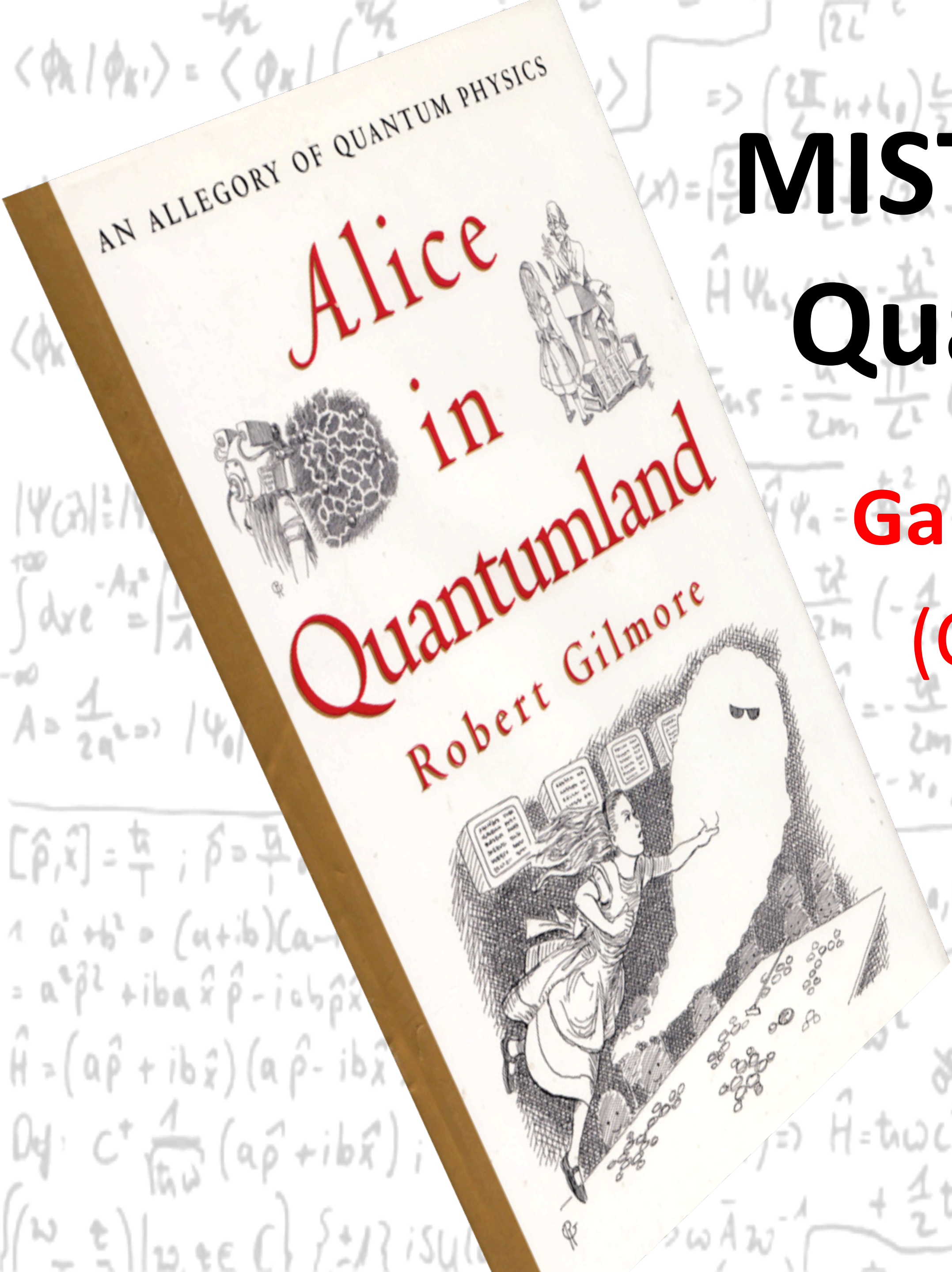

MIST in Quantumland

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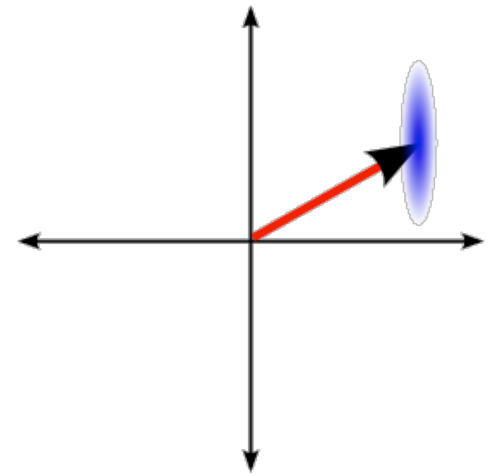
Outline

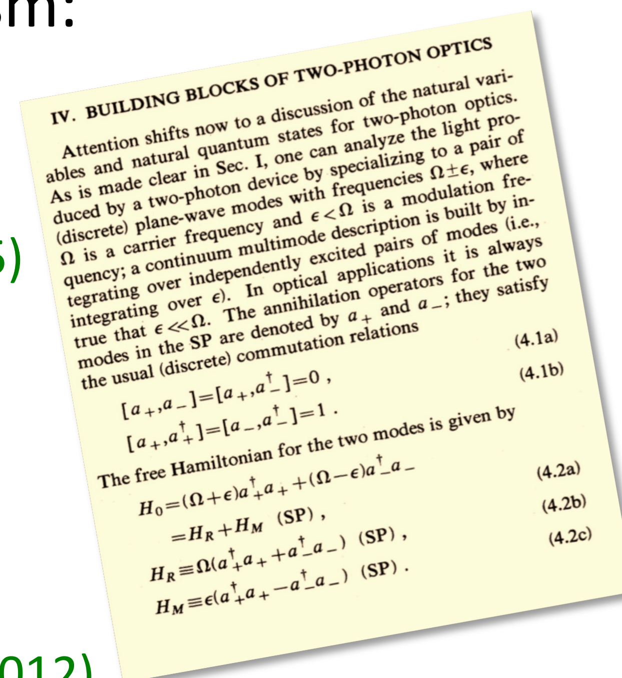
- This is not a "**What is MIST?**" talk…
  - LVC March 2014 (LIGO-G1400322)
  - LIGO seminar (LIGO-G1400061)
  - MIST web page
    http://sourceforge.net/projects/optics-mist/
  - G. Vajente, *Class. Quantum Grav.* **30,** 075014 (2013)
  - G. Vajente, *Class. Quantum Grav.* **31,** 075005 (2014)
- Radiation pressure is simulated in MIST since September 2013
- In the last version, **quantum noise** computation is also included
- This talk is about **how** quantum noise is computed in MIST

