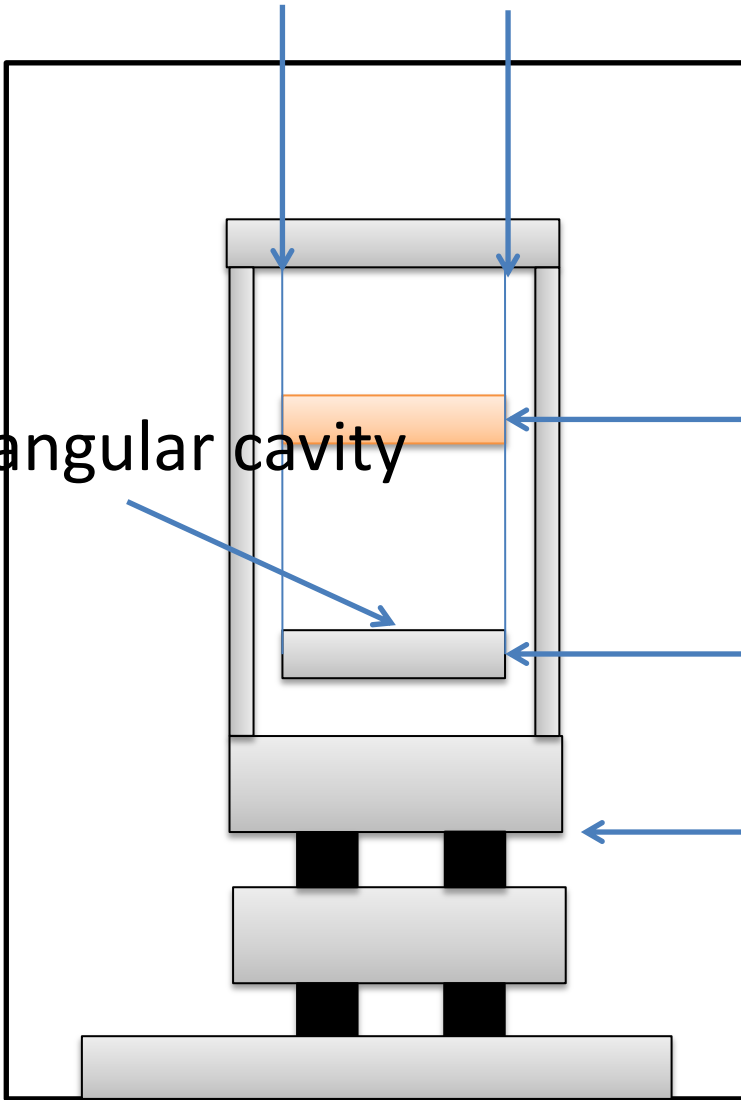
Vacuum tank ( $\sim 1e-3\text{Pa}$ )

Ring-like middle mass  
(Copper,  $>1\text{Hz}$ )

Suspended platform  
( $>1\text{Hz}$ )

Double stack ( $>4\text{Hz}$ )

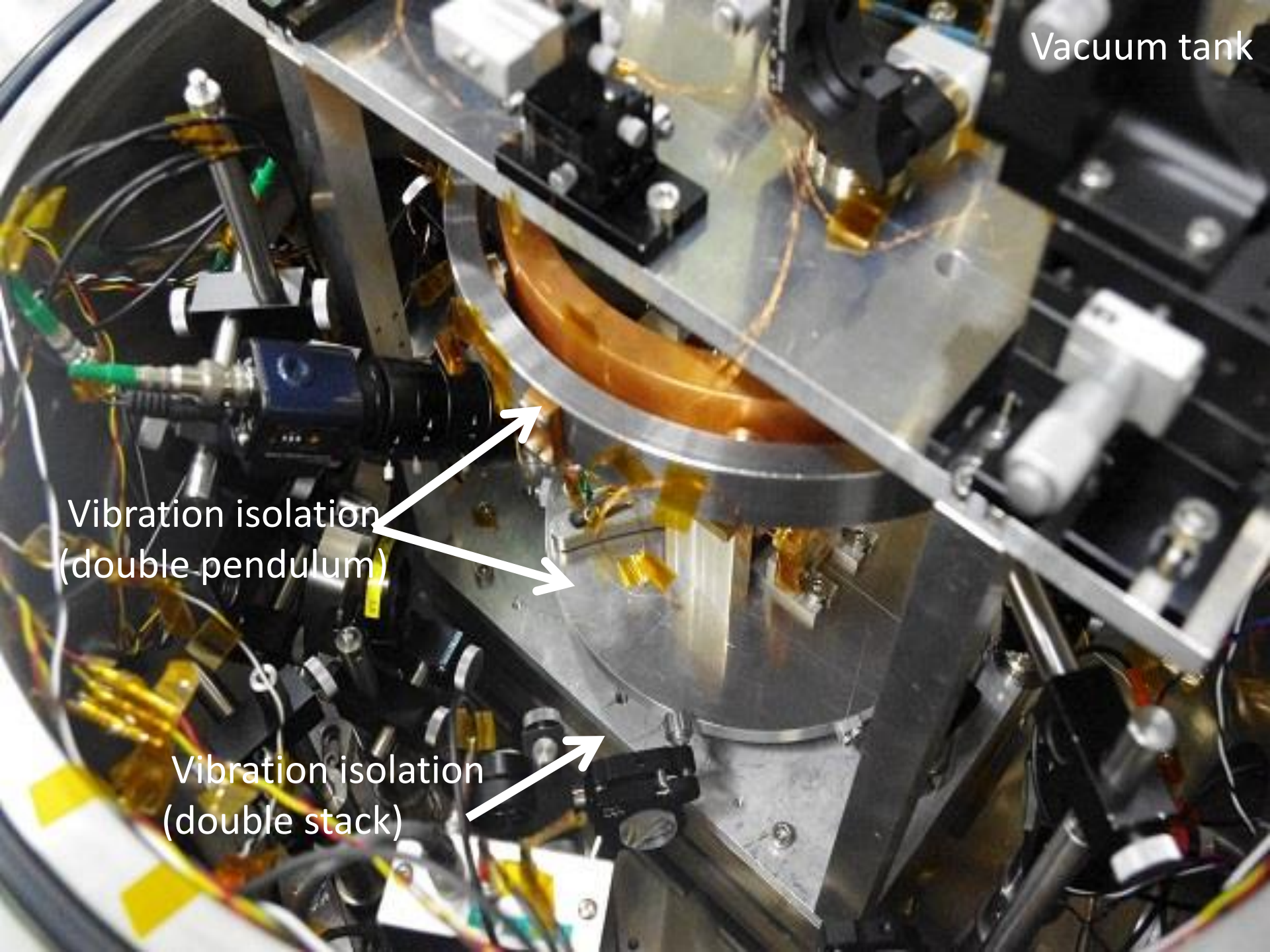
Triangular cavity

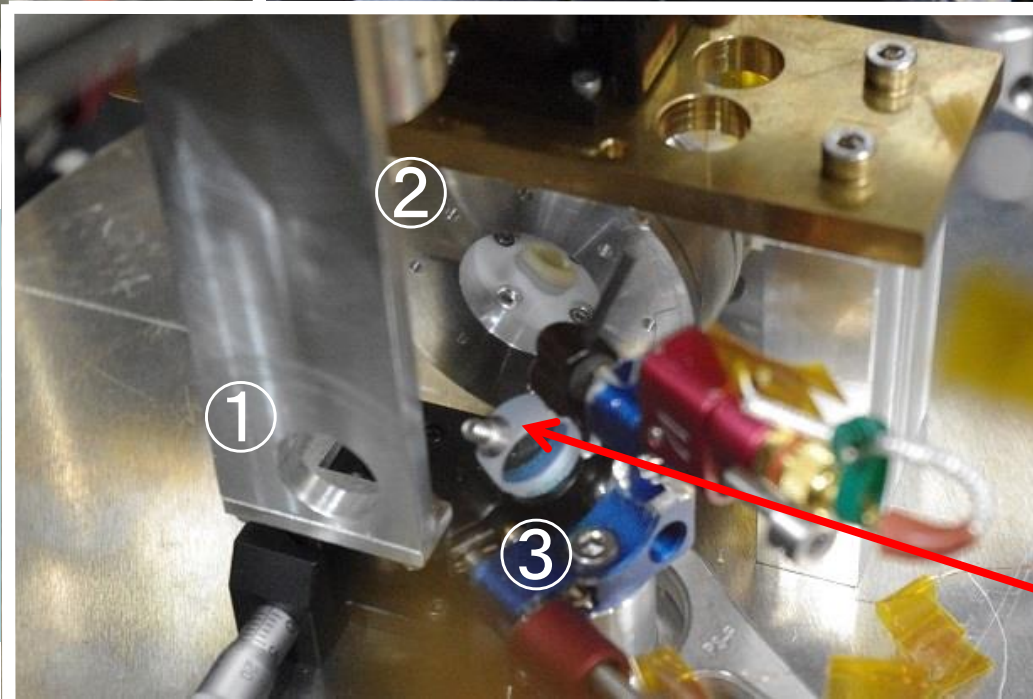
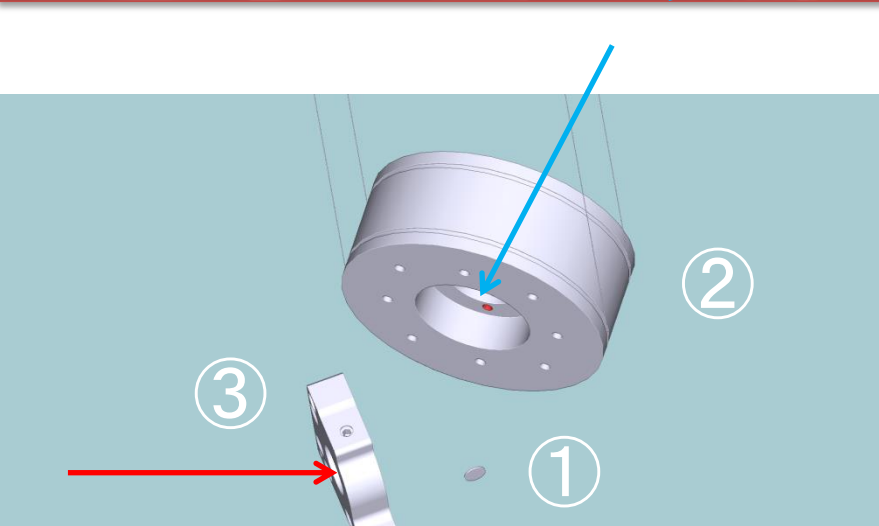
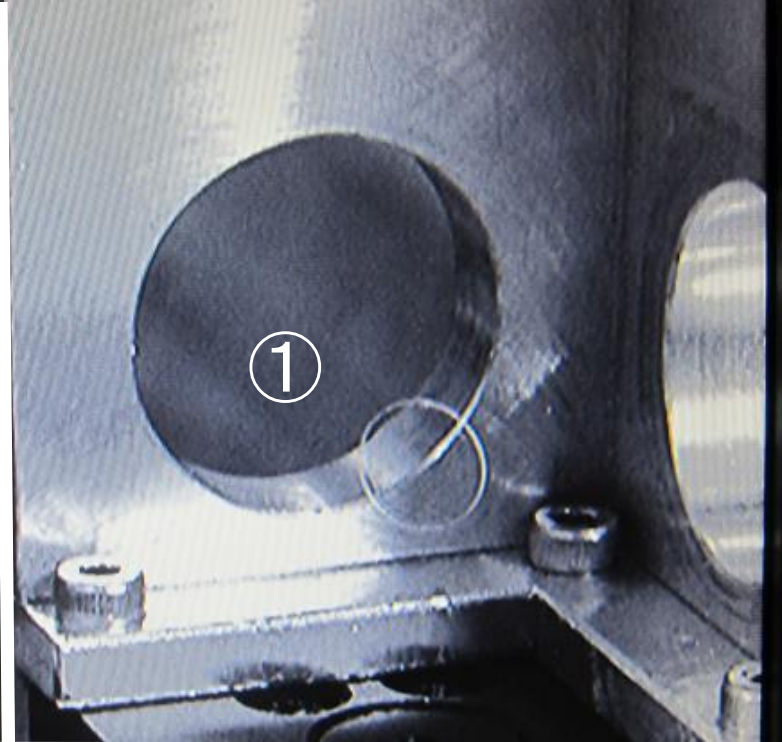
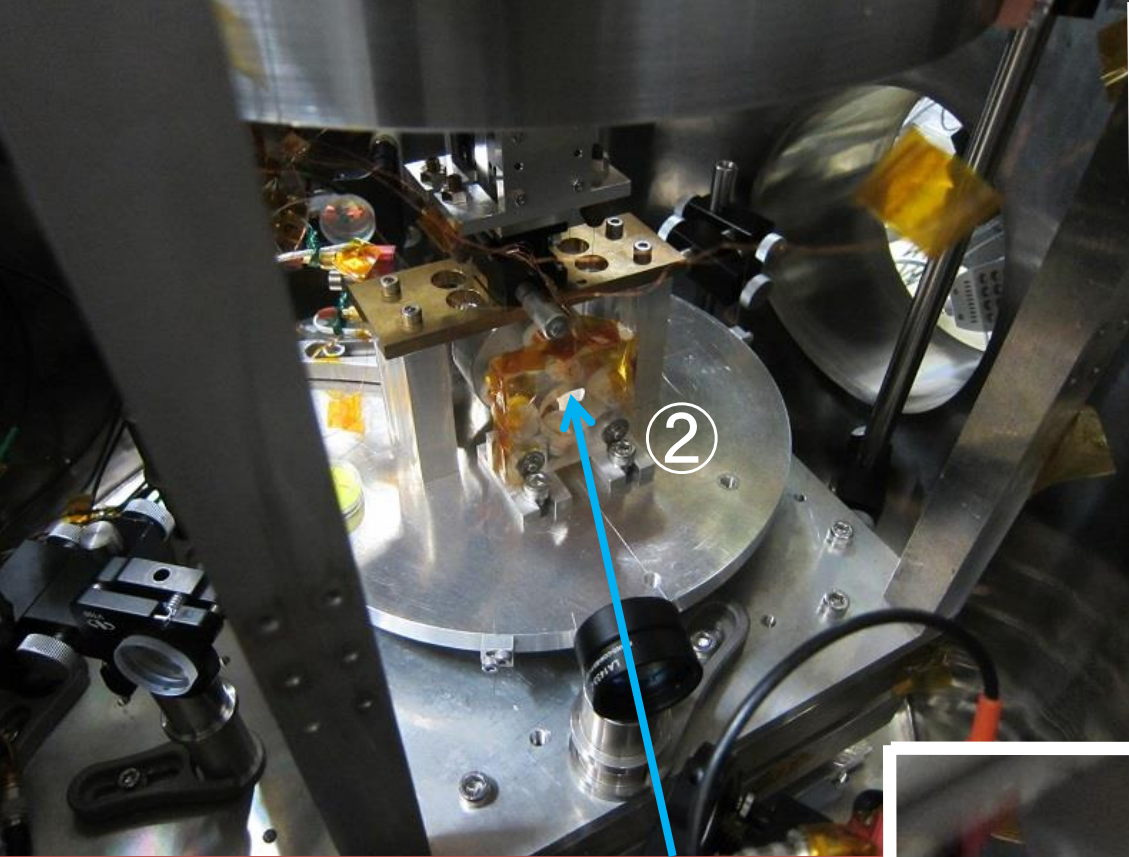


