Effect of heat fluxes on thermal noise of mechanical oscillators

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www.rarenoise.lnl.infn.it

Thermal noise in IFOs

In the community of GW detectors,
thermal noise is regarded as a difficul to beat noise source that limits the sensitivity

To beat thermal noise in the IFOs:

- low mechanical losses of mirror masses
- low suspension (pendulum) resonances
- high frequency of mirror internal resonances
- low temperatures

However the study of thermal noise has noble birth:
Einstein used the brownian motion as proof of the existence of atoms

In this talk I will discuss how the study of thermal noise in the IFOs can give us new insights into one very hot topic of Statistical Mechanics, ie into fundamental Physics

Fluctuation-dissipation theorem

Main result of statistical mechanics: relation between the spontaneous fluctuations and the response to external fields of physical observables

$$S(\omega) = 4k_B T \Re\left[Z(\omega) \right]$$

Relation between a property of a system at equilibrium (ie the fluctuations around mean value) and a parameter that characterizes an irreversible process (ie the dissipation).

Fluctuation: variation of a physical obesrvable arount its mean value
Dissipation: how the system responds to an external excitation

Very powerful:

1) Can get nonequilibrium data from equilibrium observations:
   allows one to predict the average response to external perturbations, without applying any perturbation (eg approach of molecular dynamics)

2) Predict amount of fluctuations from macroscopic measurements (of the dissipation)

Fluctuation-dissipation theorem /2

$$S(\omega) = 4k_B T \Re\left[Z(\omega) \right]$$

This result sits on the thermodynamic equilibrium hypothesis
and on the Energy Equpartition principle

T is the equilibrium temperature

Situations in which equilibrium fails are hot topic in Statistical Mechanics

If local thermodynamic equilibrium holds then one can still use the above with T= T(x)

If local equilibrium fails?

The search for FD-like relations for systems driven far from equilibrium
has been an active area of research for many decades.

Problem of Temperatures

Indeed, when LTE fails, one faces the problem of even defining what Temperature is:

in non-equilibrium systems the concept of global temperature is not always well-defined & different definitions lead to different results.

Thus, in the nonequilirium , great care should be taken

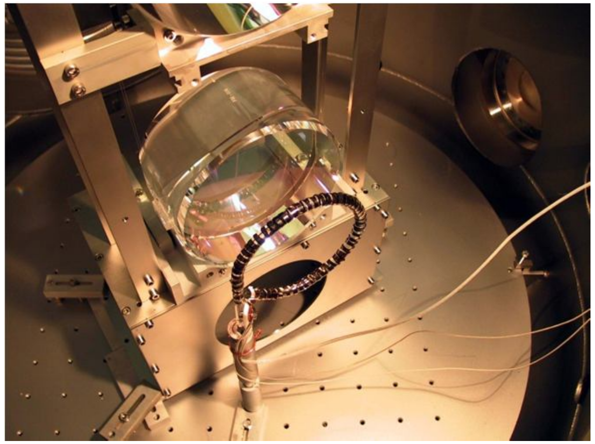
Why do we bother?

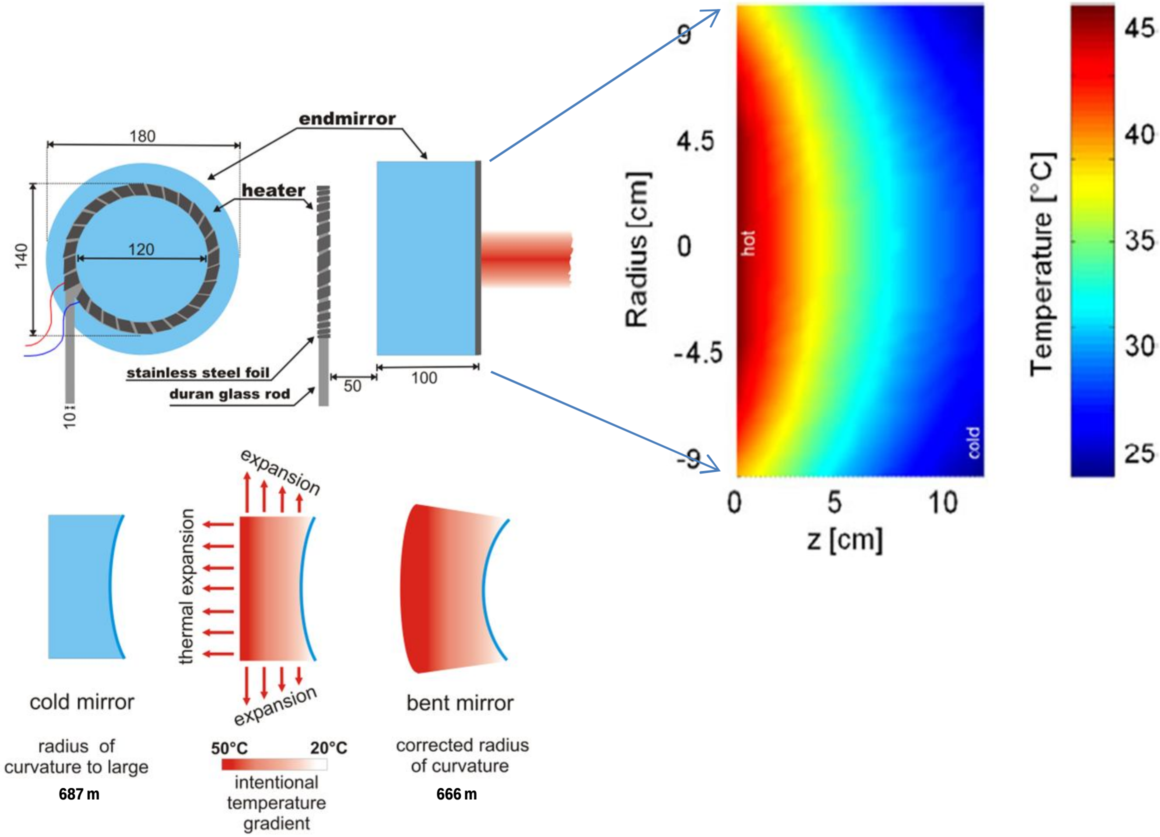
Non equilibrium in GW interferometers

Heat fluxes due to laser power dissipated in the mirror

hunderds of mW estimated to be lost in the mirrors in the Advanced IFOs

Thermal compensation systems drive IFOs even farther from equilibrium



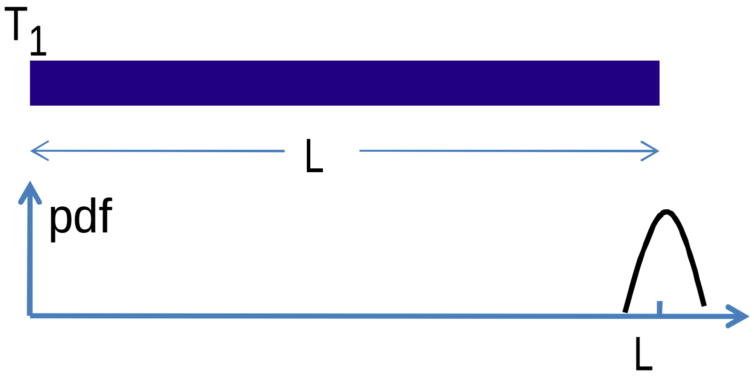


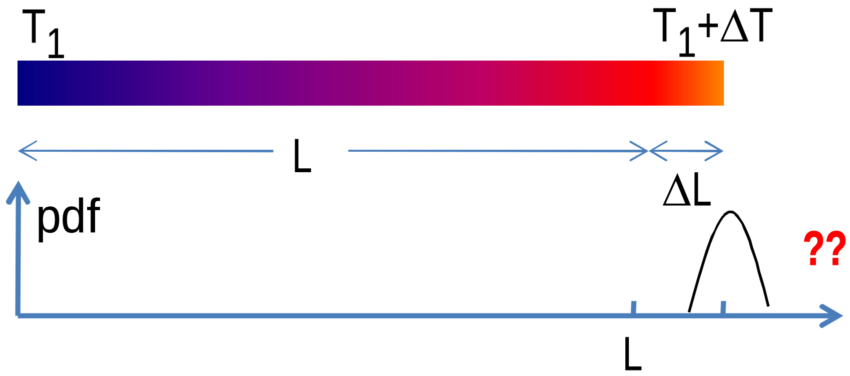
How to size the problem

Question:
how to estimate the spontaneous vibration fluctuations ('thermal noise') in non-equilibrium systems?