Quantum noise bibliography

- We often explain quantum noise in the framework of squeezed light with the “ball-on-the-stick” description



- Mathematically, quantum noise is often described in the “two-photons” formalism:
  - Seminal papers
    - Caves et al. Phys. Rev. A 31, 3068 (1985)
    - Schumaker et al. Phys. Rev. A 31, 3093 (1985)
  - A more modern approach
    - Kimble et al. Phys. Rev. D 65, 022002 (2001)
    - Corbitt et al. Phys. Rev. A 72, 013818 (2005)
  - A good (and long) review
    - Danilishin et al. Living Rev. Relativity 15, 5 (2012)



IV. BUILDING BLOCKS OF TWO-PHOTON OPTICS

Attention shifts now to a discussion of the natural variables and natural quantum states for two-photon optics. As is made clear in Sec. I, one can analyze the light produced by a two-photon device by specializing to a pair of (discrete) plane-wave modes with frequencies $\Omega\pm\epsilon$, where Ω is a carrier frequency and $\epsilon<\Omega$ is a modulation frequency; a continuum multimode description is built by integrating over independently excited pairs of modes (i.e., integrating over ϵ). In optical applications it is always true that $\epsilon\ll\Omega$. The annihilation operators for the two modes in the SP are denoted by a_+ and a_-; they satisfy the usual (discrete) commutation relations

$$[a_+,a_-]=[a_+,a_-^\dagger]=0\,, \quad (4.1a)$$

$$[a_+,a_+^\dagger]=[a_-,a_-^\dagger]=1\,. \quad (4.1b)$$

The free Hamiltonian for the two modes is given by

$$H_0=(\Omega+\epsilon)a_+^\dagger a_+ +(\Omega-\epsilon)a_-^\dagger a_- \quad (4.2a)$$

$$=H_R+H_M \quad (\text{SP})\,, \quad (4.2b)$$

$$H_R\equiv\Omega(a_+^\dagger a_+ + a_-^\dagger a_-) \quad (\text{SP})\,, \quad (4.2c)$$

$$H_M\equiv\epsilon(a_+^\dagger a_+ - a_-^\dagger a_-) \quad (\text{SP})\,.$$

Two-photons vs sidebands formalisms

- MIST and most simulations treat audio frequency motions in the "audio sidebands" formalism

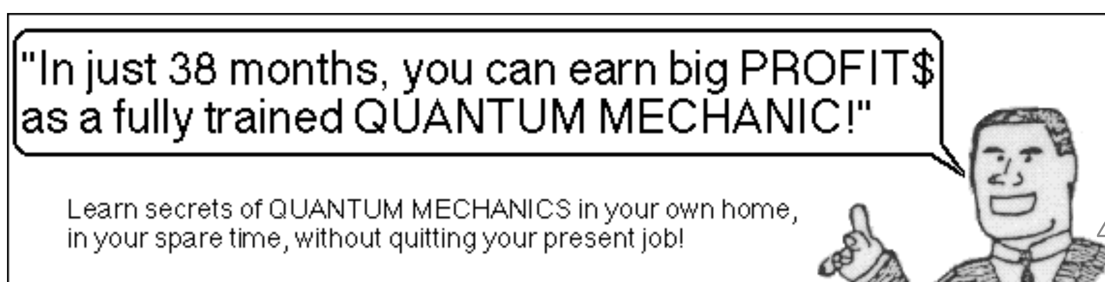
$$E(t) = e^{i\omega_0 t}\left(E_0 + E_+ e^{i\Omega t} + E_- e^{-i\Omega t}\right)$$

where ω_0 is the laser carrier frequency and Ω is the signal frequency (for example motion of a mirror)

- In quantum noise papers you see it written in a slightly more complex way:

$$E(t) = \sqrt{\frac{h\omega_0}{Ac}}\, e^{i\omega_0 t} \int_0^\infty \frac{d\Omega}{2\pi}\left[a_+(\Omega) e^{i\Omega t} + a_-(\Omega) e^{-i\Omega t}\right]$$

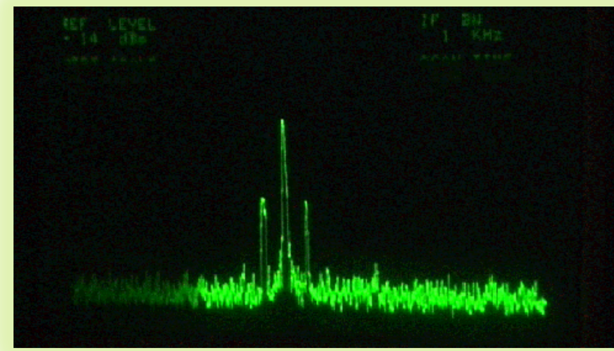
- But as long as you ignore that the a's are quantum operators, everything is fine…