Vacuum tank

Vibration isolation  
(double pendulum)

Vibration isolation  
(double stack)



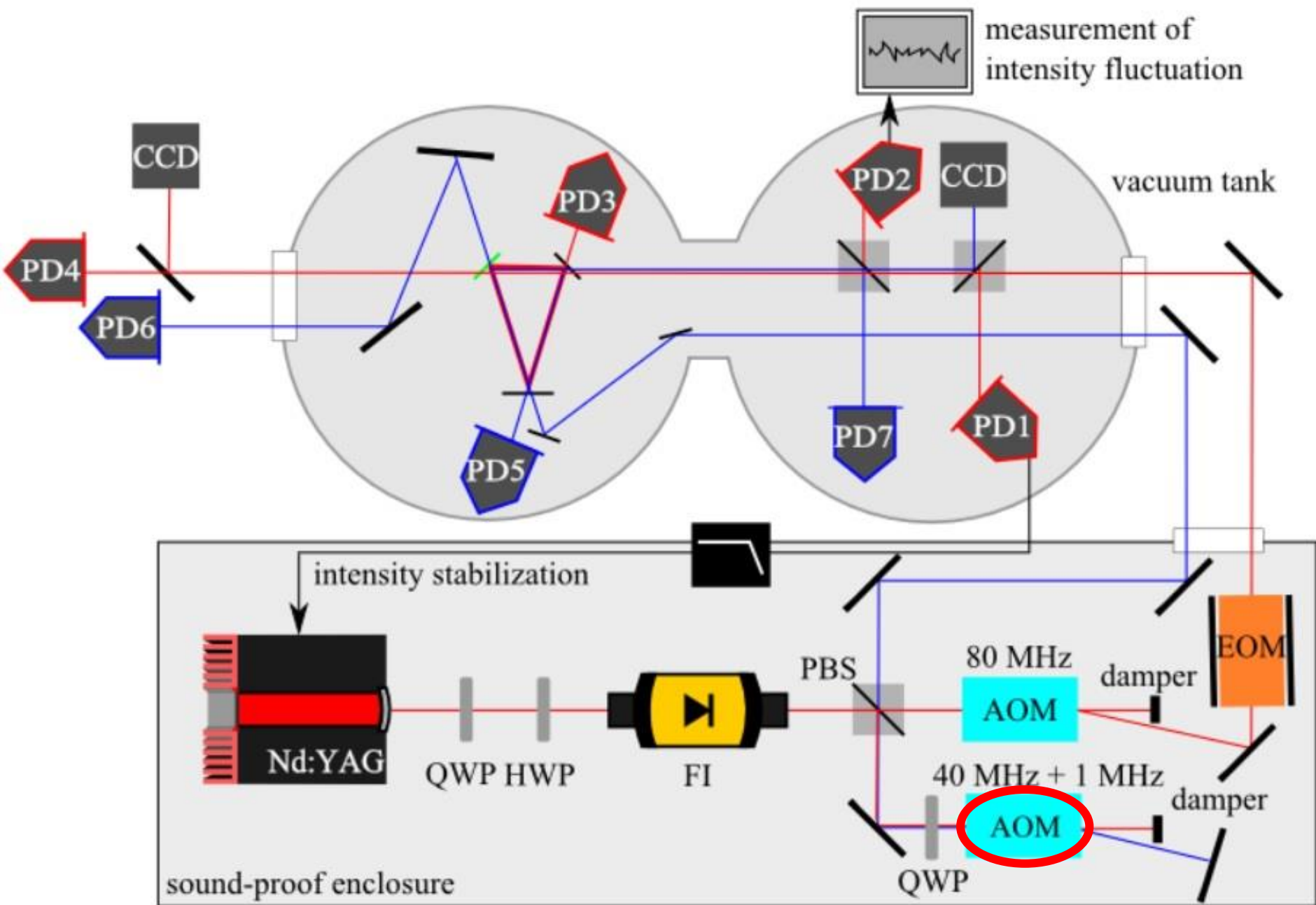




# Details of the experiment

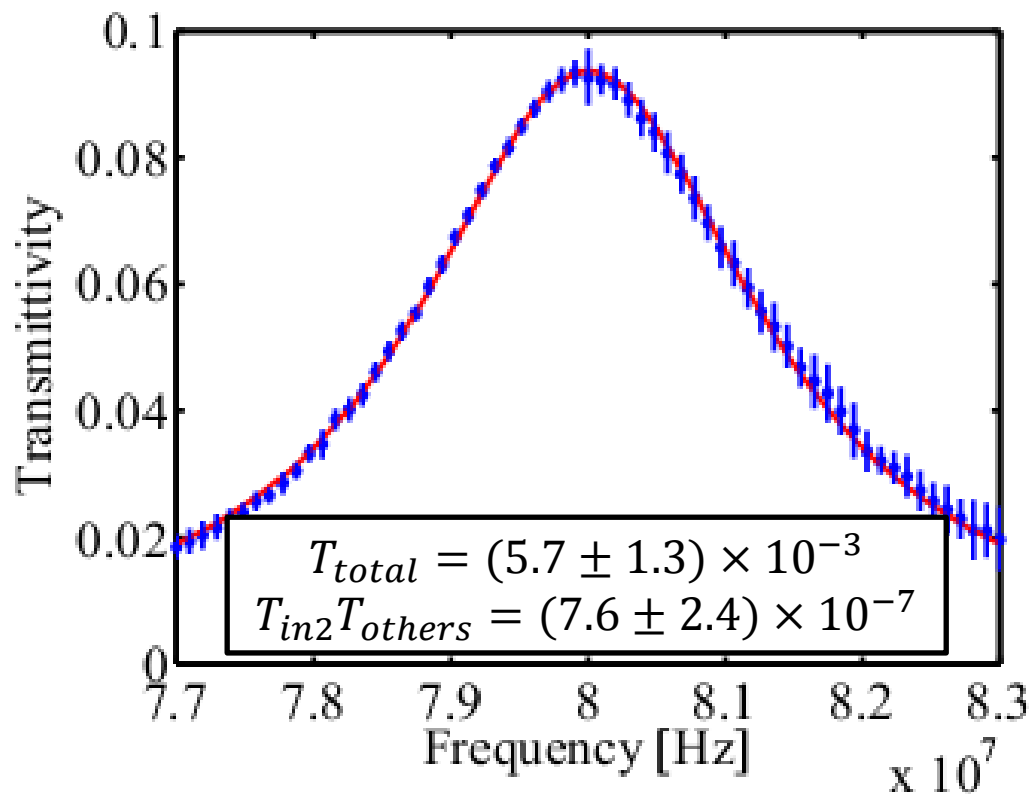
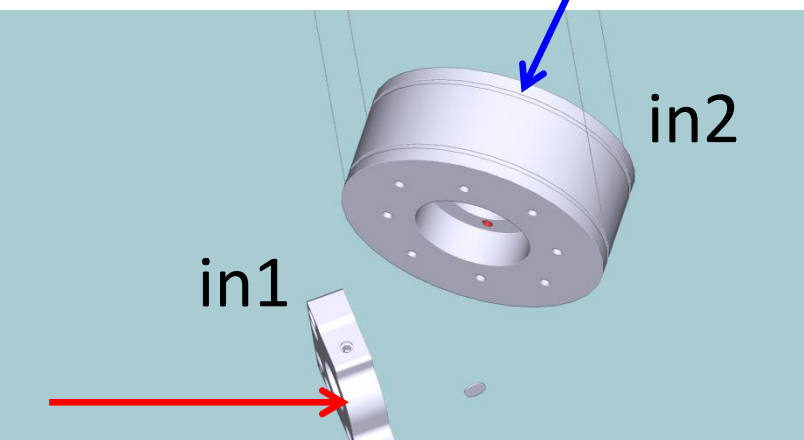
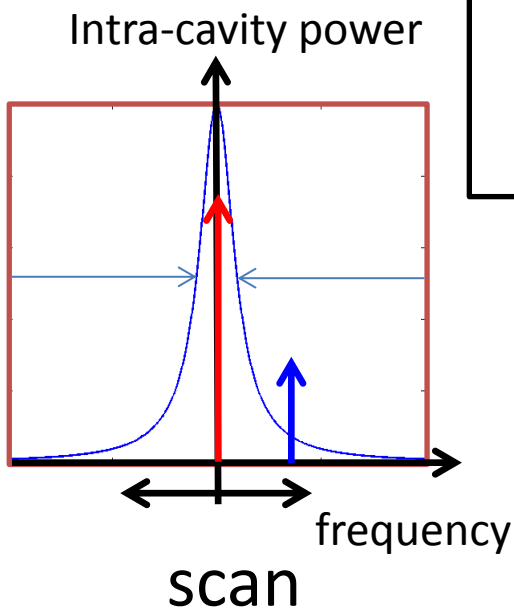
## Main experiment

- i. Setup
- ii. Measurement of the cavity linewidth  
⇒ Estimation of the **signal** level
- iii. Property measurement of the optical spring  
⇒ Estimation of the **signal** level
- iv. Quality factor of the pendulum  
⇒ Estimation of the **thermal noise** level
- v. Intensity stabilization of the laser  
⇒ Elimination of the **classical noise** of the light
- vi. calibration



cavity decay rate	$\kappa/2\pi$	Hz	$(1.52 \pm 0.03) \times 10^6$
cavity decay rate (in1)	$\kappa_{\text{in1}}/2\pi$	Hz	$(1.30 \pm 0.03) \times 10^6$
cavity decay rate (in2)	$\kappa_{\text{in2}}/2\pi$	Hz	$(3.64 \pm 2.46) \times 10^4$