Let us monitor the spontaneous length fluctuations of a rod of length L at temperature T_1

Now apply a steady-state thermal difference ΔT between the ends by flowing heat $W=dQ/dt$.
The rod expands by ΔL via thermal expansion:





Thermal noise = ??

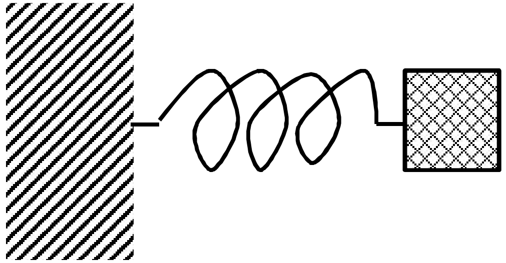
What is the variance of the noise?
-> does some sort of equipartition hold?
What is the statistical distribution of the length fluctuations?
-> is it still Gaussian?

Class. Quantum Grav. 27 (2010) 084032

The experimental strategy

We realized mechanical pieces
in Aluminum as the ‘rod’

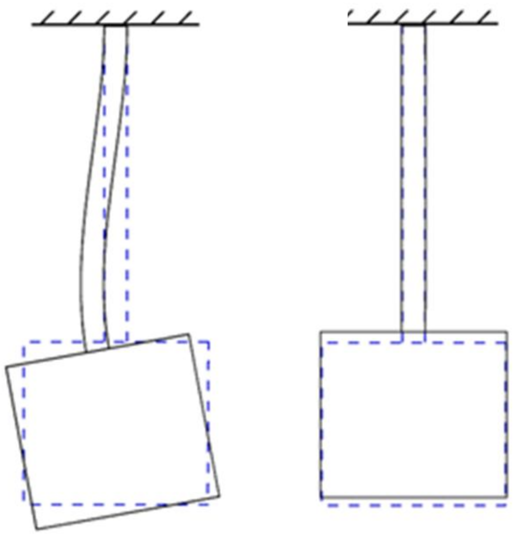
We monitored the ‘rod’ vibration in correspondance
of two elastic modes of resonance

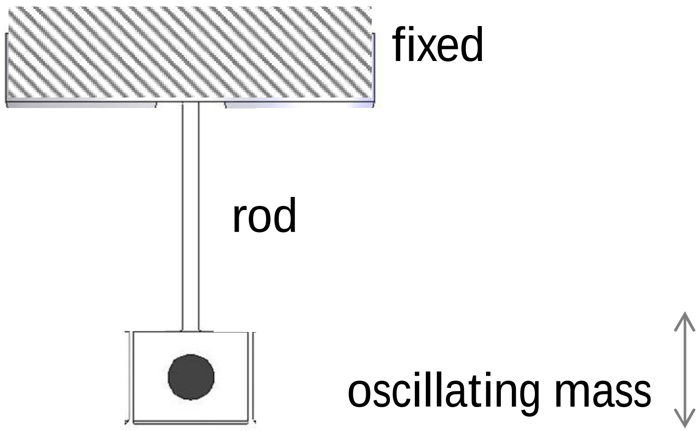


Induce NonEquilibrium Steady State (NESS) by flowing heat across the rod,
ie by setting constant thermal differences between the rod extremes.

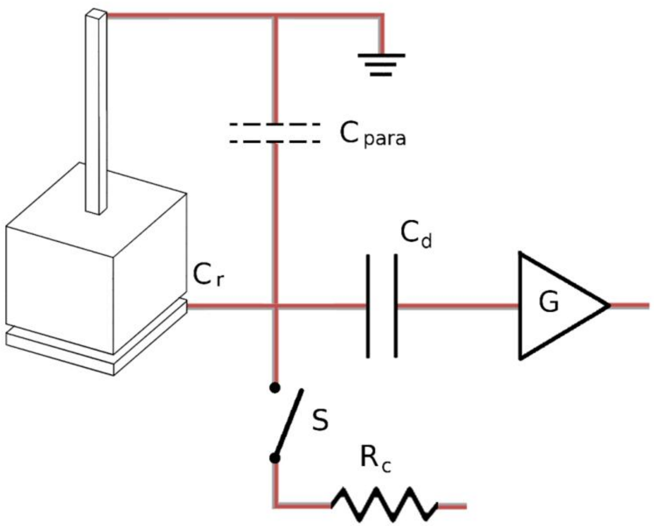
The observable: displacement fluctuations of the oscillators

Aluminum oscillator:



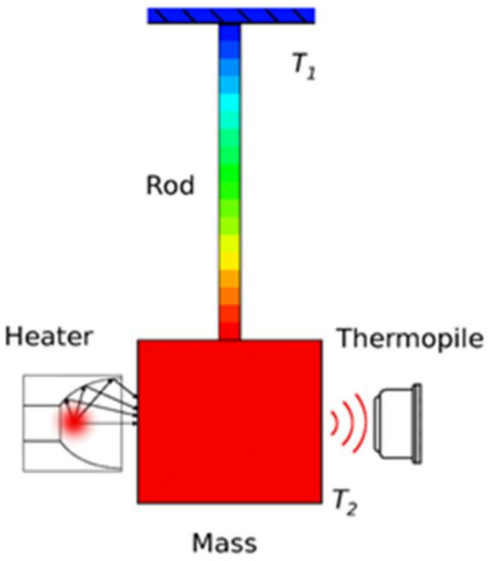


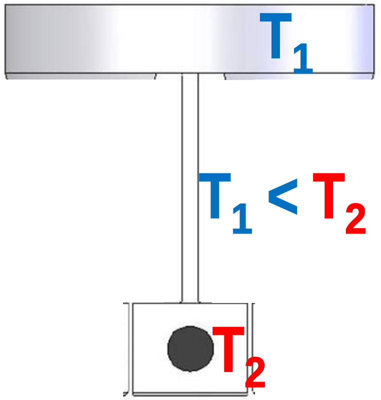
capacitive readout of oscillator vibration:



The disequilibrium: thermal differences

possibility to apply thermal difference

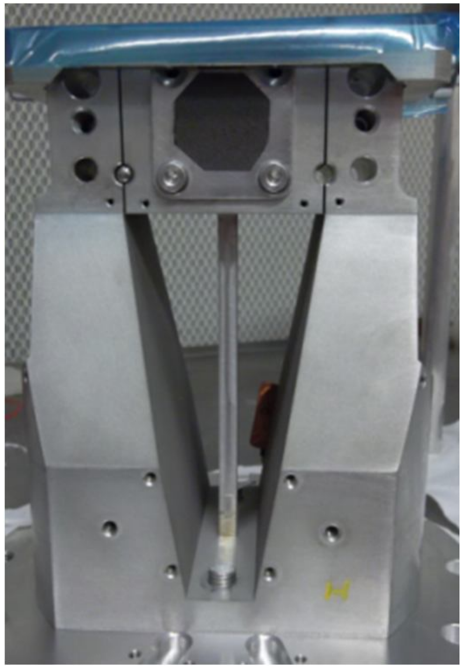




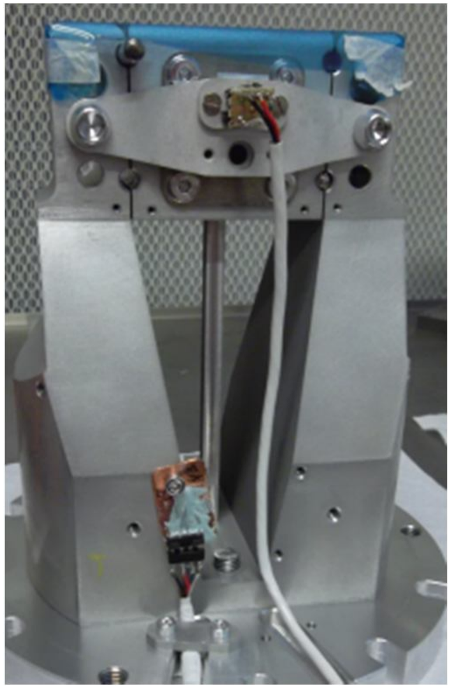
$$\Delta T = T_2 - T_1$$

Mounting the oscillator

the oscillator (upside down)

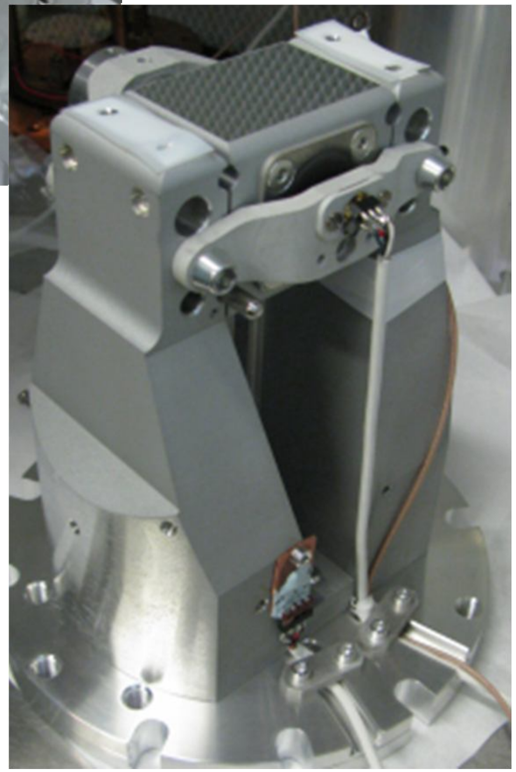


added thermometers...



... and heater

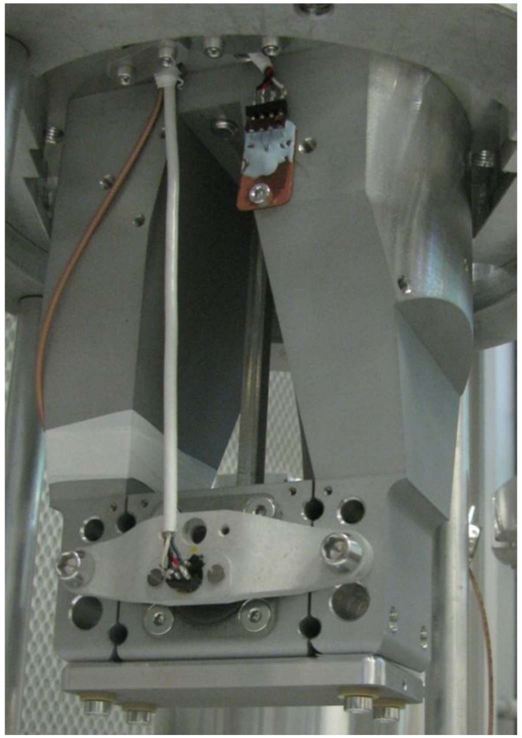
preparing for the capacitive readout



Assembling the experiment

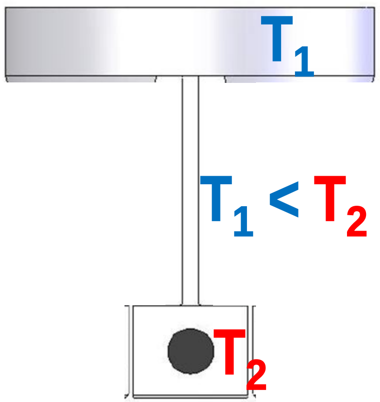
oscillator suspended by mechanical filters

the complete device



the full setup, enclosed in thermal insulator, and with active thermal control

The experimental campaign



We varied the temperature T_1 of the oscillator fixed end in a 20K interval around room temperature

Simultaneously we heated the oscillating mass, thus raising its temperature T_2 thus setting several temperature differences T_2-T_1 : $0K \leq T_2 - T_1 \leq 15K$

We consider data only taken while in steady state:

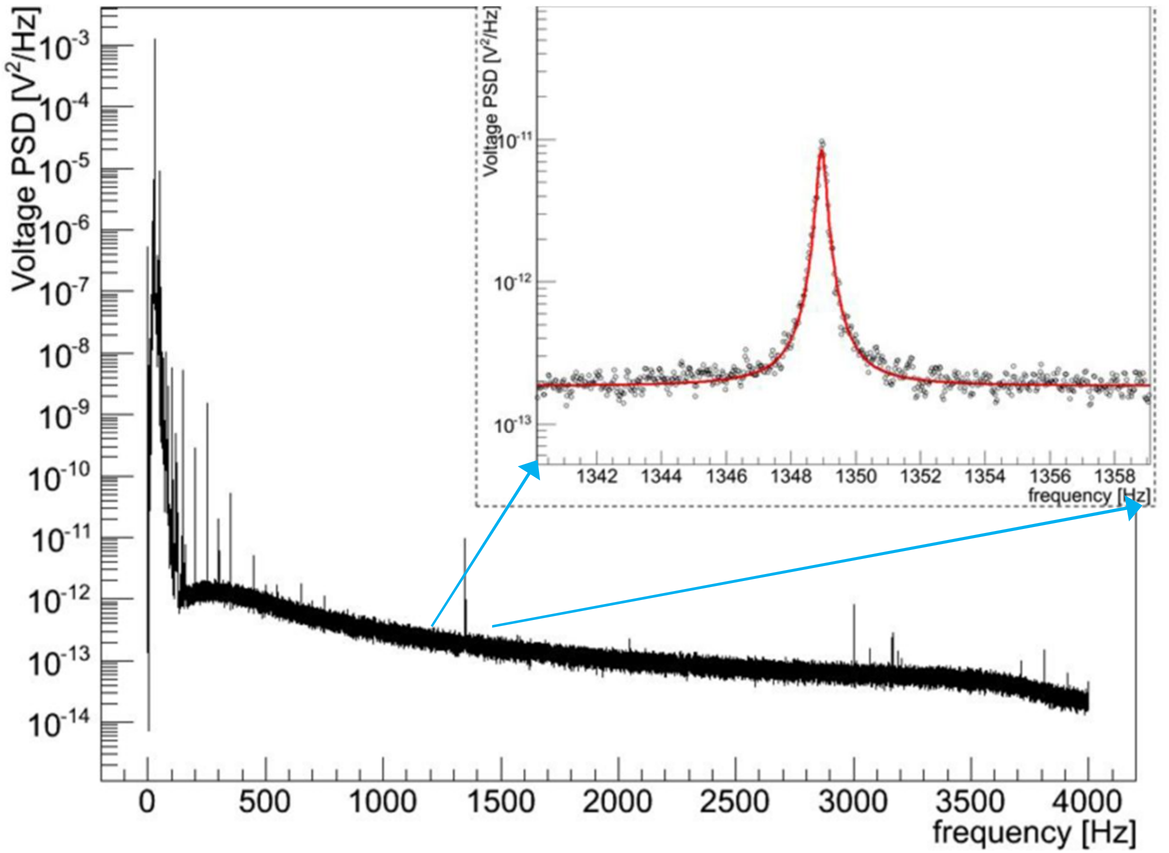
$$\sqrt{\left(\frac{1}{T_1}\frac{dT_1}{dt}\right)^2 + \left(\frac{1}{T_2}\frac{dT_2}{dt}\right)^2} < 6 \cdot 10^{-8} s^{-1}$$

→ temperature stability of <~10μK/s

We kept the experiment running for months.