Two-photons vs sidebands formalisms

- The two-photons formalisms mixes the audio sidebands, to take into account phenomena where correlated pairs of photons are created (like OPAs in squeezers)

$$a_1 = \frac{a_+ + a_-^+}{\sqrt{2}} \qquad a_2 = \frac{a_+ - a_-^+}{\sqrt{2}}$$



- This is just a linear combination of the coefficients we normally use in the audio sidebands approach

$$\begin{pmatrix} E_+ \\ E_-^+ \end{pmatrix} = \sqrt{\frac{h\omega_0}{16\pi}} \begin{bmatrix} 1 & i \\ 1 & -i \end{bmatrix} \begin{pmatrix} a_+ \\ a_- \end{pmatrix} = M \begin{pmatrix} a_+ \\ a_- \end{pmatrix}$$

- Since our IFOs are linear systems for the audio sidebands, we can compute their input-output relations using either formalism, and convert back to the other one

Quantum noise – basic computation

Quantum vacuum fields enter from all open ports of the IFO

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

The IFO acts as a linear operator and can mix the two quadratures

$$T = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

And we get some vacuum fields at the output, impinging on our PD

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

The signal at the photodiode output (at audio frequency) is a linear combination of the two quadratures

$$S = C_1\, b_1 + C_2\, b_2$$

To get the power spectral density of the noise on the signal, we have to "average" the quantum operators on the input vacuum field

$$S(\Omega)\delta(\Omega - \Omega') = \langle in | S(\Omega) S(\Omega')^+ | in \rangle$$

S_v is the vacuum cross-spectral density matrix (which is unity fo unsqueezed vacuum)

$$S(\Omega) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} T\, S_v\, T^+ \begin{bmatrix} C_1^* \\ C_2^* \end{bmatrix}$$

Quantum MIST /1

- MIST **does not use the two photons formalisms**
- Quantum vacuum fields can be injected anywhere
  - They can be converted into the corresponding audio sideband components using the "base change" matrix M
- Then MIST can propagate as usual the audio sidebands to all output ports, including radiation pressure effects

www.croplands.com.au

$$\begin{pmatrix} F_+ \\ F_-^+ \end{pmatrix} = T \begin{pmatrix} E_+ \\ E_-^+ \end{pmatrix}$$

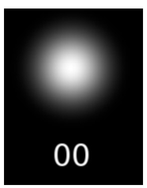
- The (audio frequency) signal on a photodetector is a linear combination of the audio sidebands

$$S(\Omega) = C_+ F_+ + F_-^+ C_-$$

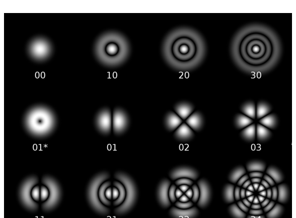
where in general the fields F are vectors in a HOM basis

Quantum MIST – higher order modes

- Each **higher order mode** in the input vacuum field is assumed to be independent of the others
- It must be propagated separately:
  - MIST computes the output of the IFO for each one of the HOMs in input separately, including all possible couplings among modes
- Each HOM in input contributes to quantum noise in output
  - All contributions are summed incoherently



$$S(\Omega) = \left[\left(C_+ \;\; C_-^T\right) T_{00\to HOMs} M\right] \cdot S_v \cdot \left[M^+ T^+_{00\to HOMs} \begin{pmatrix} C_+^+ \\ C_-^* \end{pmatrix}\right]$$