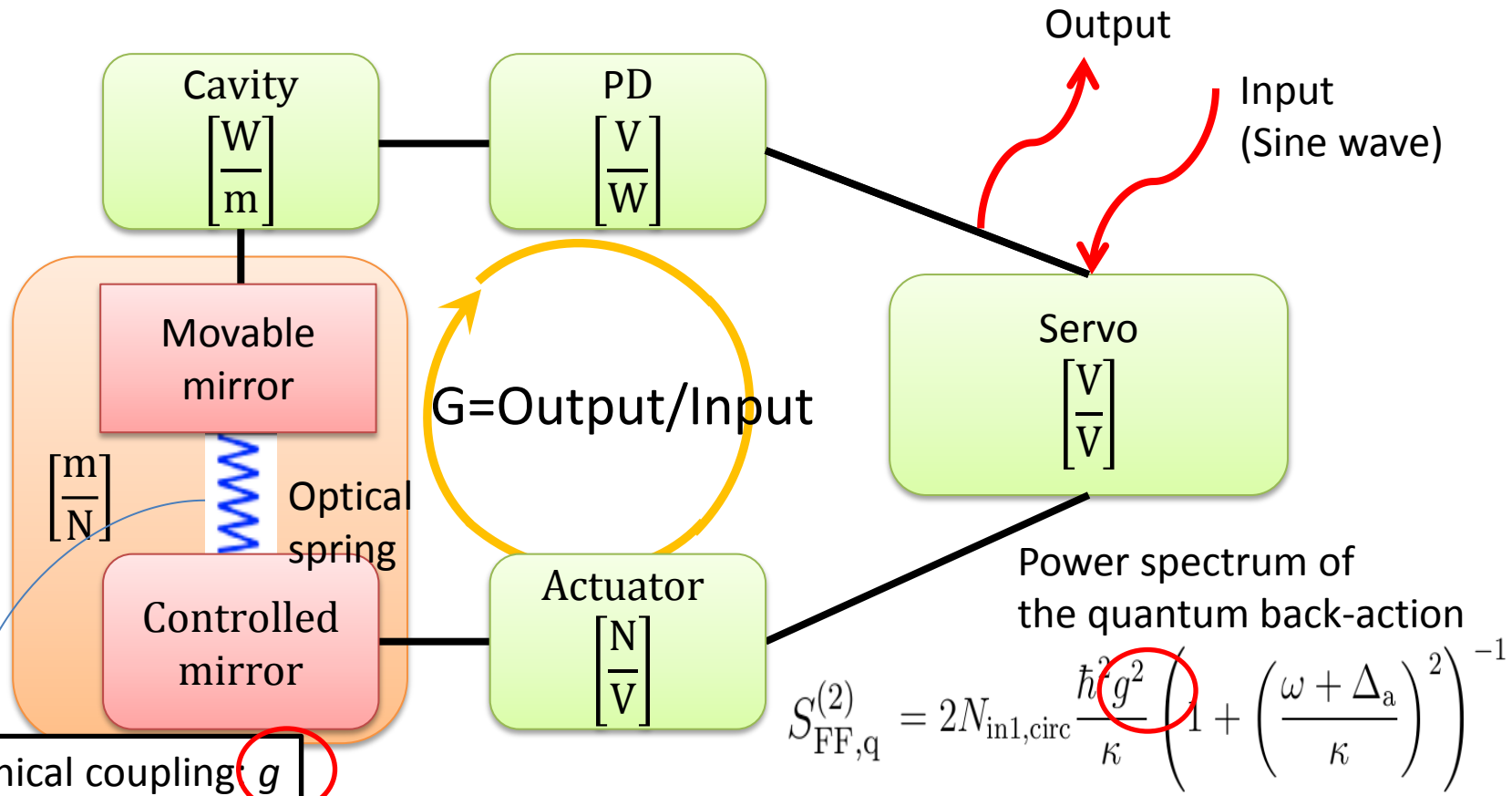
$$\begin{aligned}
 S_{\text{FF},q}^{(2)} &= \langle \delta \hat{F}_{\text{BA}}(-\omega) \delta \hat{F}_{\text{BA}}(\omega) \rangle \\
 &= 2\hbar^2 \kappa |G_a|^2 |\chi_a(-\omega)|^2 + 2\hbar^2 \kappa |G_b|^2 |\chi_b(-\omega)|^2 \\
 &= 2N_{\text{in1,circ}} \frac{\hbar^2 g^2}{\kappa} \left( 1 + \left( \frac{\omega + \Delta_a}{\kappa} \right)^2 \right)^{-1} + 2N_{\text{in2,circ}} \frac{\hbar^2 g^2}{\kappa} \left( 1 + \left( \frac{\omega + \Delta_b}{\kappa} \right)^2 \right)^{-1}
 \end{aligned}$$

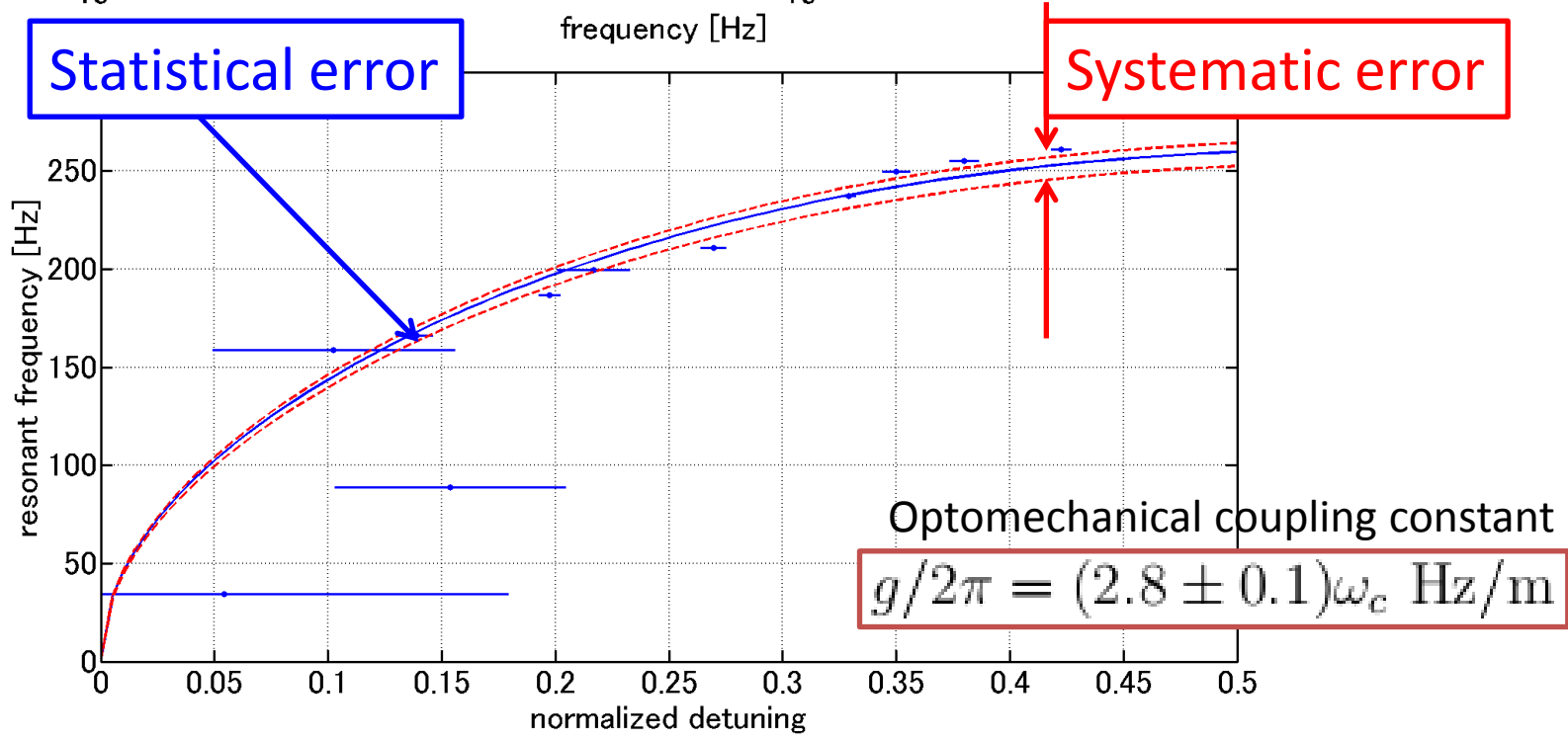
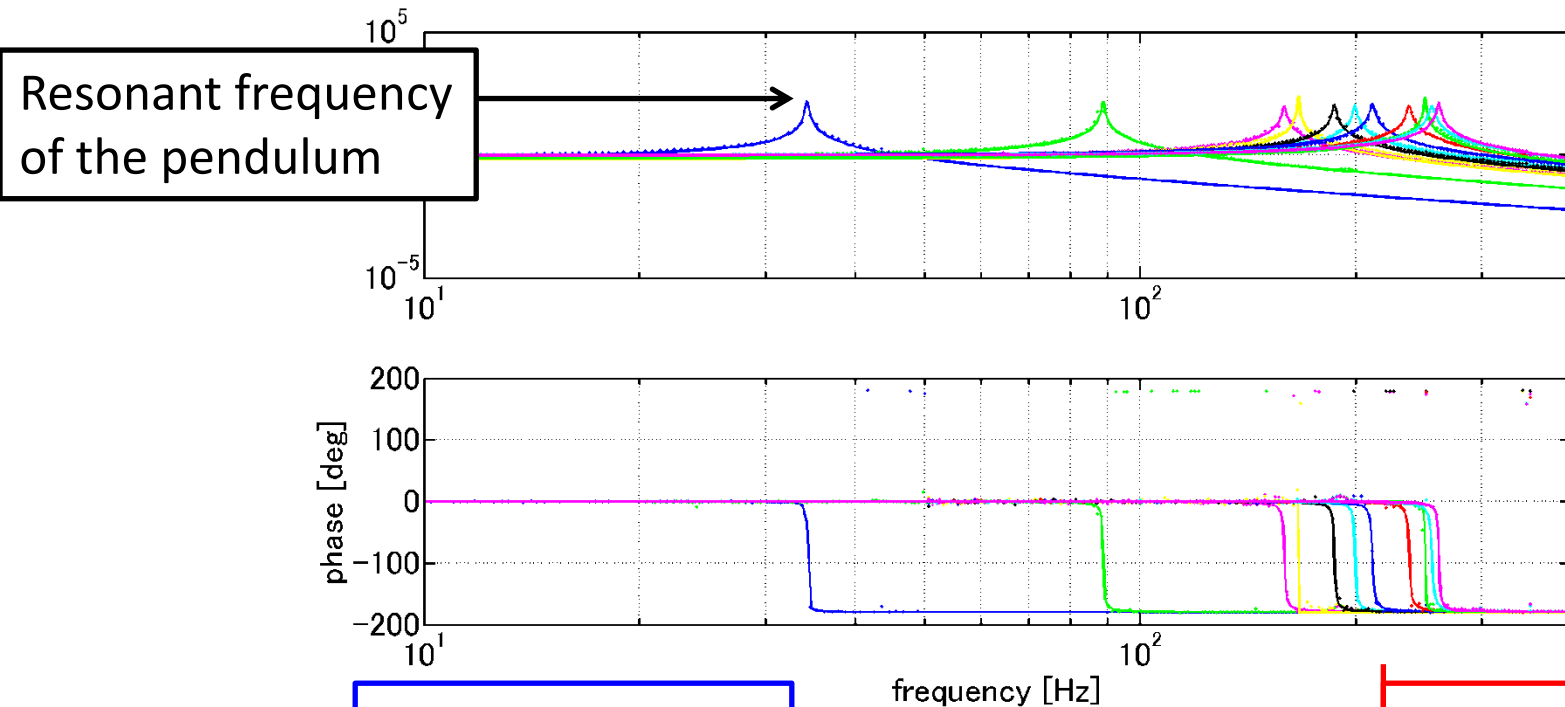


# Measurement of the optical spring

Measurement of the optical spring by the open-loop transfer function

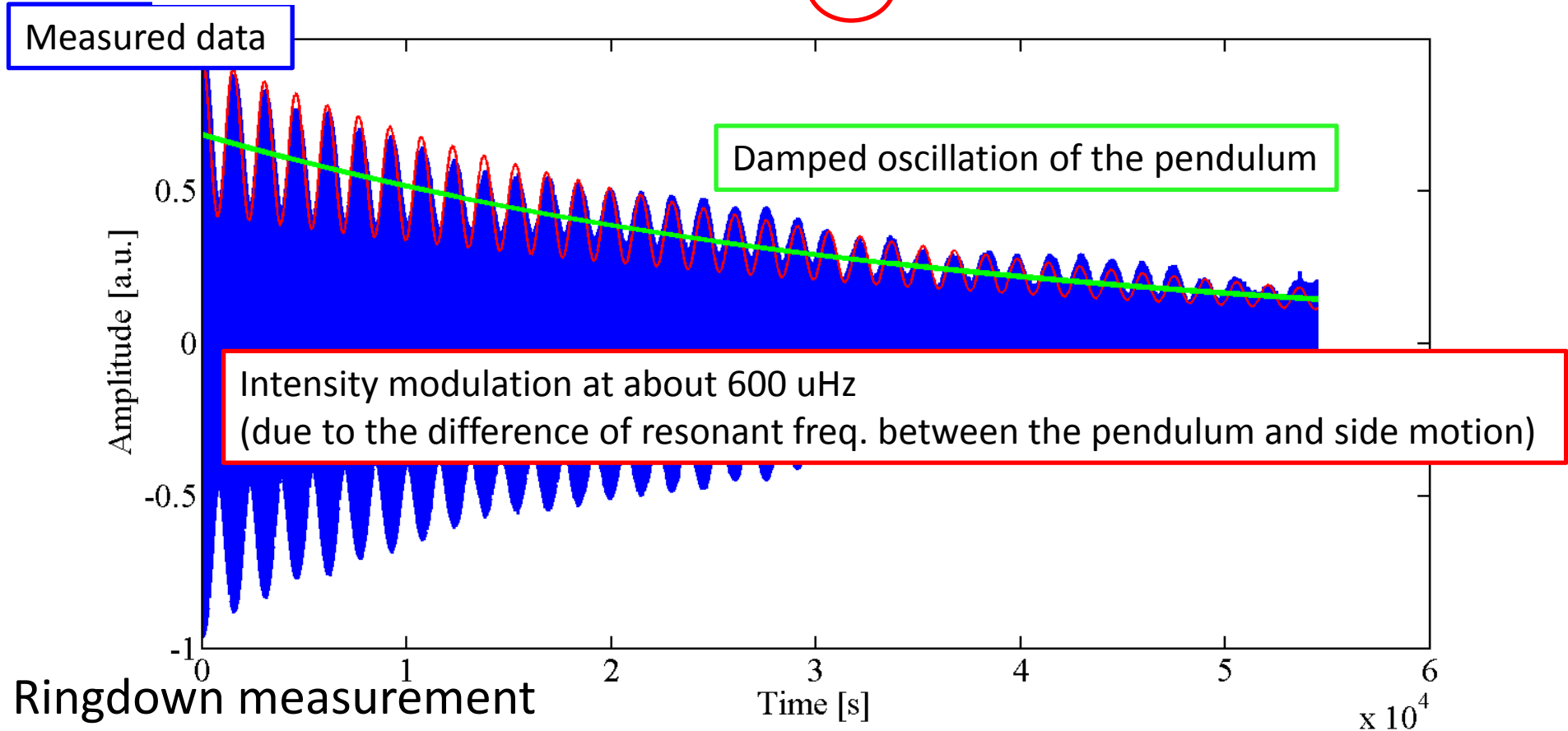
1. Disturbance input → Measurement of a response
2. Offset adjustment of a control signal (detuning adjustment) ⇒ returning to 1