Typical measurement

Fit of PSD around the resonance: $\rightarrow \omega_l \, , Q \, , \left\langle V(t)^2 \right\rangle$

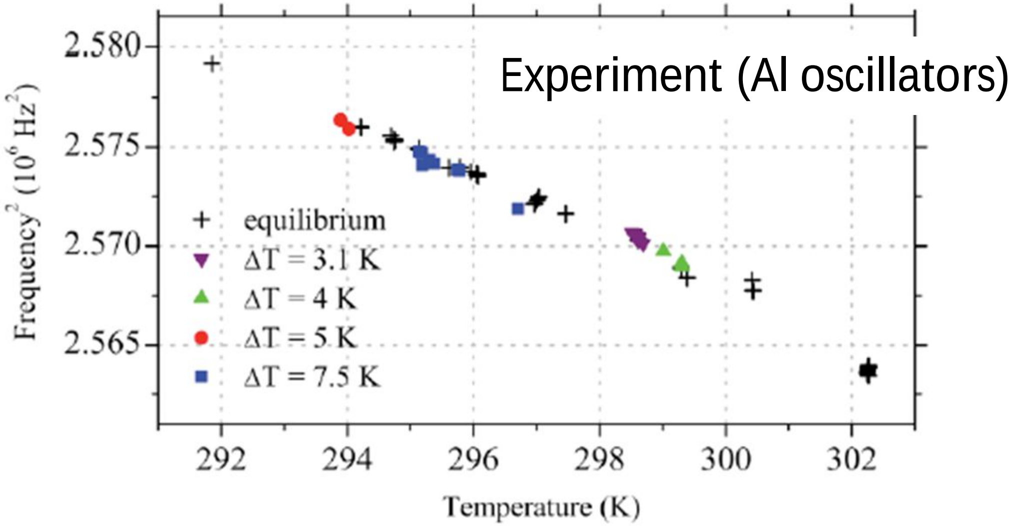


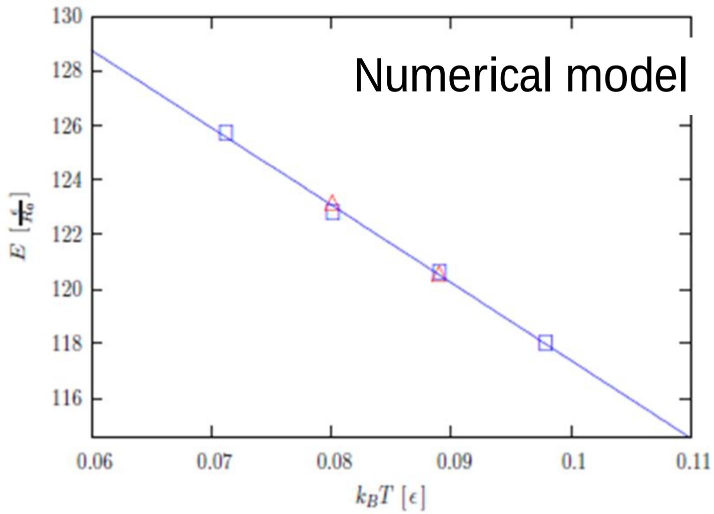
Thermal rms ~ $2 \times 10^{-14} m$

1^{st} result

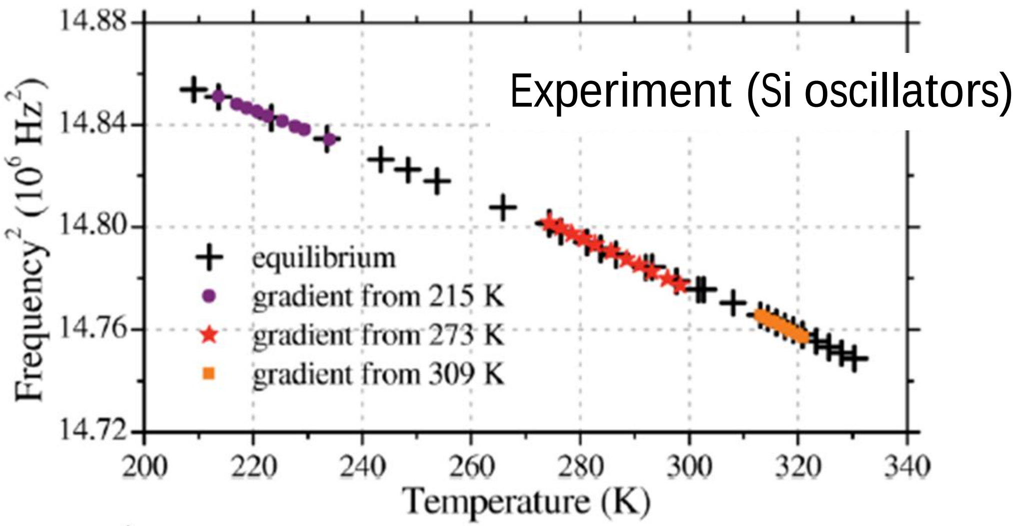
The resonant frequency and Young modulus scale with the average Temperature:
not a surprise, as if local equilibrium holds

This is confirmed by results on Silicon oscillators and by numerical results of 1dim chain of oscillators





Phys. Rev. E 85, 066605 (2012)



Typical measurement

Fit of PSD around the resonance:

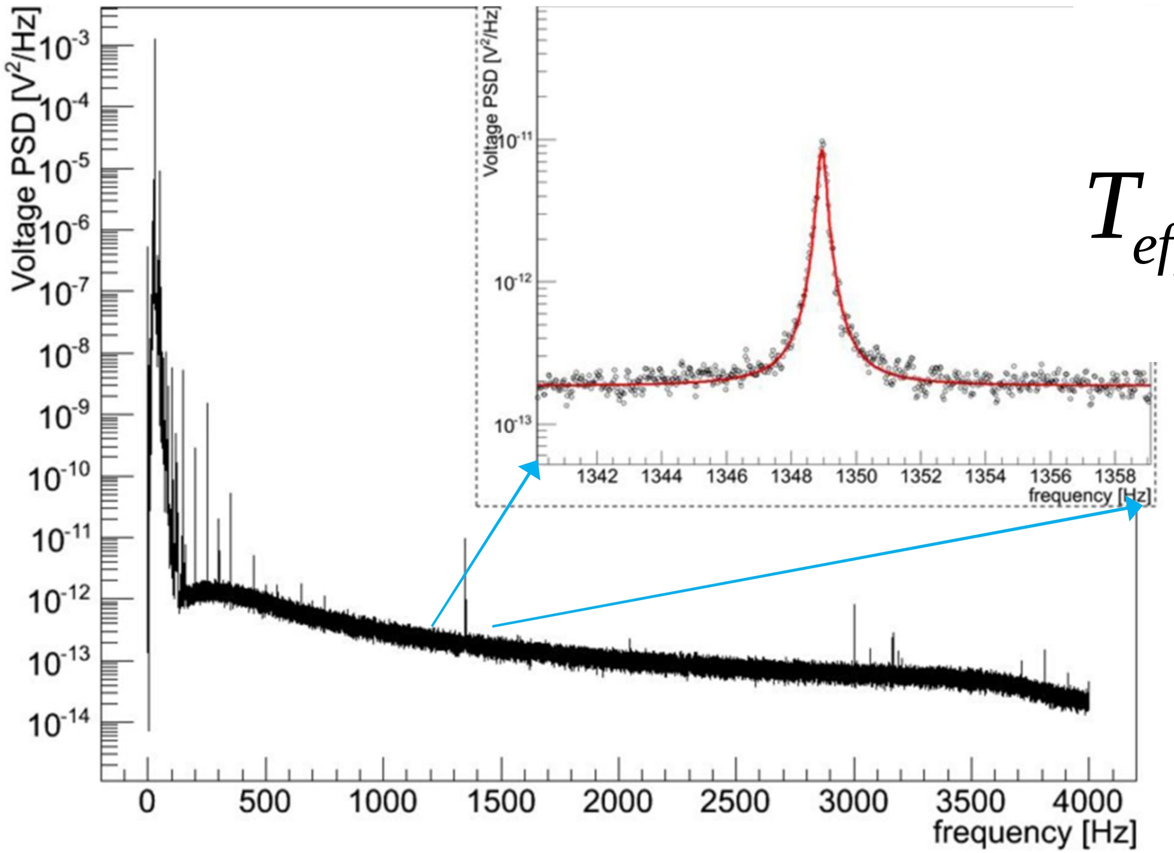
→ $\omega_l\ ,Q\ ,\langle V(t)^2\rangle$

→ conversion into mass vibration

→ energy of the mode

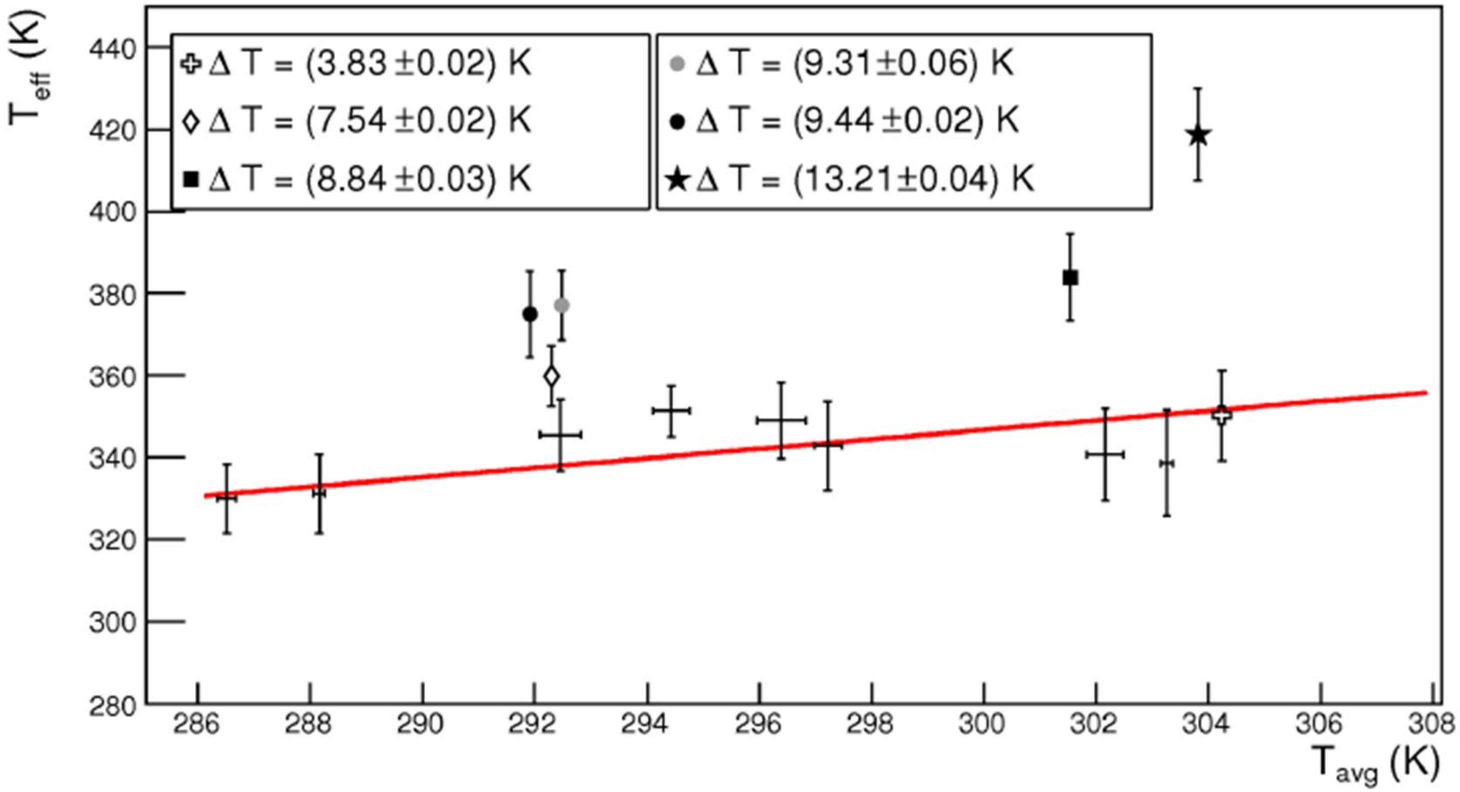
→ effective temperature T_{eff} of the mode

$$T_{eff}=\frac{m_l\,\omega_l^{\,2}\left\langle x_l\left(t\right)^2\right\rangle}{k_B}$$



Thermal rms ~ $2\times10^{-14}m$

Effective temperature



Intensity of the fluctuations increases with the heat flux (ie with ΔT)

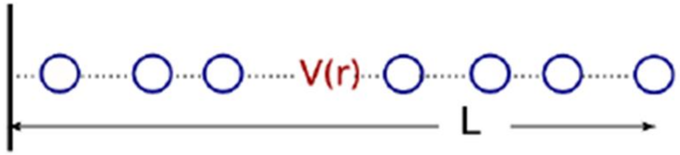
$$\left\langle x_l(t)^2 \right\rangle \; > \; \frac{k_B T}{m_l \omega_l^2} \quad \rightarrow \quad T_{eff} > T$$

T is the physical temperature,
T_{eff} approximates it well only at equilibrium.

J. Stat. Mech. (2013) P12003

Numerical model

In parallel with the experiment, we developed molecular dynamics model:



L as the observable

First and second neighbors with Lennard-Jones potentials.
Left clamped, immediate neighbors thermostated at T_1.
Right free, and two rightmost particles thermostated $T_2 \geq T_1$
$(T_1 + T_2)/2$ fixed, $\Delta T = T_2 - T_1$.