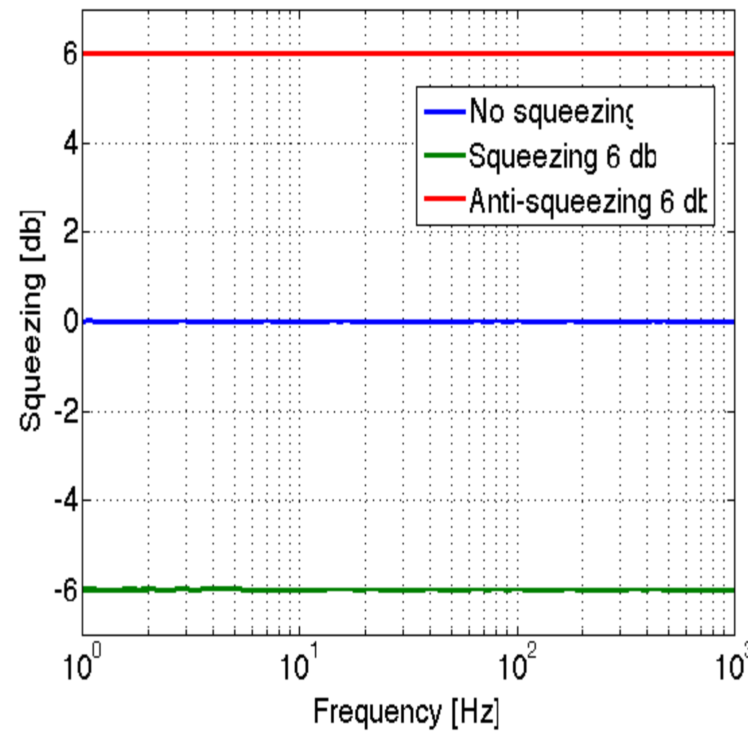
$$S(\Omega) = C_+ F_+ + F_-^+ C_-$$

One contribution from each HOM of the input vacuum field

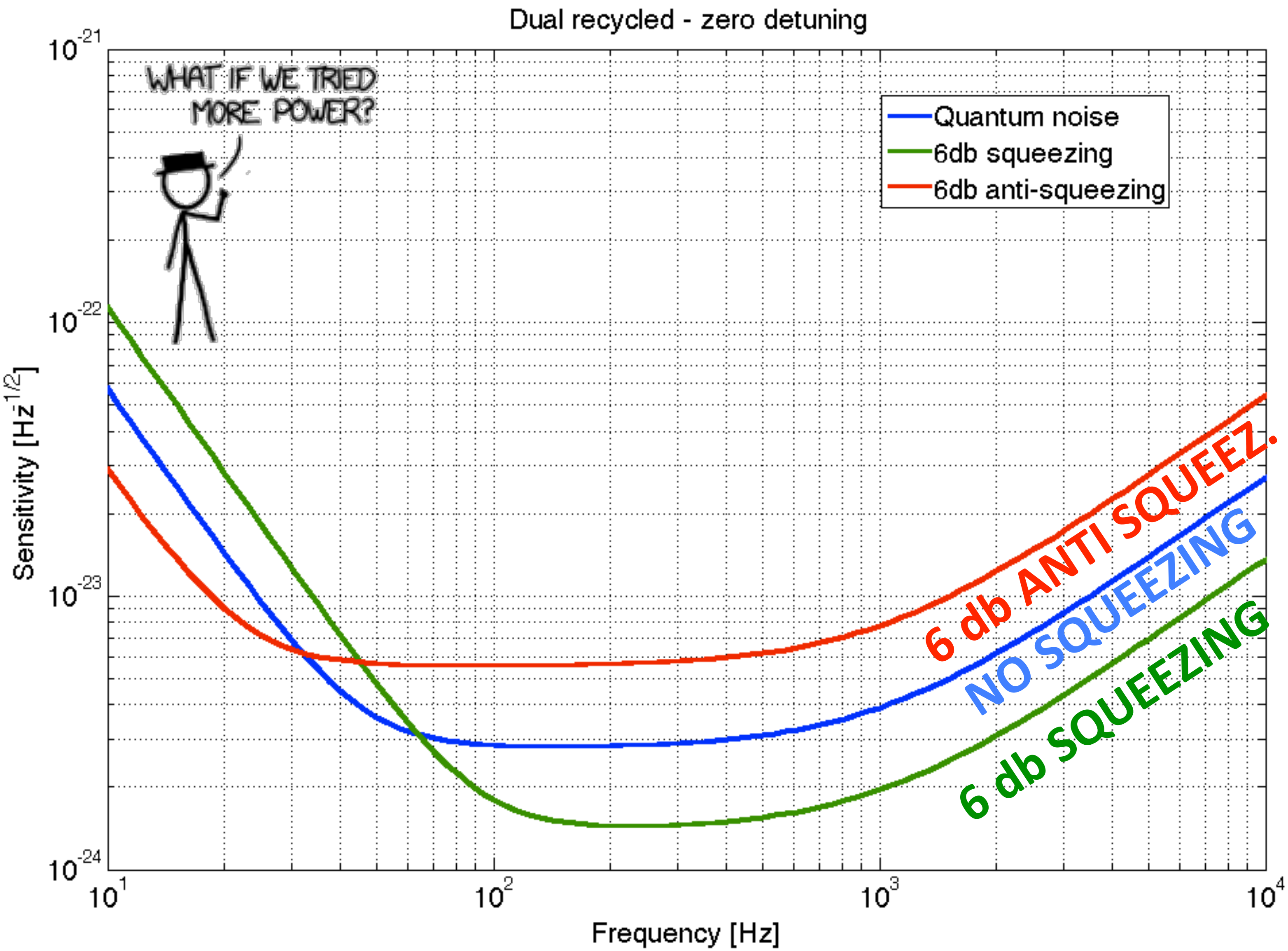
Quantum MIST /3

- Presently, the user must decide at which ports quantum vacuum fields must be injected
  - MIST does not include automatic injection of vacuum at all lossy ports
  - In the future this will be an option, together with automatic computations of losses due to clipping
- Arbitrary squeezing factor and angle can be injected in any point inside the optical system