# Quality factor of the pendulum

- **Thermal fluctuating force**  $S_{\text{FF,th}}^{(2)} = 4k_{\text{B}}T\gamma_{\text{m}}m$ .
- Damped oscillation  $\rightarrow$  life time  $\leftrightarrow$  mechanical Q-value  
 $\Rightarrow Q_{\text{m}} = 4.7 \times 10^5$  ( $Q_{\text{m}} = \omega_{\text{m}}/2\gamma_{\text{m}}$ )

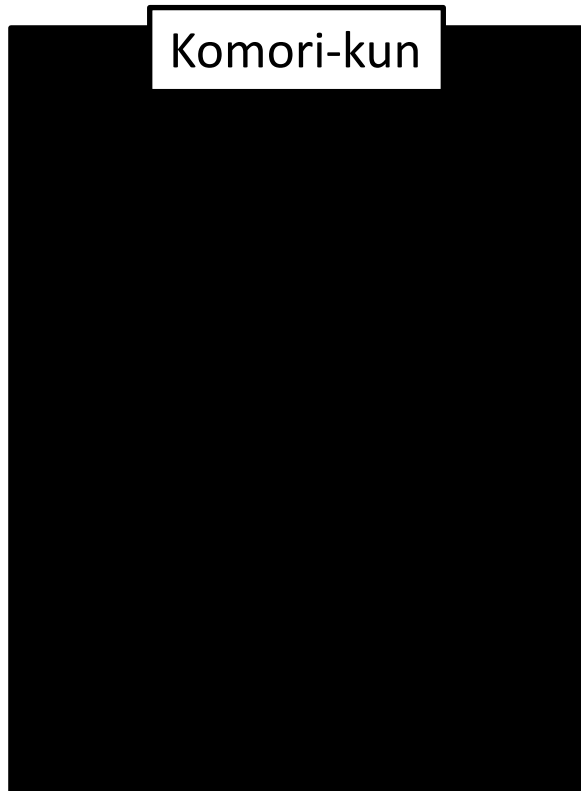


# Error of the quality factor

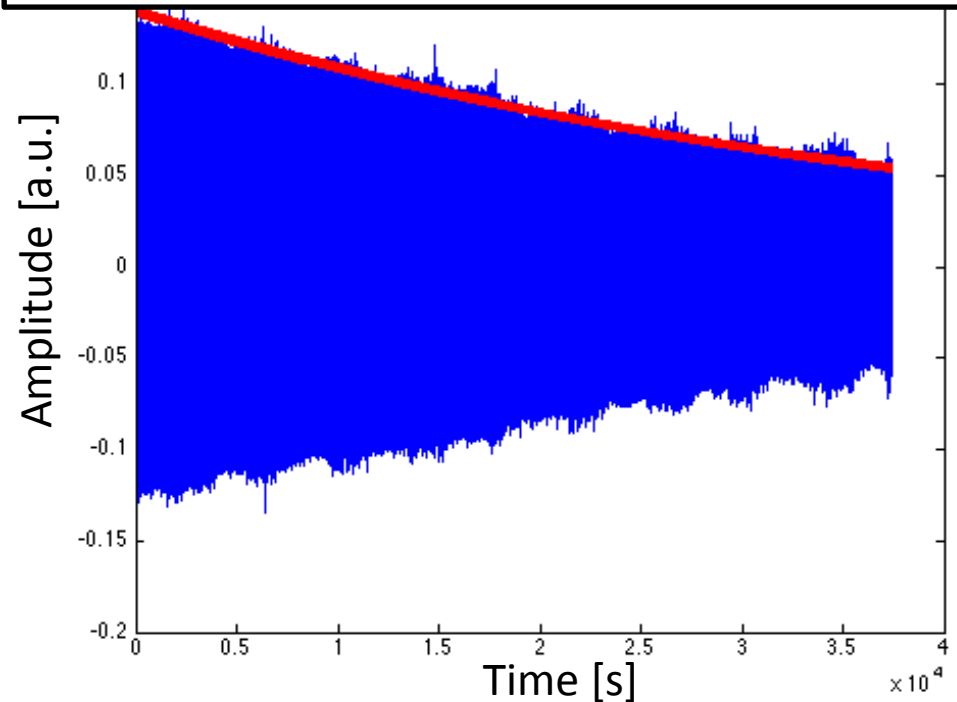
Q-factor is currently measured to obtain its error by Kentaro Komori.

Improvement for reducing the side motion:

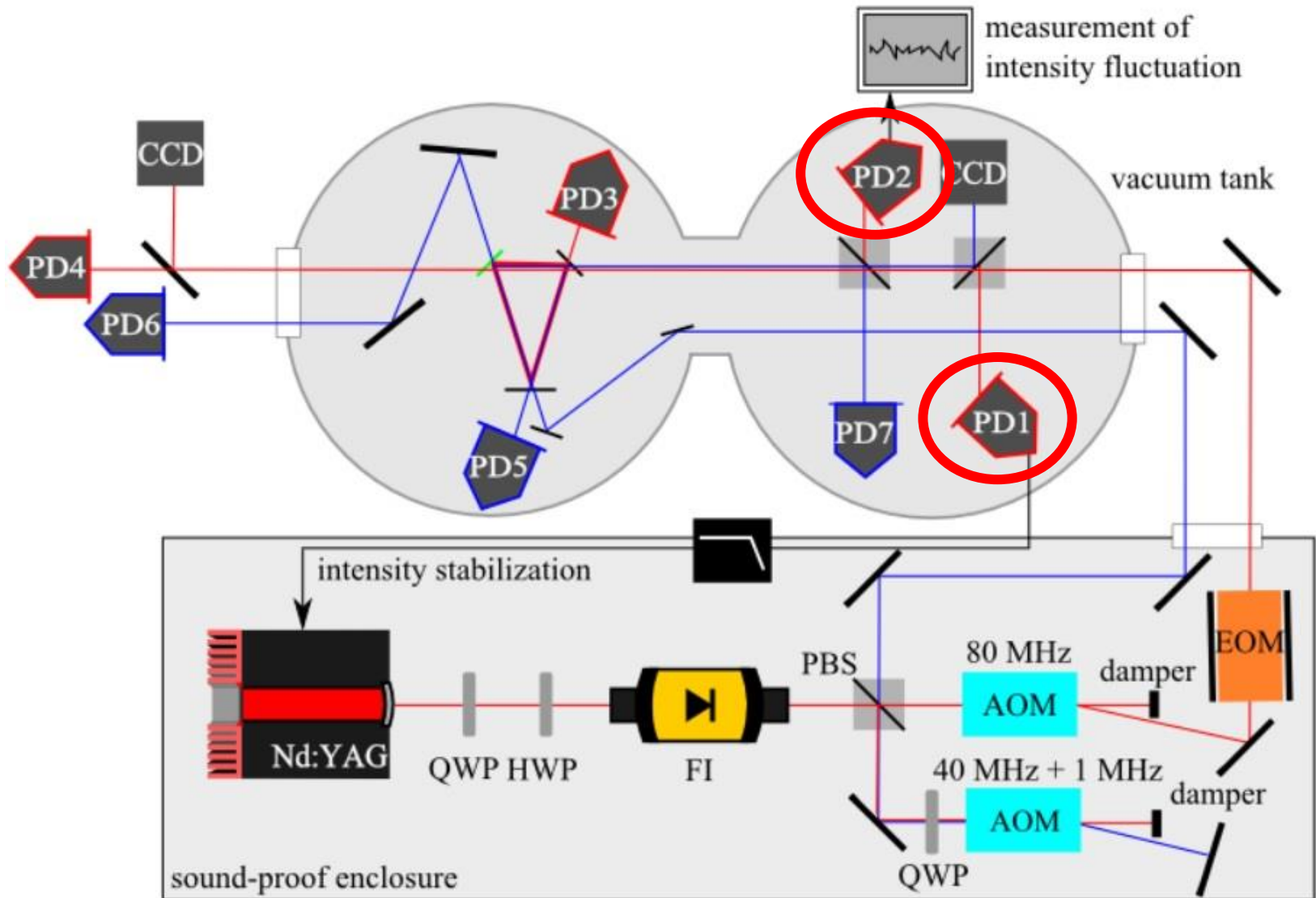
- i. the direction of the incident laser on the suspended mirror
- ii. excitation of the pendulum



Preliminary result:  $Q \sim 270,000$   
(Similar but low-quality pendulum was used)



# Intensity stabilization of the laser



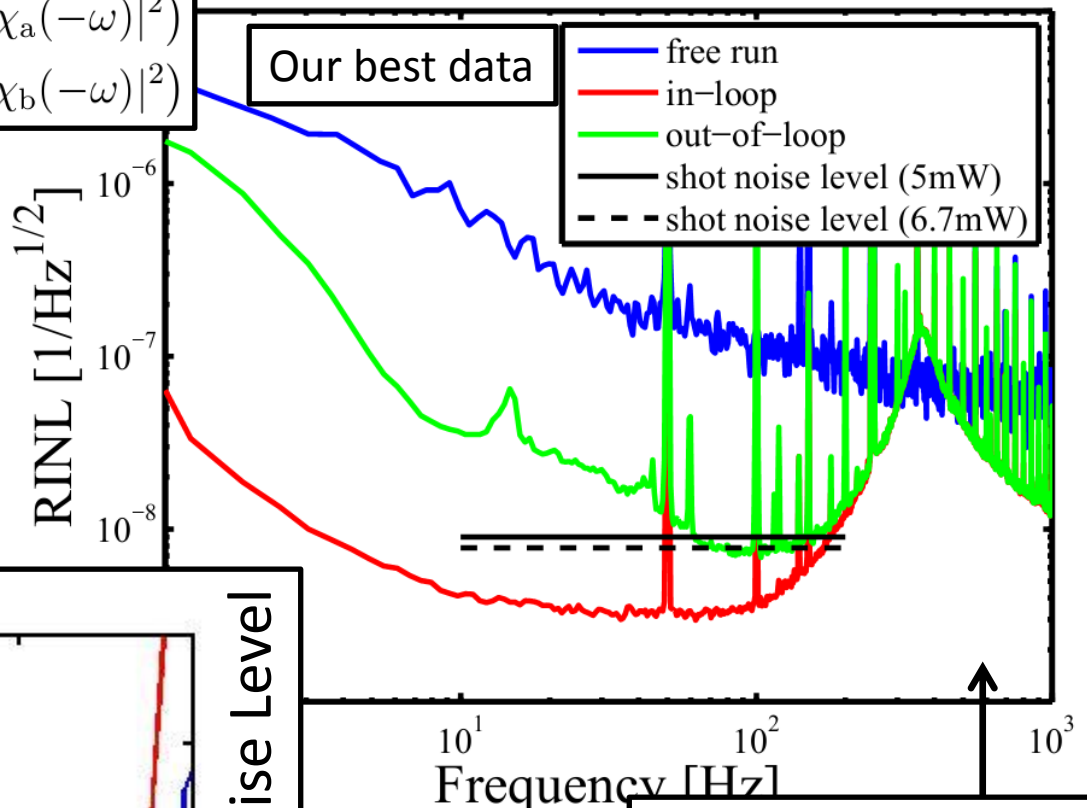


$$S_{\text{FF},c}^{(2)} = 2(B_{\text{in}1} - 1)\hbar^2\kappa_{\text{in}1}|G_a|^2 (|\chi_a(\omega)|^2 + |\chi_a(-\omega)|^2) + 2(B_{\text{in}2} - 1)\hbar^2\kappa_{\text{in}2}|G_b|^2 (|\chi_b(\omega)|^2 + |\chi_b(-\omega)|^2)$$