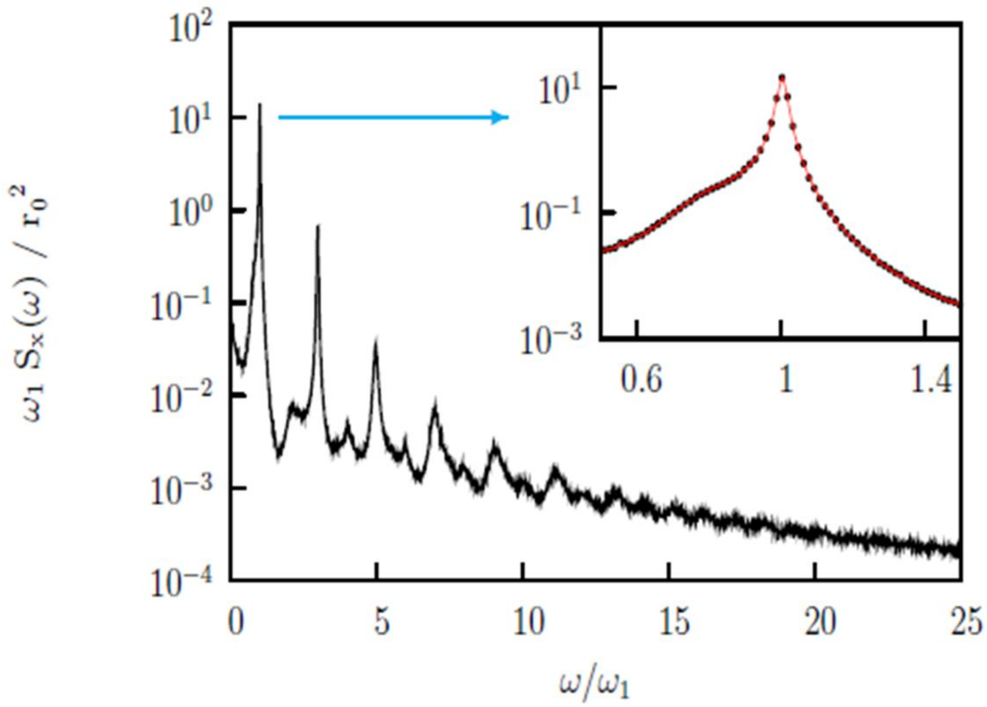
1D model reproduces (real) thermo-elastic properties,
at equilibrium and nonequilibrium:
the NESS gives results equivalent an equilibrium at the average temperature (no surprise)

P. De Gregorio et al., PRB 84, 224103 (2011)
LC et al., PRE 85, 066605 (2012)

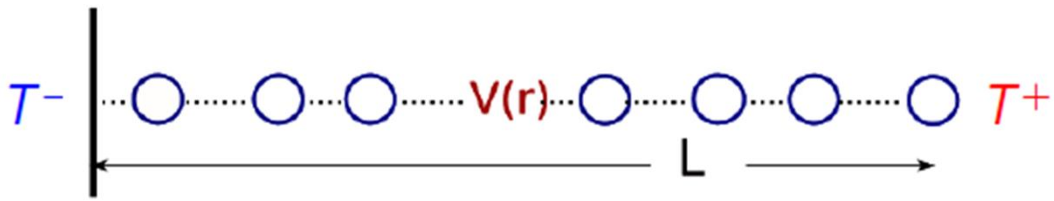
Analysis of numerical data

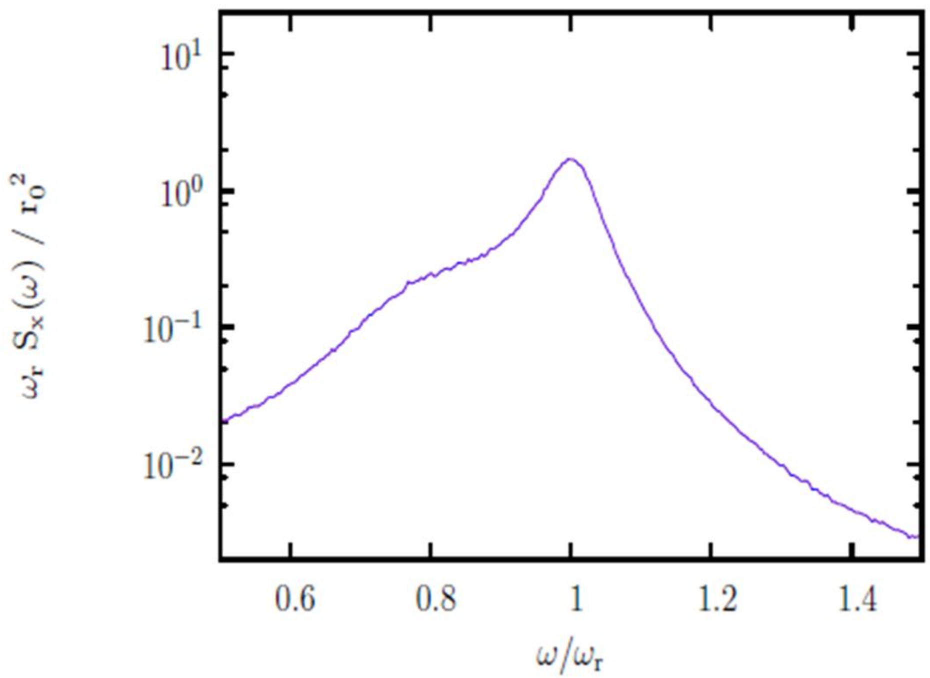
Similarly to experiment
we run equilibrium and nonequilibrium simuations:

Fourier analysis of time evolution of total length of the chain
-> longitudinal modes of vibration of the chain



Effects of growing gradients, at same average T





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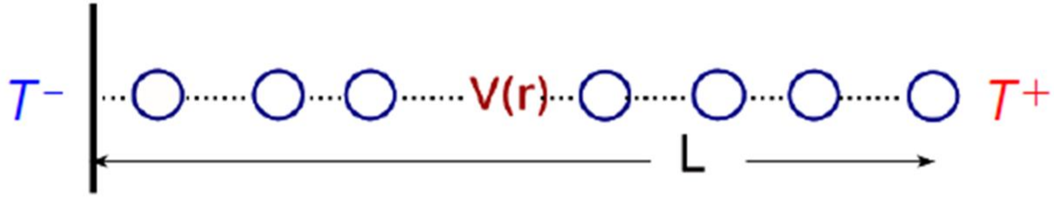

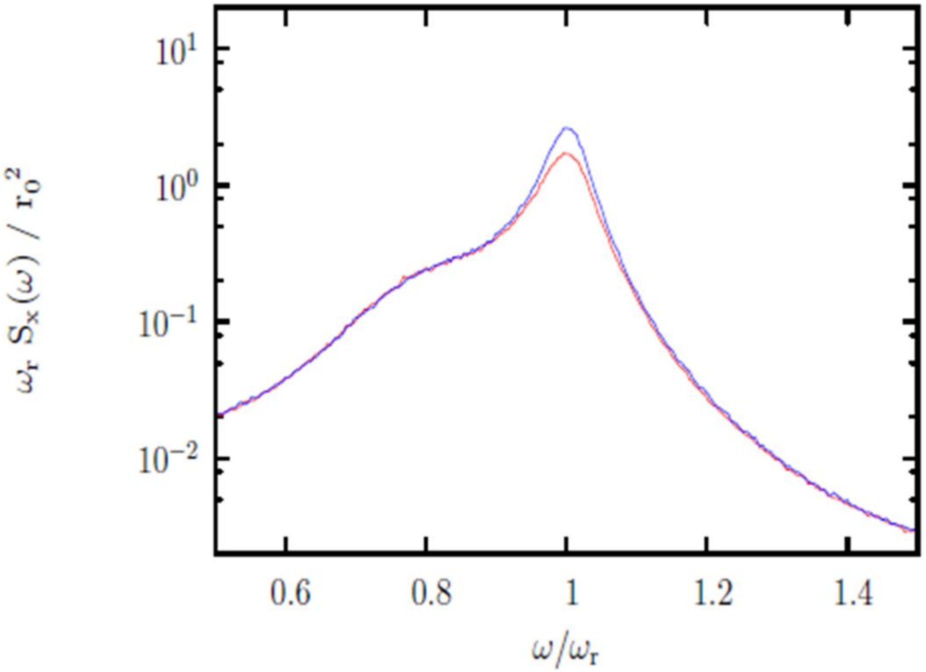

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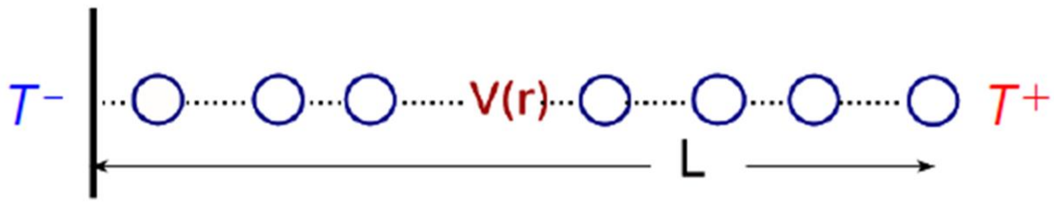