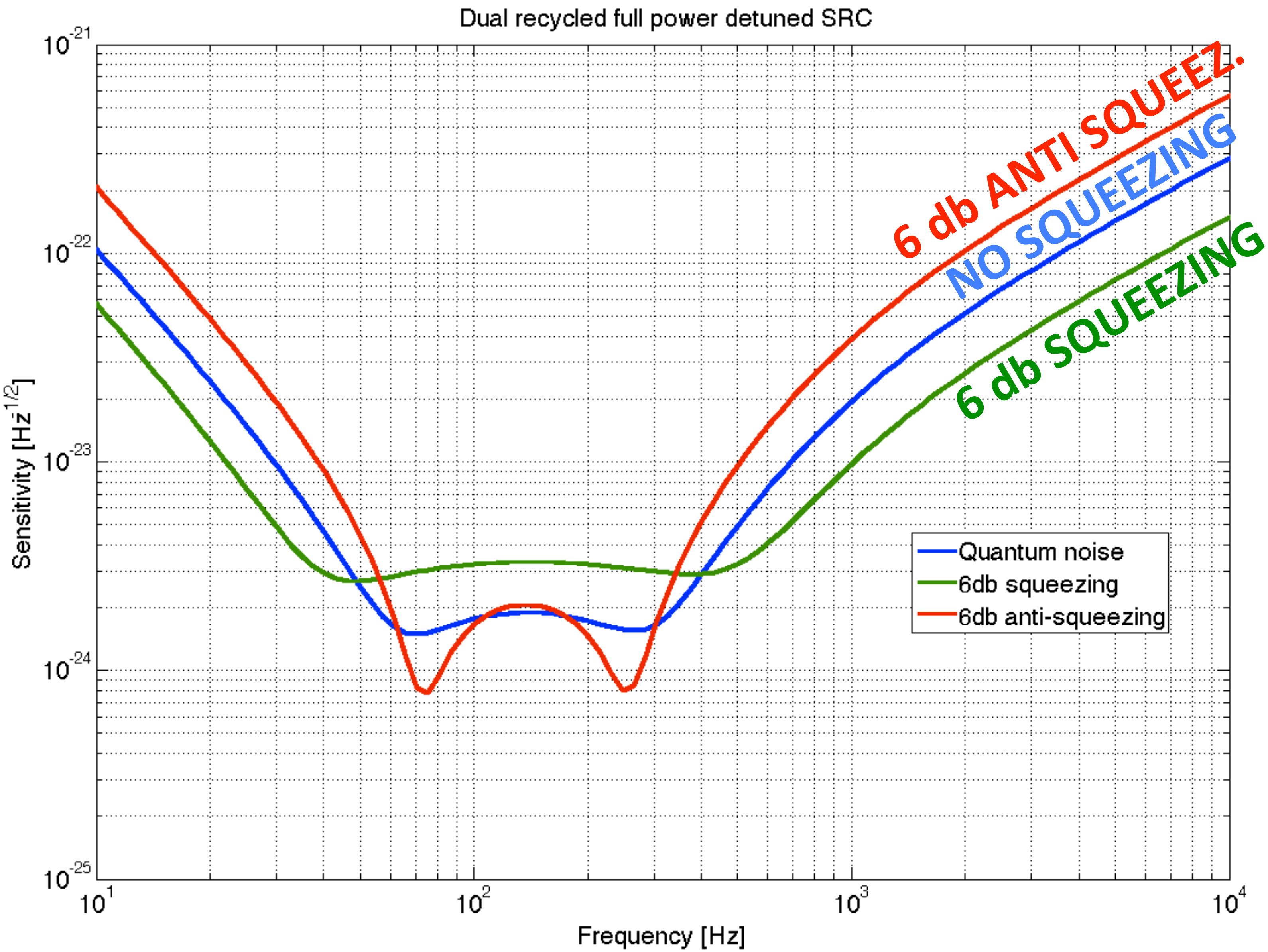
```
vacuum name node
    db=squeezing-factor-db
    angle=rotation-angle-deg
    w=vacuum-mode-beam-size
    R=vacuum-mode-curvature
    ampl=amplitude-relative
    from=object-to-set-direction
    into=object-to-set-direction
```