Back-action force is enhanced by the classical noise

simultaneous measurement



Our best data

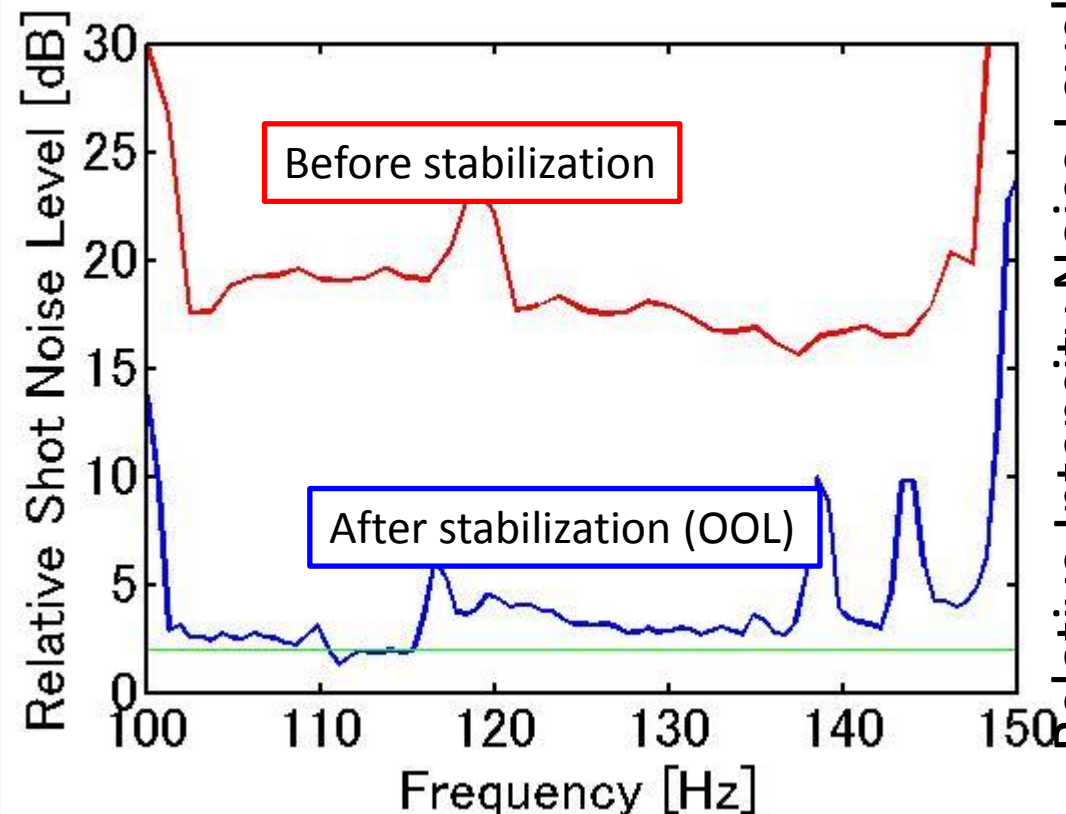
- free run
- in-loop
- out-of-loop
- shot noise level (5mW)
- - - shot noise level (6.7mW)

turbomolecular pumps is switched off

turbomolecular pumps is switched on

Requirement

$$1.4 \times 10^{-8} [1/\sqrt{\text{Hz}}]$$



Before stabilization

After stabilization (OOL)

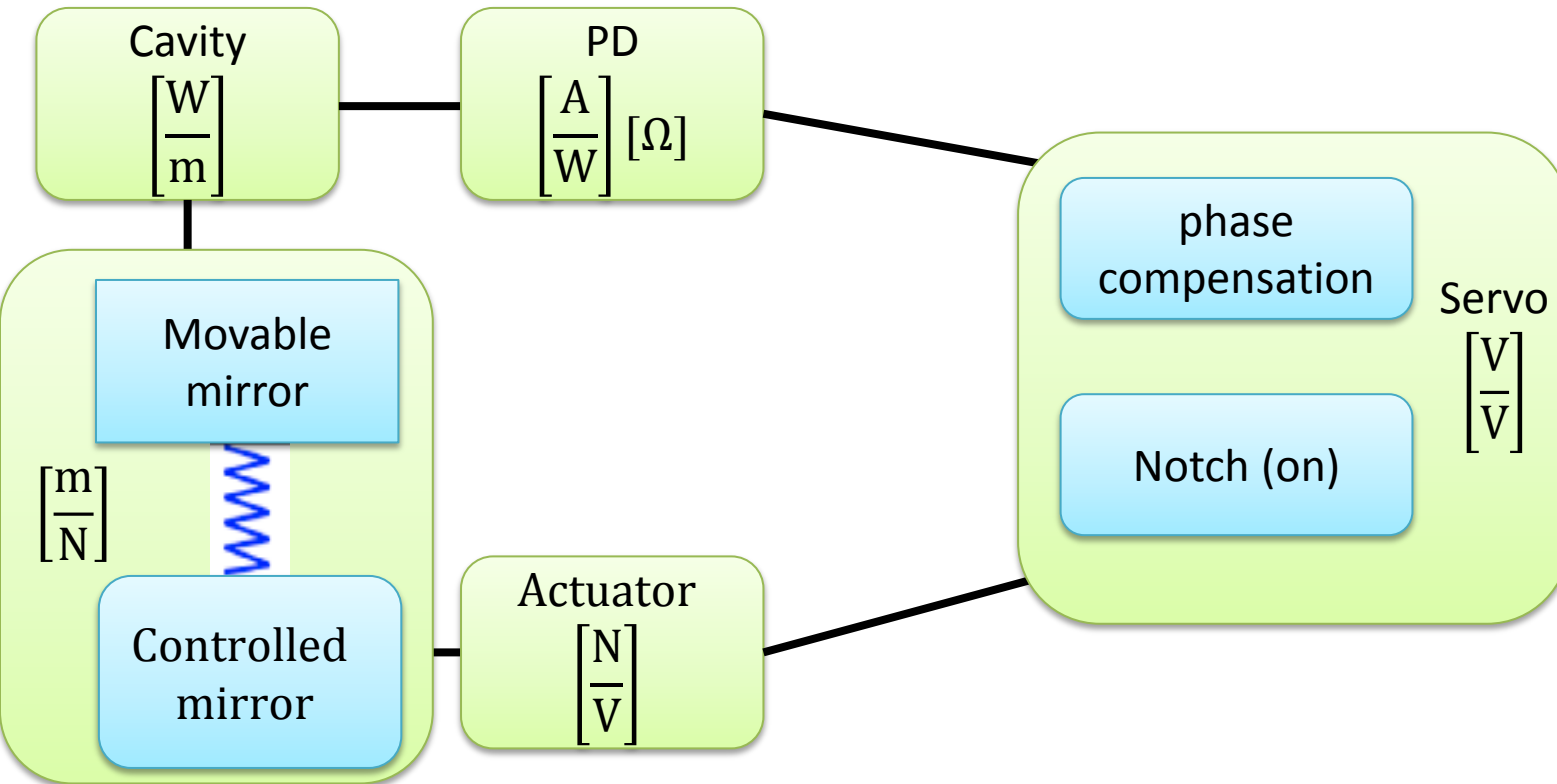
Relative Intensity Noise Level

# Details of the experiment

## Main experiment

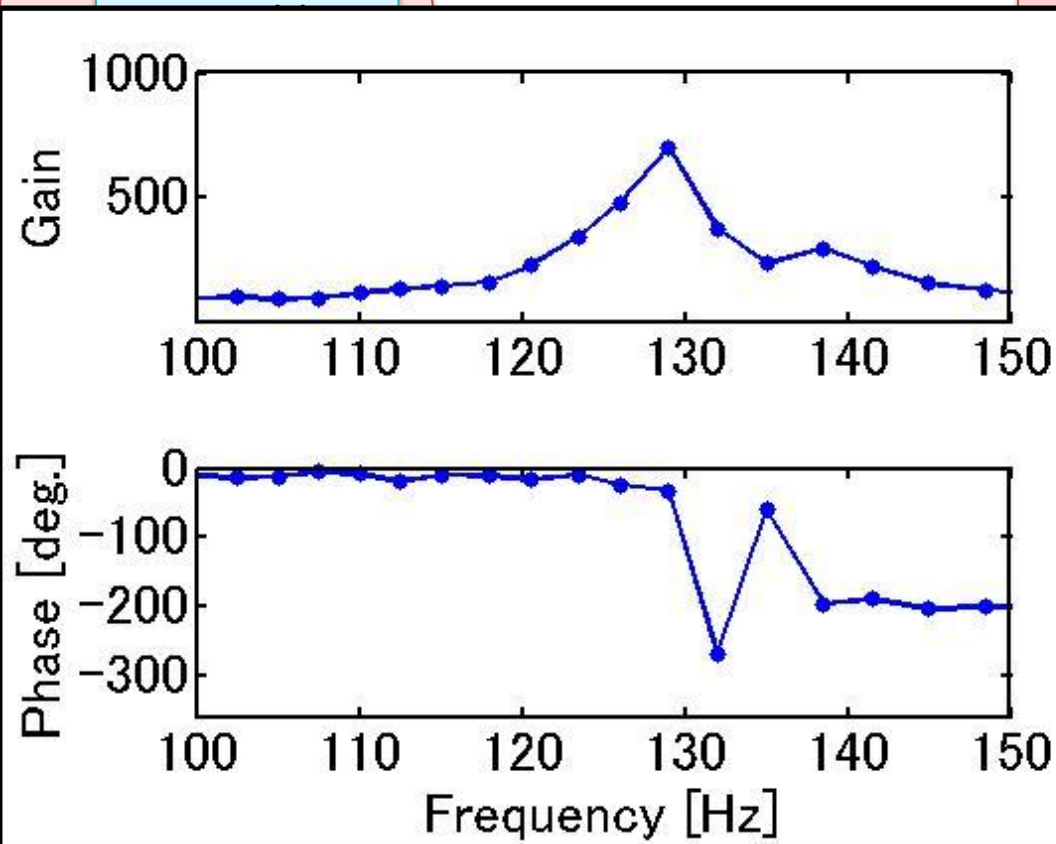
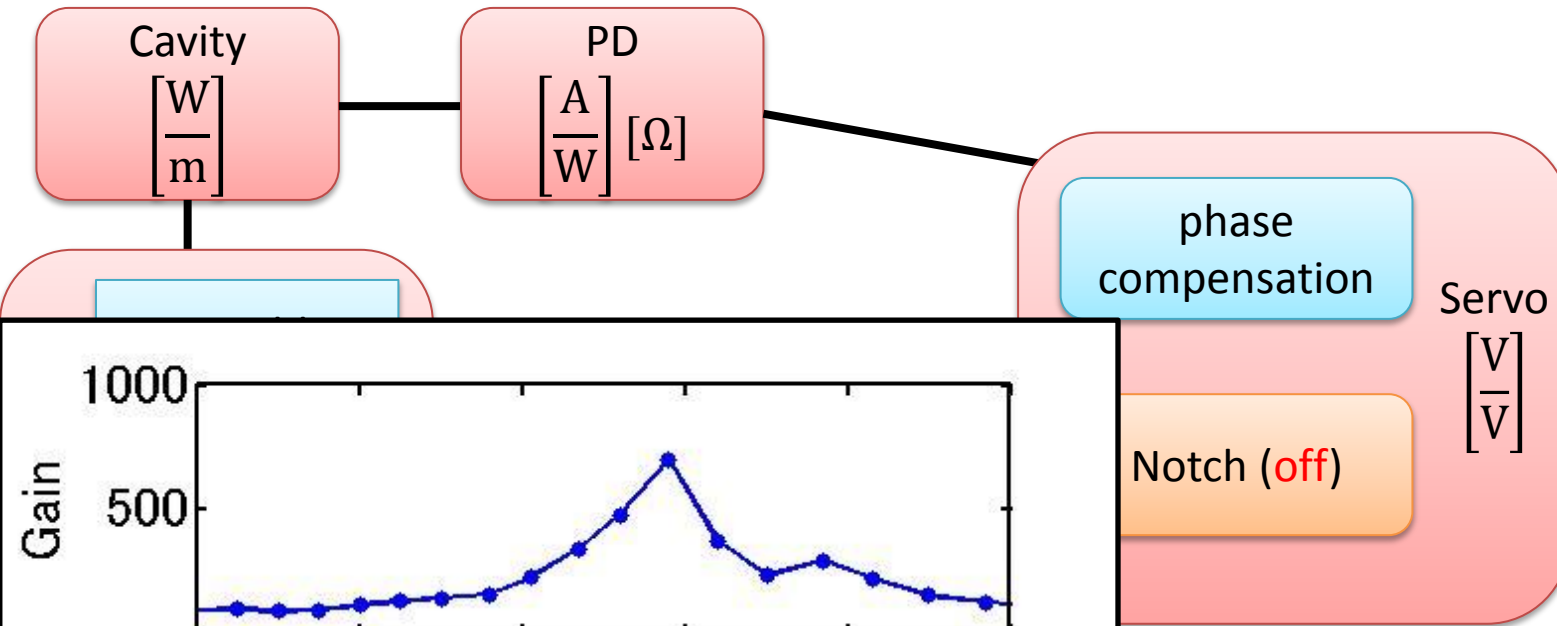
- i. Setup
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- v. Intensity stabilization of the laser  
⇒ Elimination of the classical noise of the light
- vi. Calibration