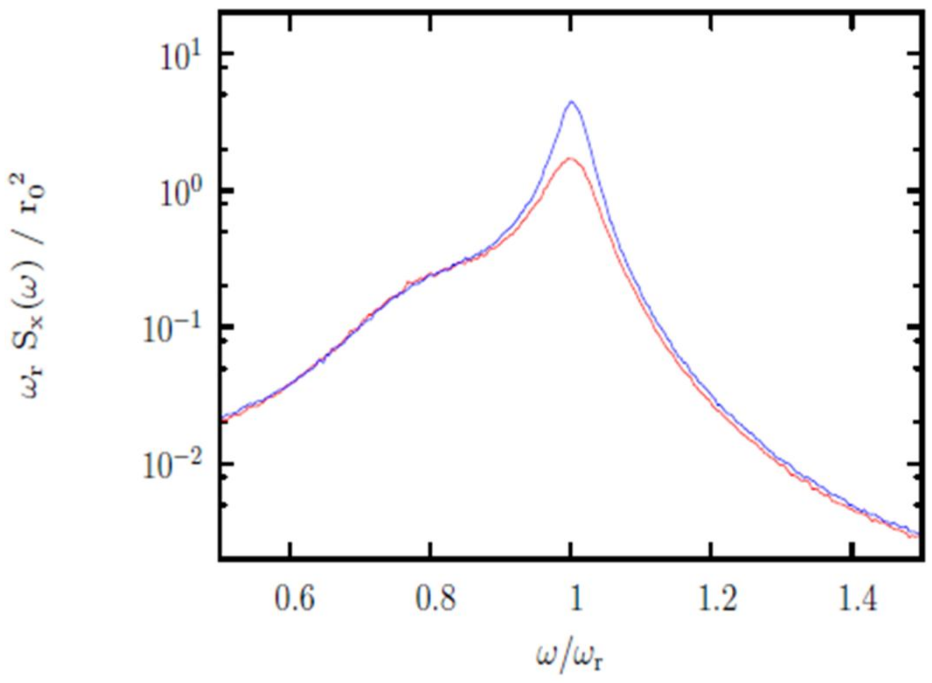

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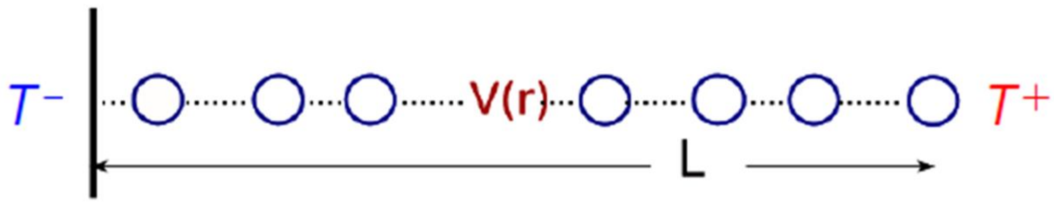

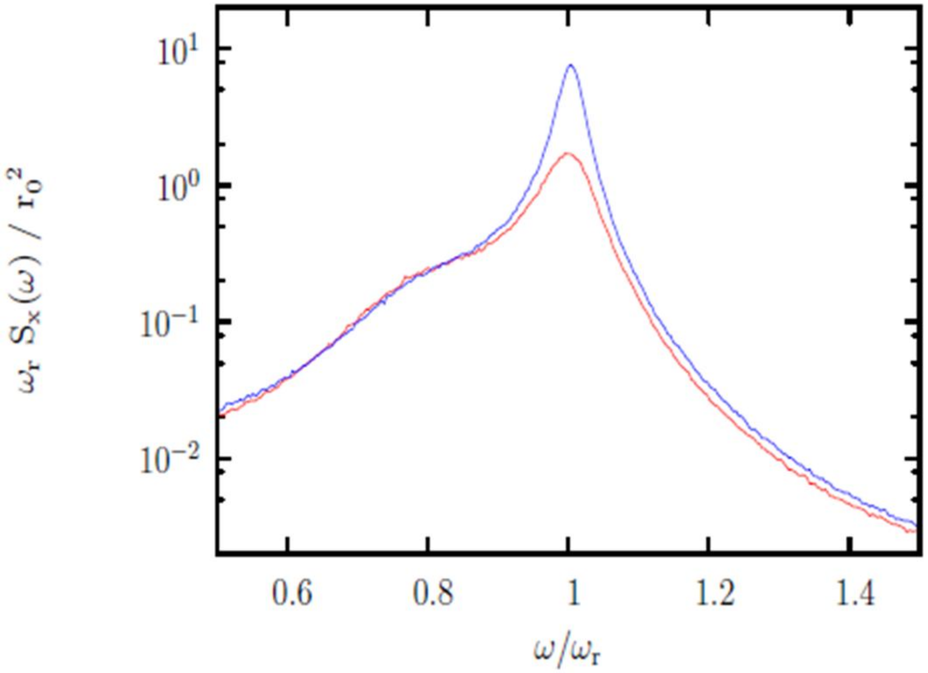

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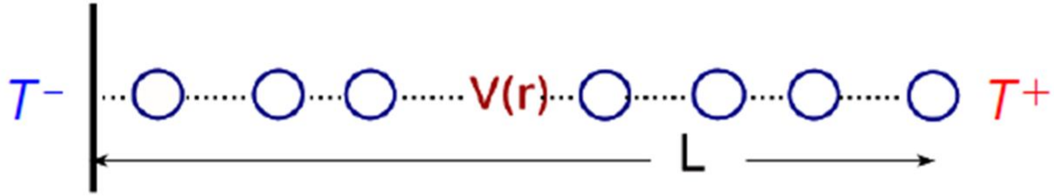

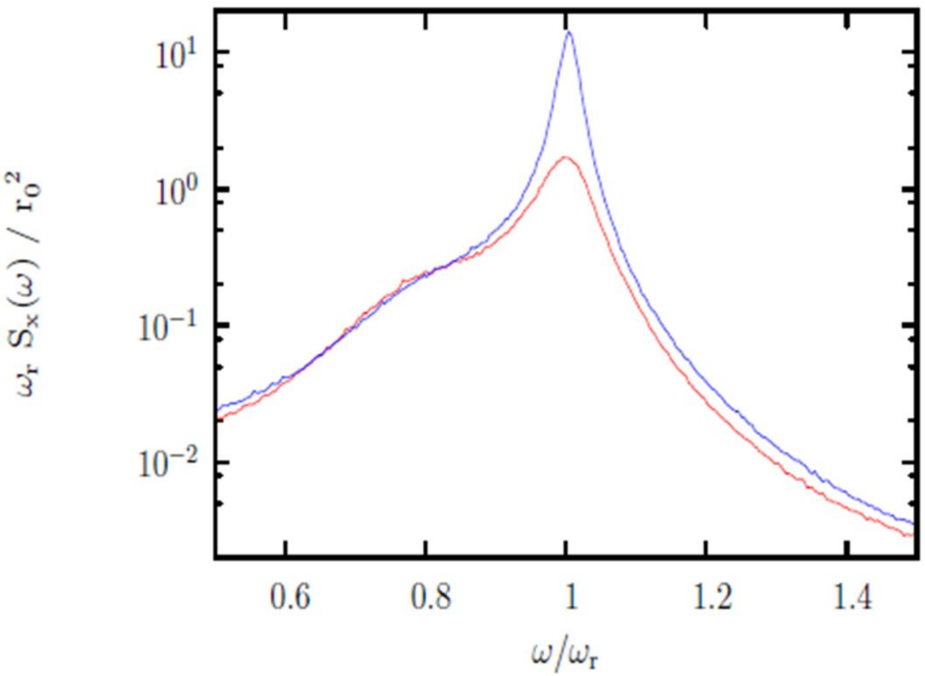

Area below the peak is a measure of $\mathbf{T_{eff}}$.
It grows despite $\mathbf{T_{average}}$ fixed.