Dual recycling with squeezing /1

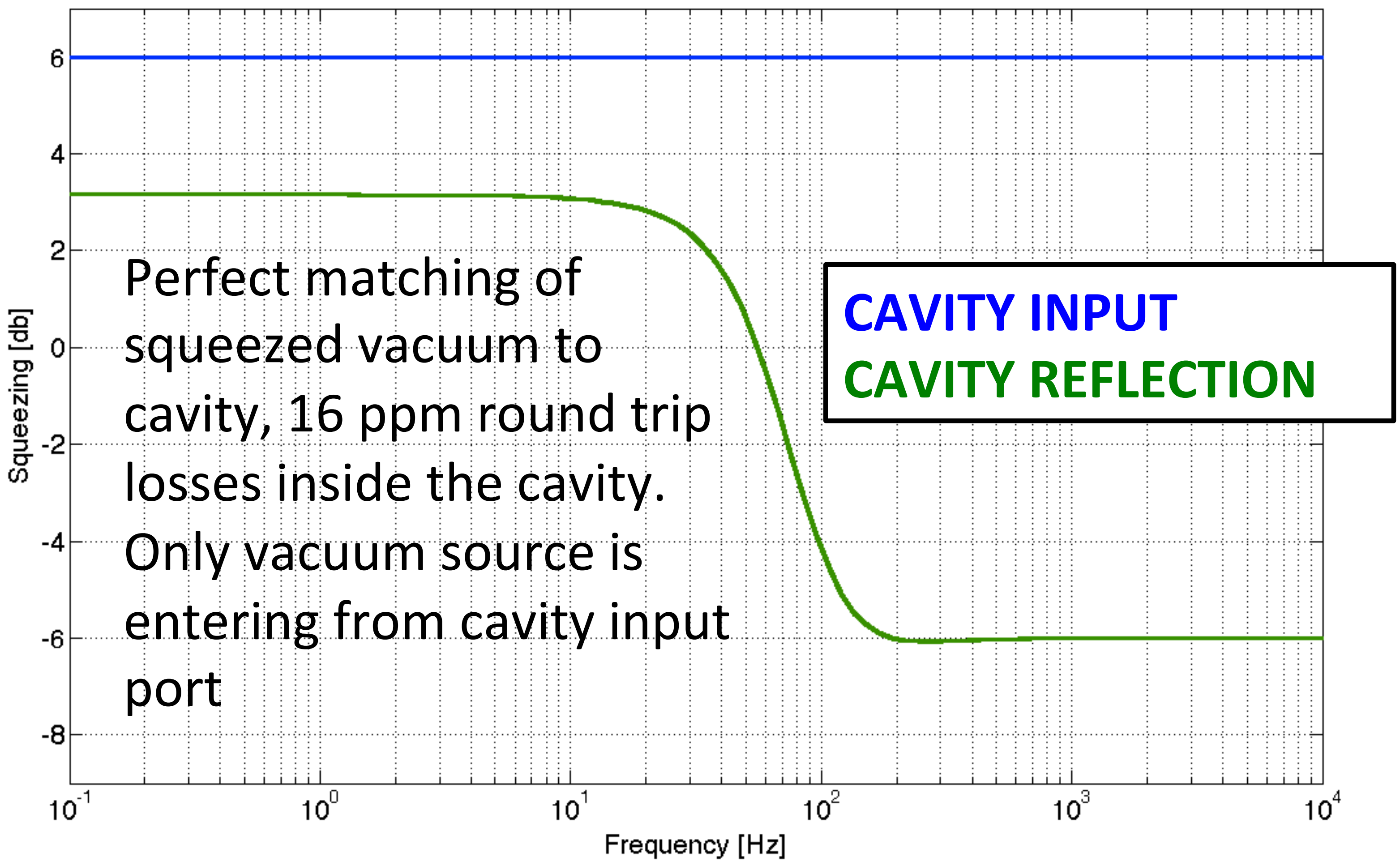


Dual recycling with squeezing /2

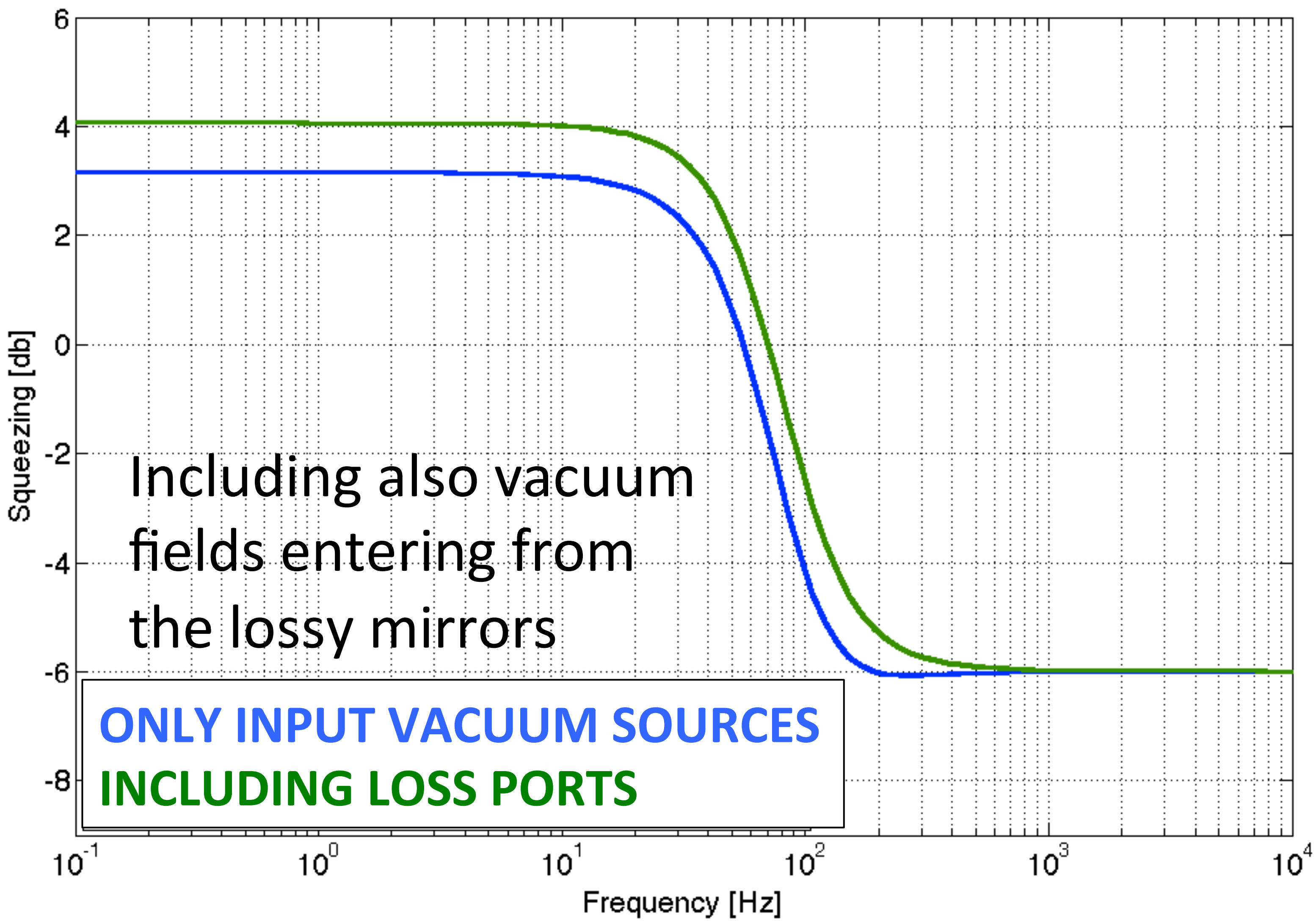