# Calibration

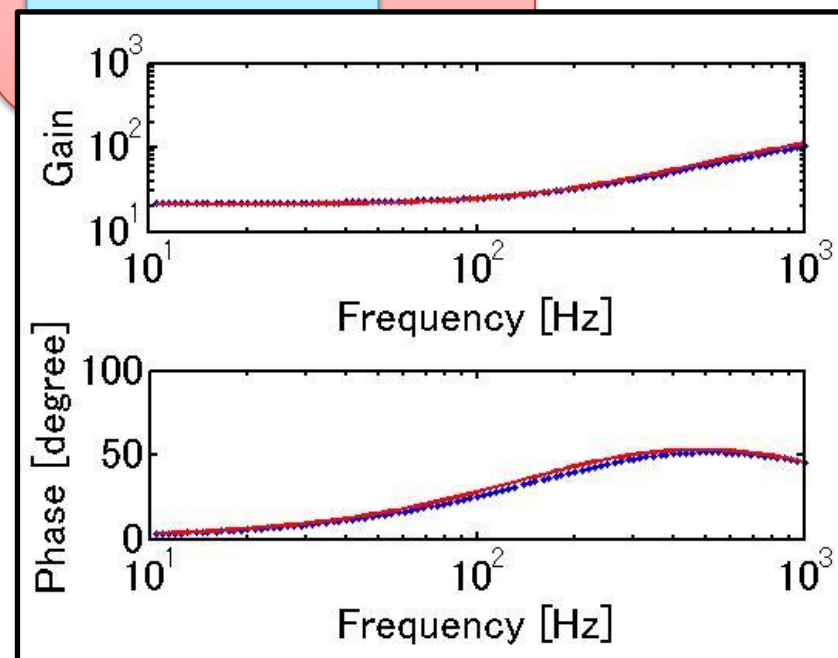
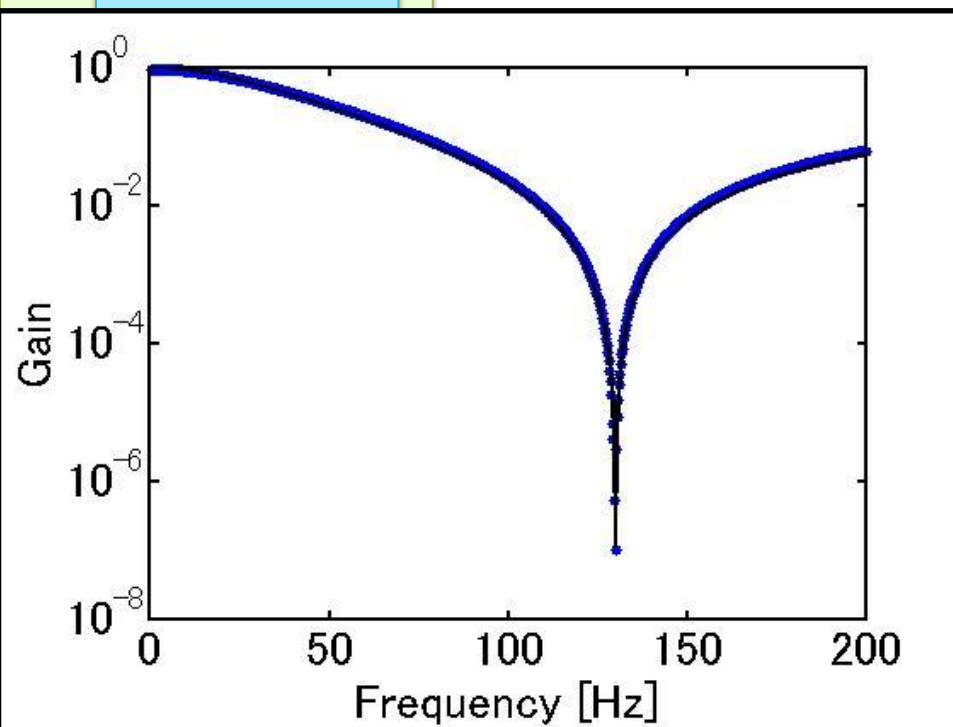
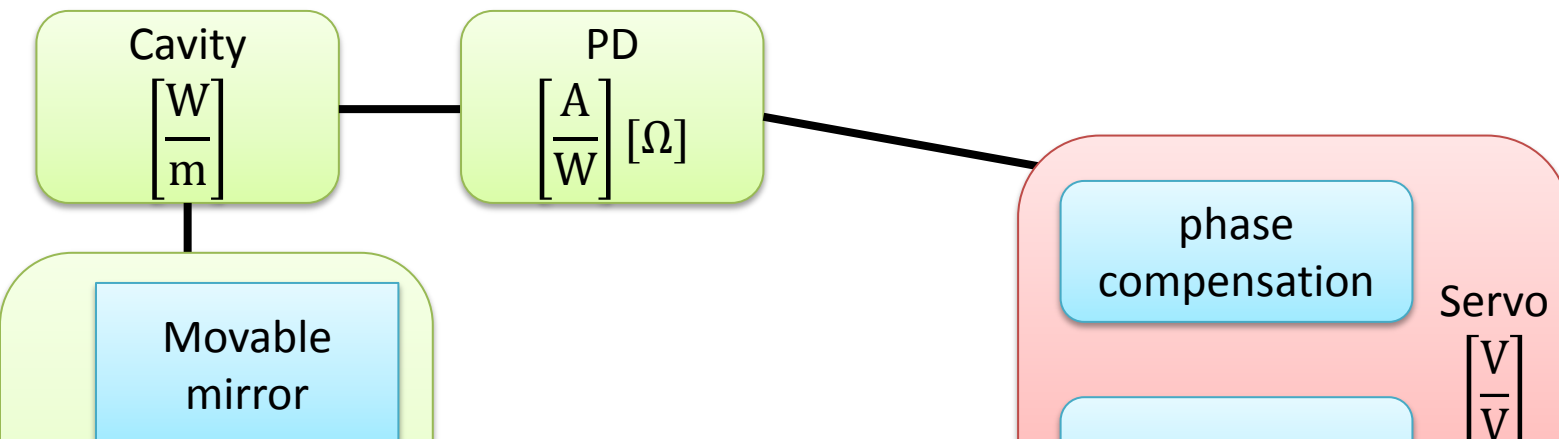


Open-loop transfer function

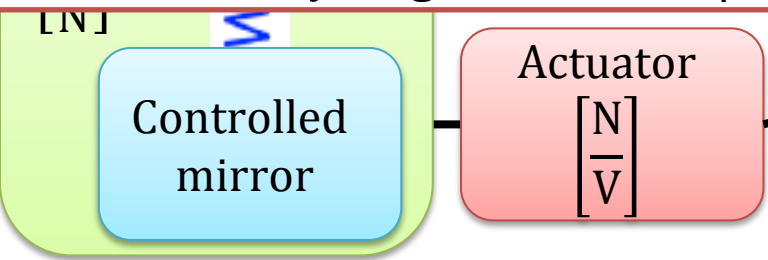
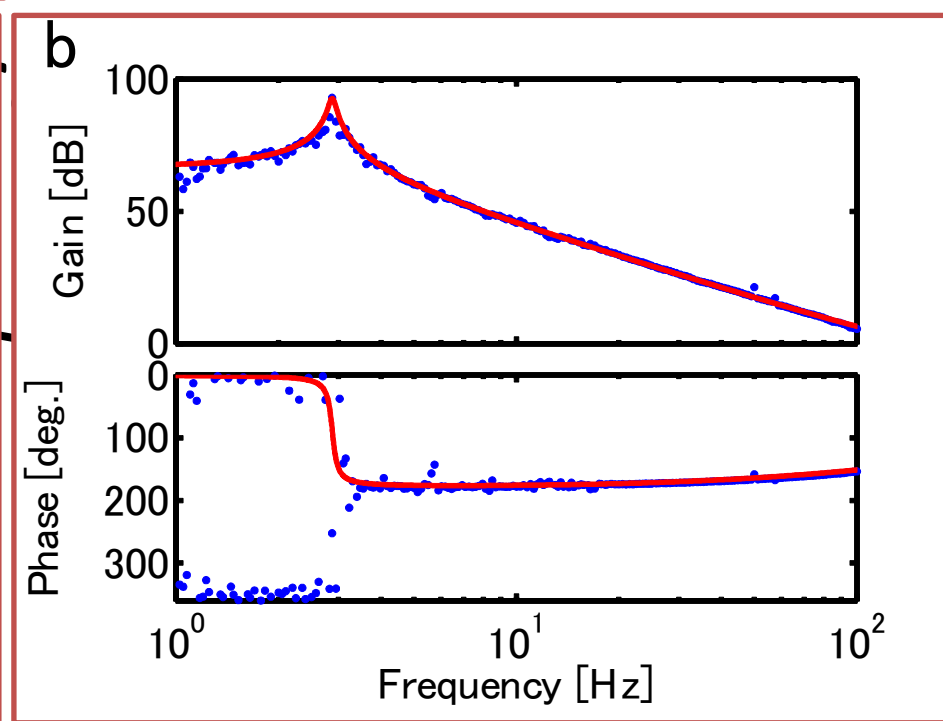
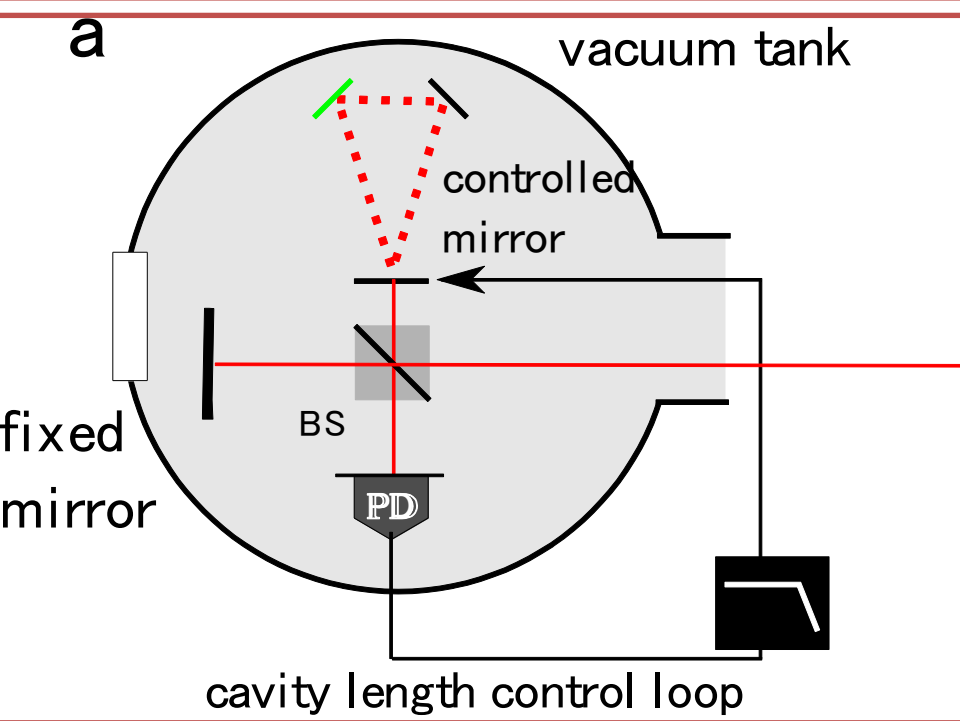
# Calibration