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Heat flow as a correlation

Given the low losses, the dynamics is as if the sum of independent damped oscillators forced by thermal noise.
-> H defining the Boltzmann weight is diagonal in the normal mode variables.

$$P_{EQ}(\mathbf{x}, \mathbf{v}) = \frac{e^{-H(\mathbf{x},\mathbf{v})/k_B T}}{Z} \qquad H = \frac{1}{2}\sum_i \mu_i(\omega_i^2 x_i^2 + v_i^2)$$

In NESS, the heat flux is commonly defined via cross terms $x_i \, v_j$

thus

a current J≠ 0 means correlation between modes

Modified Boltzmann factor

A possibility [Miller, Larson, PRA 1979; Kato, Jou PRE 2001] is to write in the Boltzmann factor:

$$H / k_B T \rightarrow H / k_B T + \gamma J$$

By increasing the heat flux while maintaining $T_{average}$, one changes only the second term, ie the cross-terms.

After mathematics, one gets: $$< x^2 >_{NESS} = f\left(T, \Delta T\right)$$

Thus in NESS the effective temperature T_{eff} depends also on ΔT, ie T_{eff} is not anymore a good thermometer

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$R_{NEQ/EQ}$

Define the ratio:

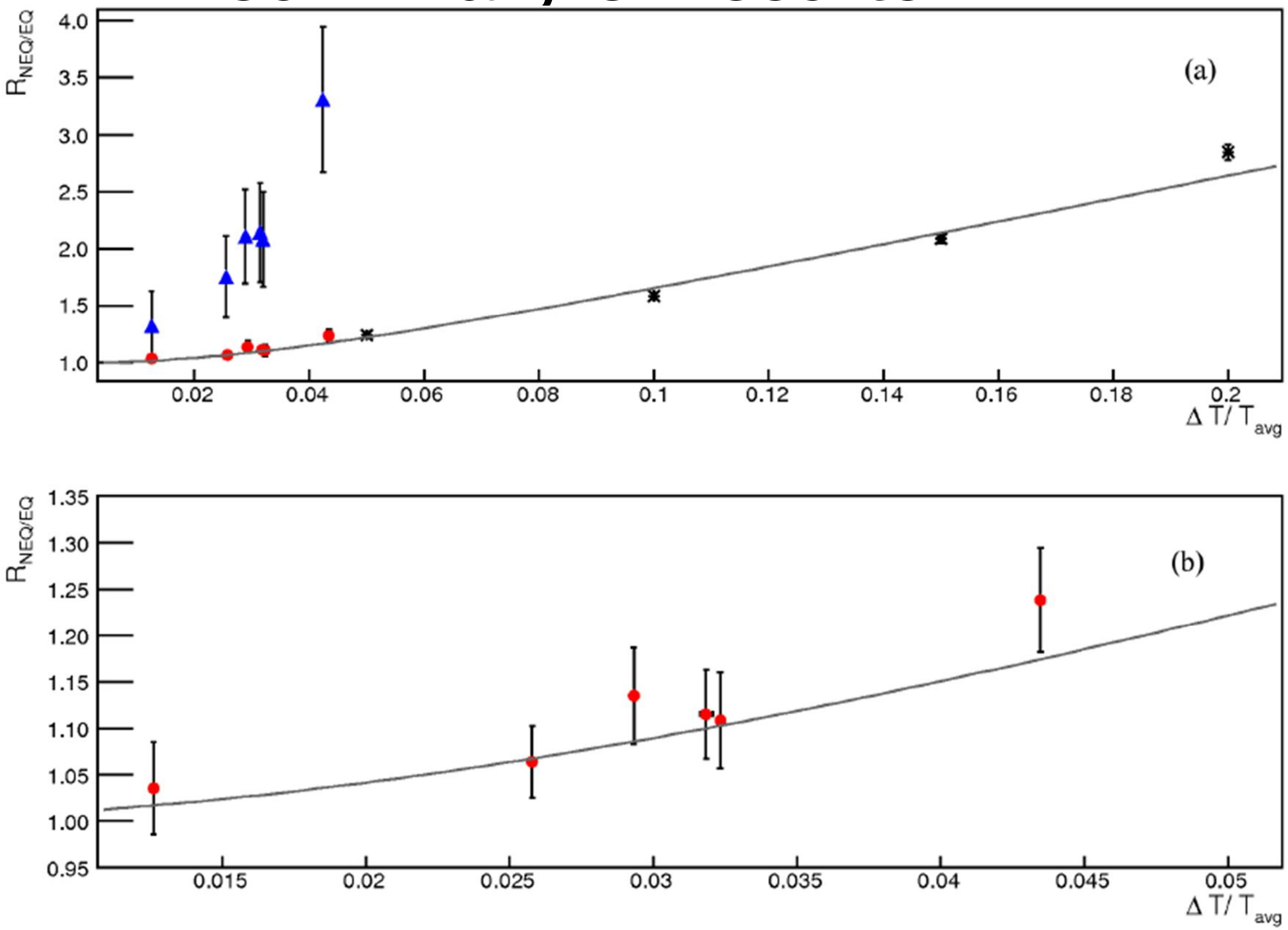
$$R_{NEQ/EQ} = \frac{T_{eff}\big|_{NEQ}}{T_{eff}\big|_{EQ}}$$

For T fixed and small ΔT: $R_{NEQ/EQ} - 1 \propto (\Delta T/T)^2$

Using this quantity we can compare:

- experimental data
- numerical data
- theoretical curve

Summary of results



● longitudinal mode (experim)

▲ transverse mode (experim)

× longitudinal mode (numerical)

— fit of numerics with theoretical model (1 parameter)

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Comments

The average energy of the mode does not scale with the average temperature

This differs from the behavior of the resonant frequency and Young modulus.

The T_{eff} result reveals **lack of Energy Equipartition**:
the low frequency modes have $T_{eff} > T$ (even the maximum T in the system) while the very high frequency modes have temperatute T(x)

Thus different modes have different energy :
energy is not equally distributed among the modes ≡ no energy equipartition

We have also studied the **statistical distribution** of the mode energies:
we found no deviations from the exponential distribution, up to the 4° order of momentum (likely to be published in PRE): consistent with our theoretical model

Concluding remarks

These results are similar to those found with a system of electromechanical oscillators actively cooled below thermodynamic temperature
by active feedback (ie Auriga detector) PRL 103, 010601 (2009)

We are now oberving lack of energy equipartition in out of equilibrium steady-states of 1d chain of oscillators (analytical and numerical model).

Energy ripartition different from equilibrium equipartition seems to emerge easily.

For IFOs care should be take when assigning a Temperature to infer noise density via the Fluctuation-Dissipation Theorem