A filter cavity /1

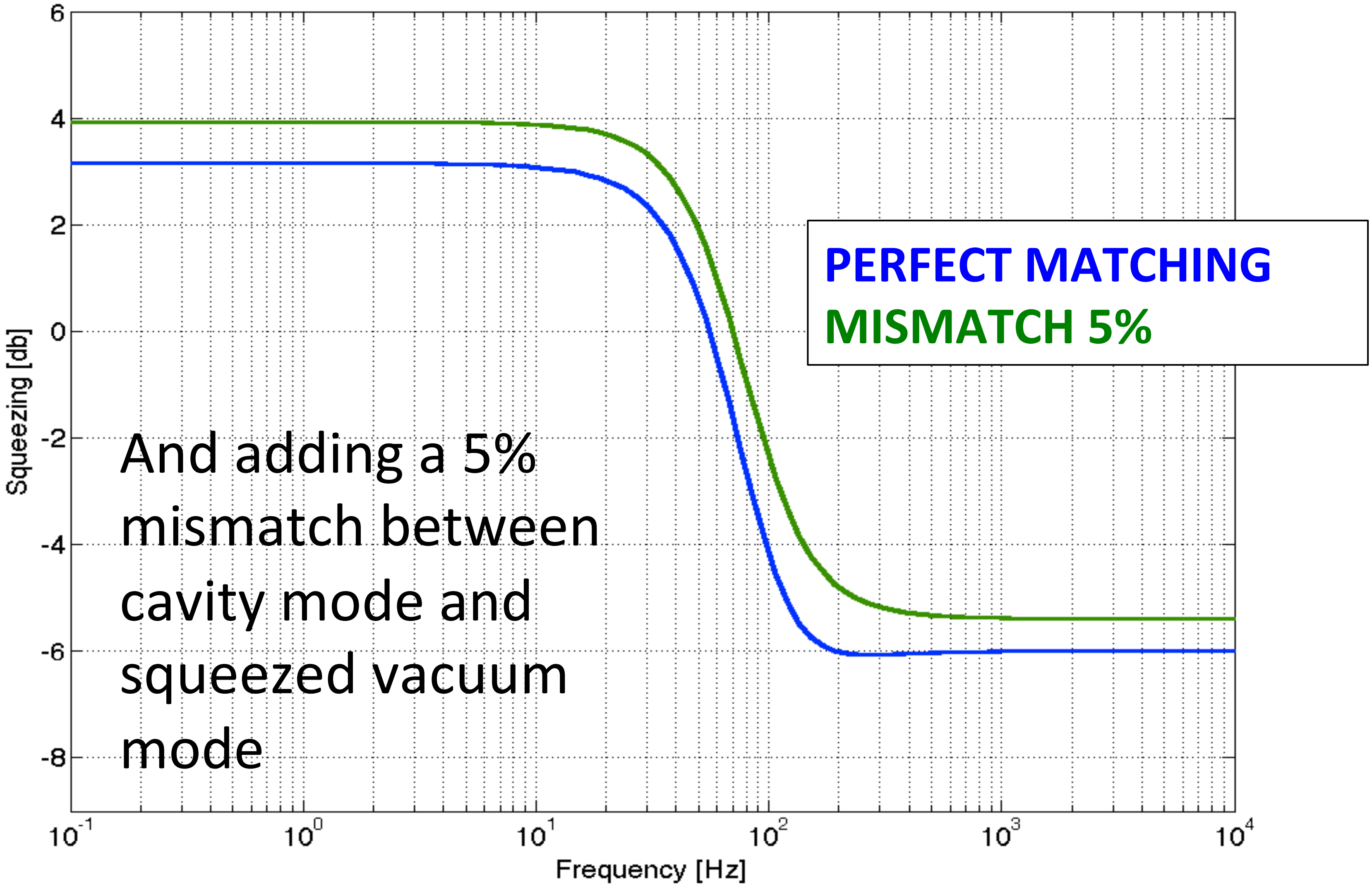
- Parameters from *Evans et al. Phys. Rev. D 88, 022002 (2013)*