# Calibration

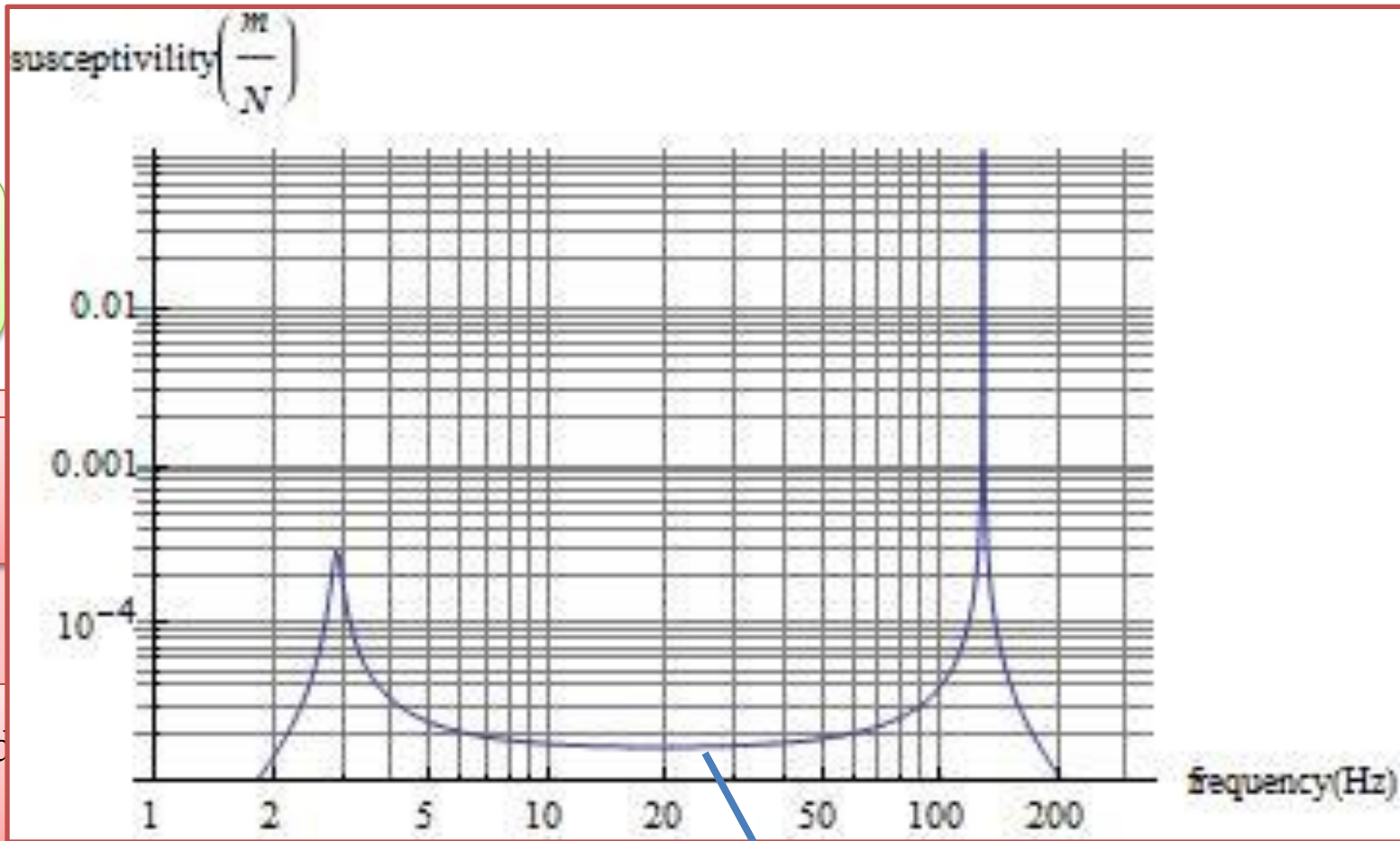
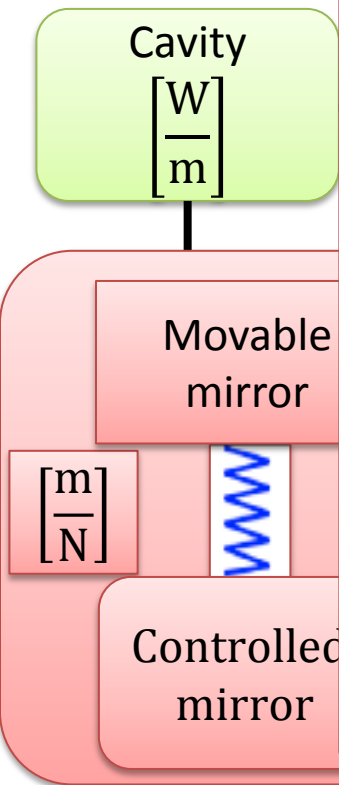


# Open-loop function for locking a MI



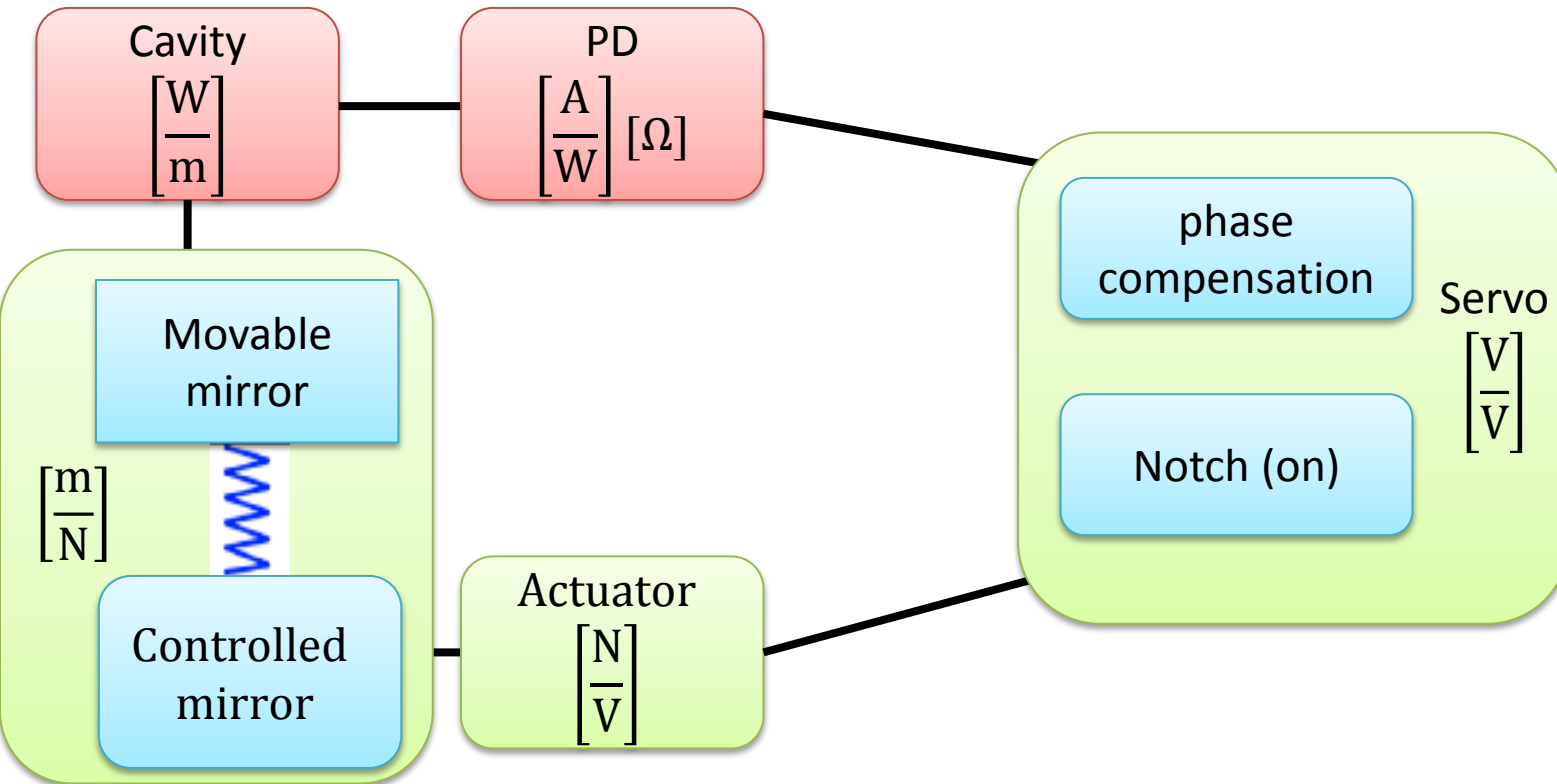
$$(1.4 \pm 0.1) \times 10^{-6} \left[ \frac{N}{V} \right]$$

# The model of a coupled oscillator



$$1.5 \times 10^{-5} \left[ \frac{m}{N} \right]$$

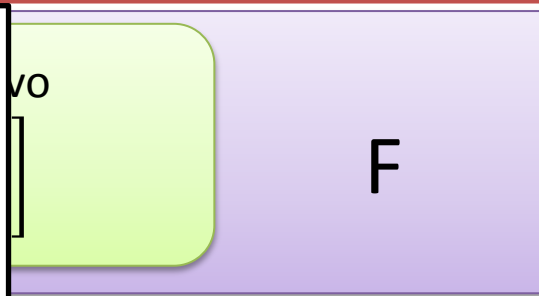
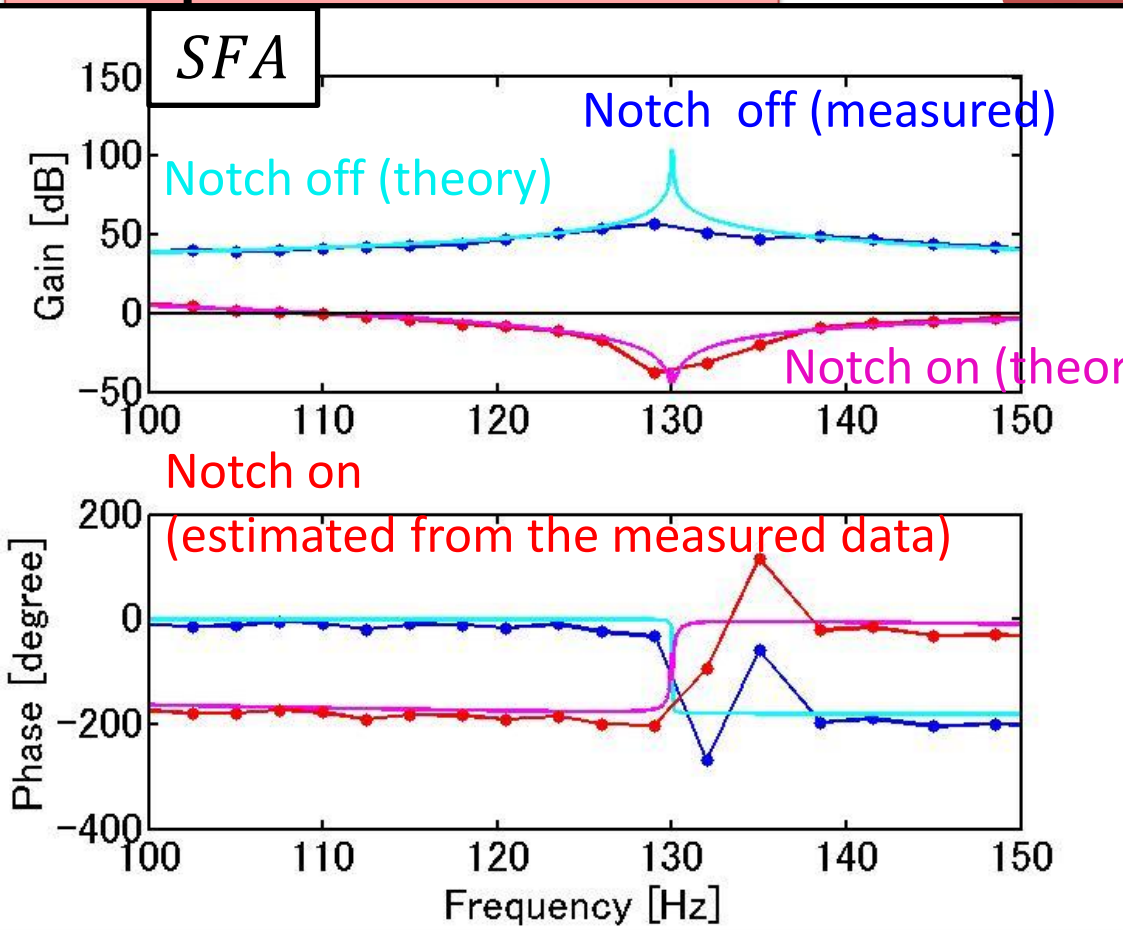
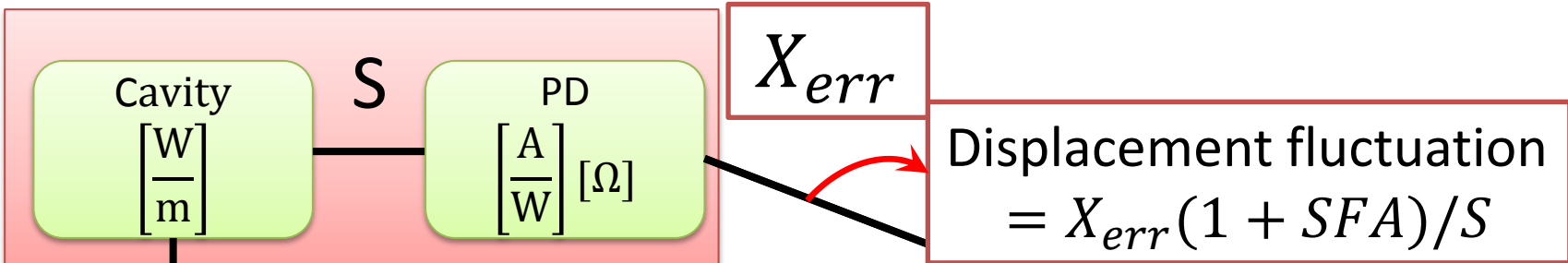
# Calibration



$$(8.5 \pm 0.6) \times 10^{-11} \left[ \frac{V}{m} \right]$$



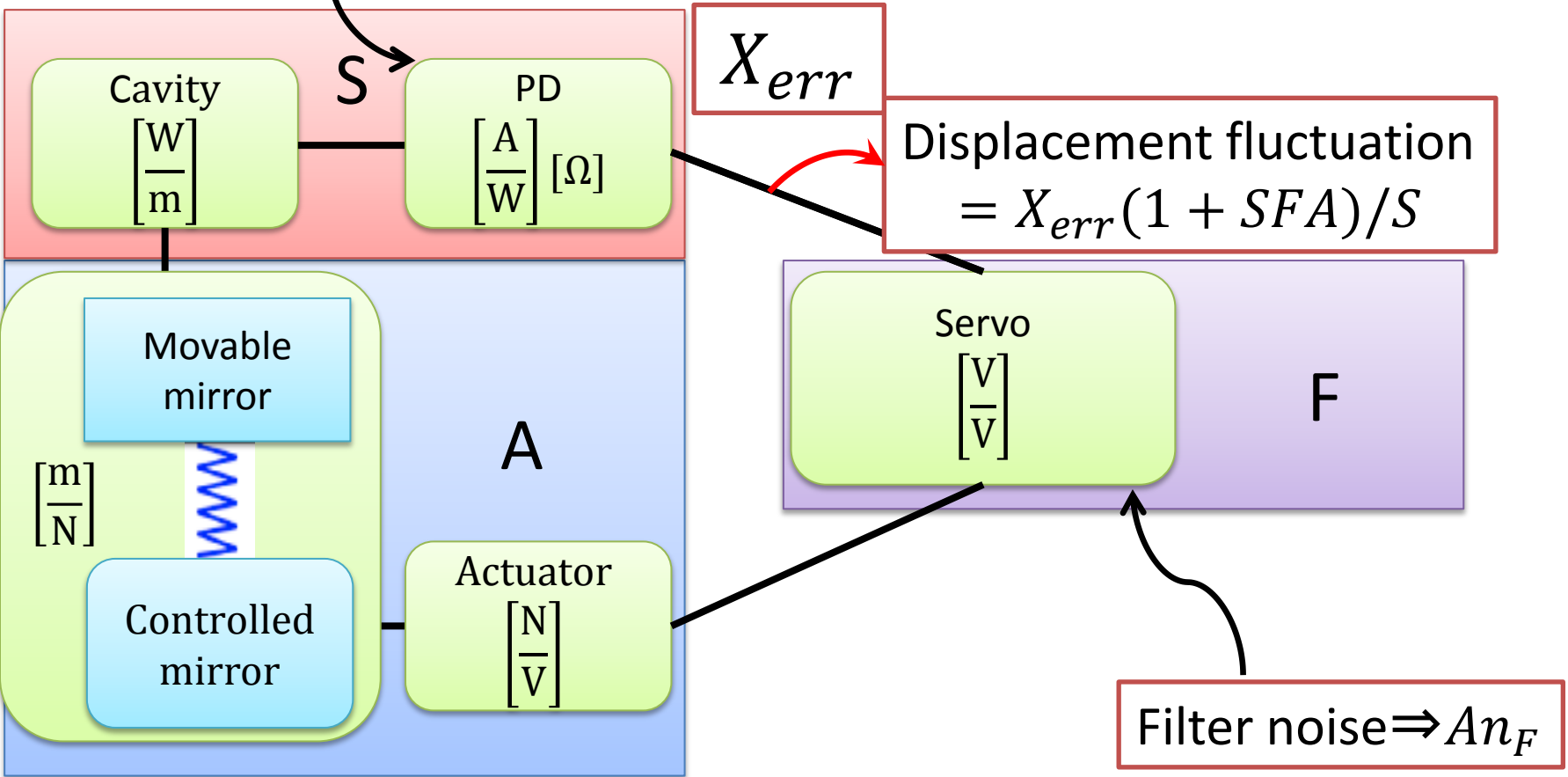
# Calibration

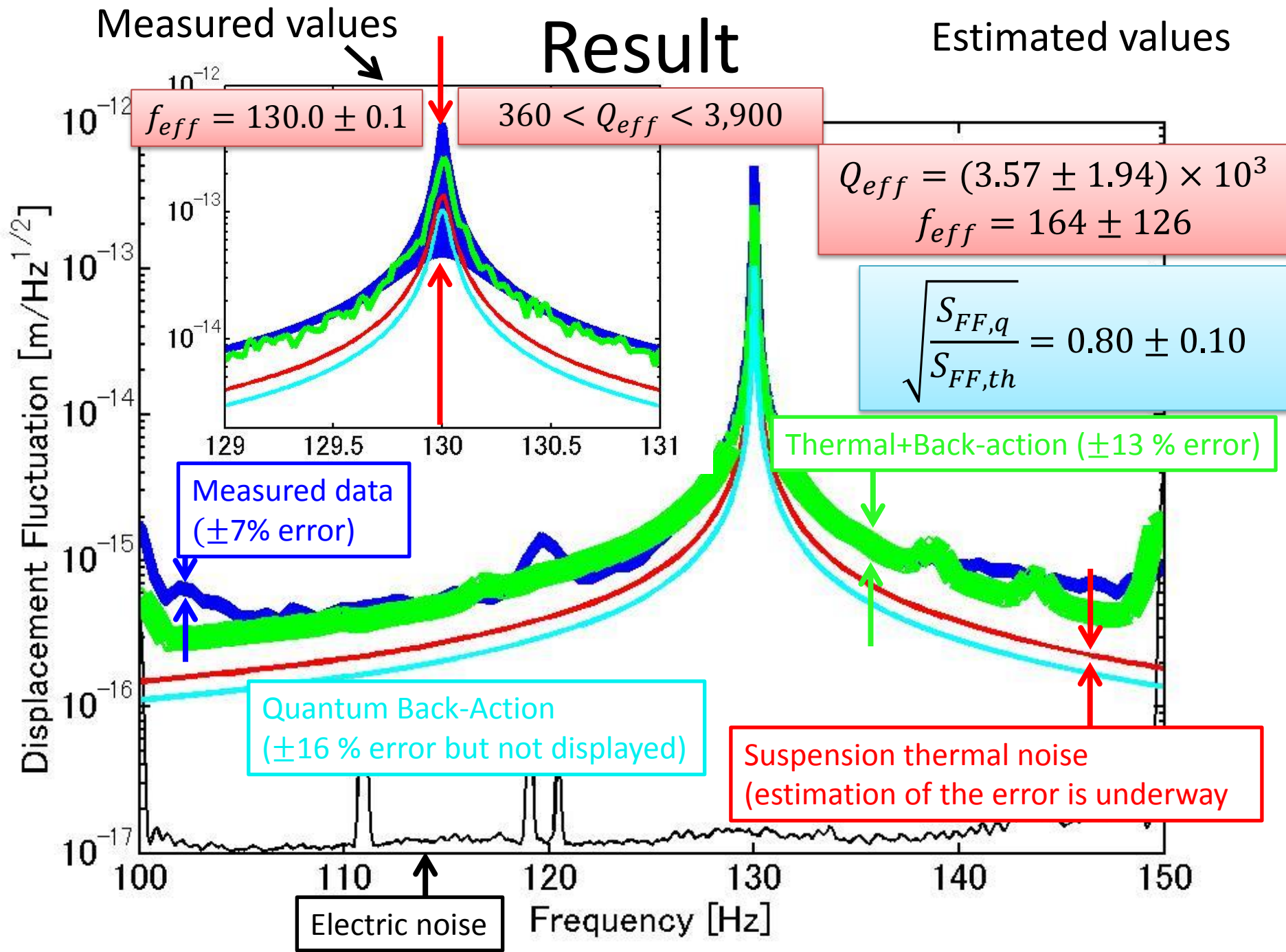


$$S = (8.5 \pm 0.6) \times 10^{-11} \left[\frac{\text{V}}{\text{m}}\right]$$

Sensor noise  $\Rightarrow n_s/S$

# Calibration





# Summary of parameters

Table 6.1: parameters

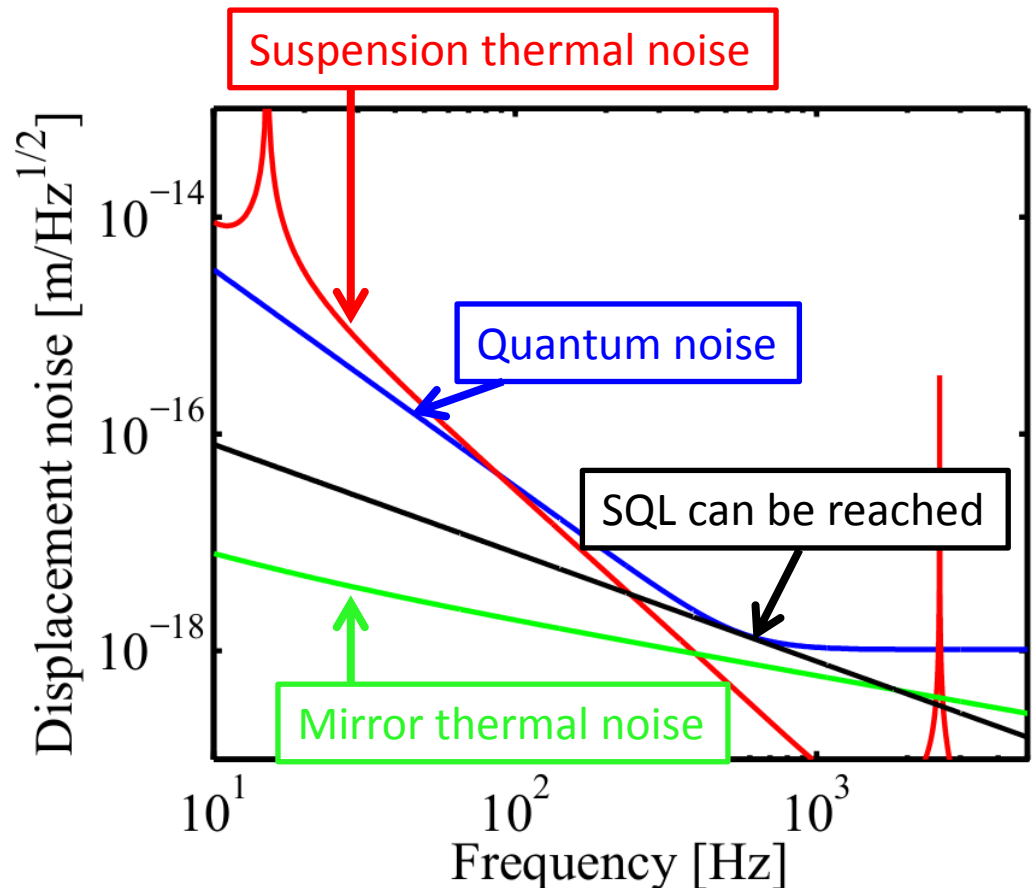
parameter	symbol	unit	measured value	estimated value
input power (d)	$P_{\text{in1}}$	mW	$5.0 \pm 0.5$	-
input power (s)	$P_{\text{in2}}$	W	$0.10 \pm 0.01$	-
intracavity power (d)	$P_{\text{in1,circ}}$	W	-	$3.00 \pm 0.34$
intracavity power (s)	$P_{\text{in2,circ}}$	W	-	$0.61 \pm 0.42$
photon number (d)	$N_{\text{in1,circ}}$	-	-	$(4.78 \pm 0.66) \times 10^9$
photon number (s)	$N_{\text{in2,circ}}$	-	-	$(9.87 \pm 7.00) \times 10^8$
normalized detuning (d)	$\Delta_{\text{a}}/\kappa$	-	$0.048 \pm 0.005$	-
normalized detuning (s)	$\Delta_{\text{b}}/\kappa$	-	$1.30 \pm 0.07$	-
optomechanical coupling	$g/2\pi$	Hz/m	$(2.8 \pm 0.1)\omega_{\text{c}}$	(design) $2.84\omega_{\text{c}}$
cavity decay rate	$\kappa/2\pi$	Hz	$(1.52 \pm 0.03) \times 10^6$	-
cavity decay rate (in1)	$\kappa_{\text{in1}}/2\pi$	Hz	$(1.30 \pm 0.03) \times 10^6$	-
cavity decay rate (in2)	$\kappa_{\text{in2}}/2\pi$	Hz	$(3.64 \pm 2.46) \times 10^4$	-
finesse	$\mathcal{F}_{\text{p}}$	-	$(1.10 \pm 0.02) \times 10^3$	-
effective resonant frequency	$f_{\text{eff}}$	Hz	$130 \pm 0.1$	$164 \pm 126$
effective quality factor	$Q_{\text{eff}}$	-	360-3,900	$(3.57 \pm 1.94) \times 10^3$
quality factor	$Q_{\text{m}}$	-	$4.7 \times 10^5$	$< 3.8 \times 10^6$
thermal fluctuating force	$\sqrt{S_{\text{FF,th}}^{(2)}}$	N/ $\sqrt{\text{Hz}}$	-	$(1.44 \pm 0.02) \times 10^{-16}$
quantum back-action force	$\sqrt{S_{\text{FF,q}}^{(2)}}$	N/ $\sqrt{\text{Hz}}$	-	$(1.08 \pm 0.07) \times 10^{-16}$
actuator efficiency	$A$	N/V	$(1.4 \pm 0.1) \times 10^{-6}$	-
slope of the PDH signal	$H$	V/m	$(8.5 \pm 0.6) \times 10^{-11}$	-
signal to noise ratio	-	-	-	$0.80 \pm 0.10$

# Next Step

- Reaching the SQL using MZ interferometer

1. Mass 5 mg  $\Rightarrow$  15 mg
2. Diameter 3  $\mu$ m  $\Rightarrow$  1  $\mu$ m
3. Length 5 cm  $\Rightarrow$  2 cm
4. Improve of clamping (Q-factor will be 8-times)
5. Other material (e.g. CNTF) (test is underway)

Q-factor will be increased from  $4.7 \times 10^5$  to  $2 \times 10^7$   
( $2 \times 9 / 2.5 \times 8 \approx 58 > 40$ )



# Conclusion

- **Triangular cavity** has potential to reach the SQL.
- At the moment, the ratio of quantum back-action to thermal noise is estimated to be  **$0.80 \pm 0.10$** .

Next  $\Rightarrow$  More validity of the measurement  
e.g.) error of the Q-factor,  
freq. noise,  
dependence of the laser power, and  
correlation measurement

Improve of the pendulum

$\Rightarrow$  **reaching the SQL**