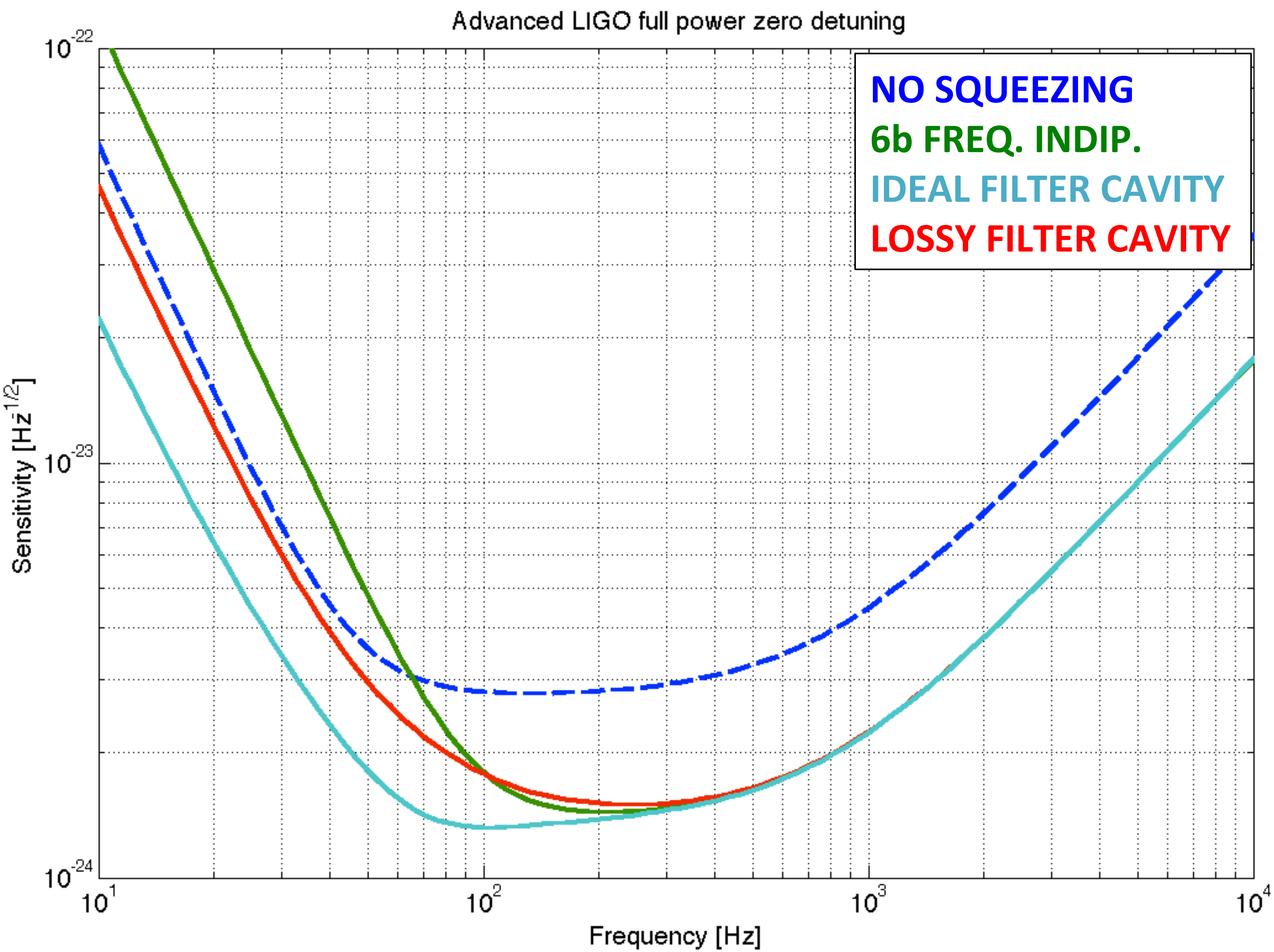
A filter cavity /2



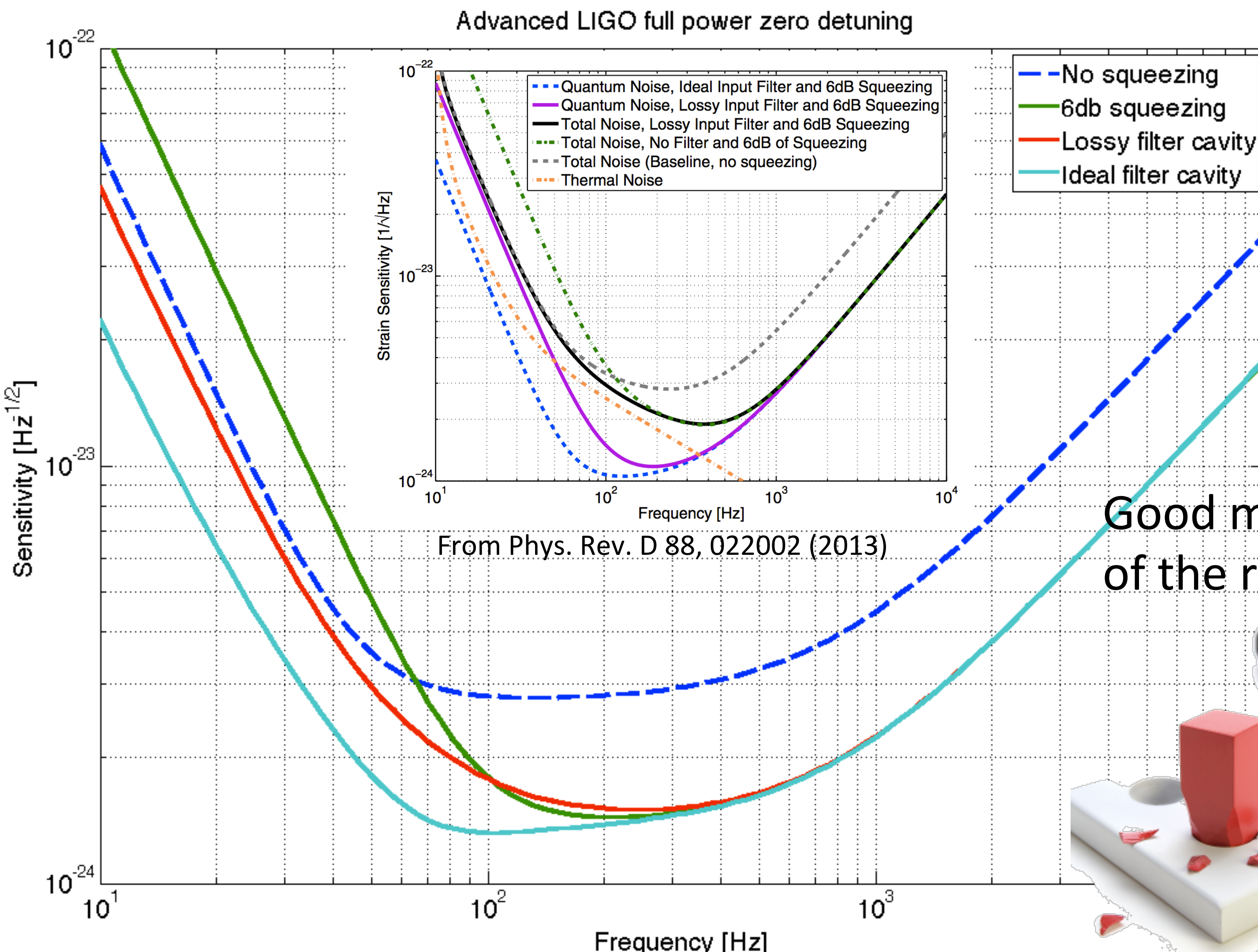
A filter cavity /3



Advanced LIGO with filter cavity

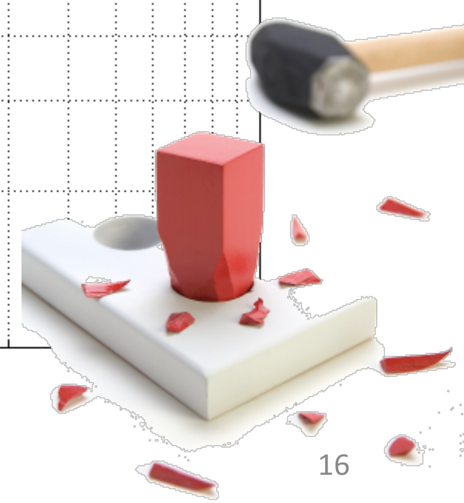


Advanced LIGO with filter cavity



From Phys. Rev. D 88, 022002 (2013)

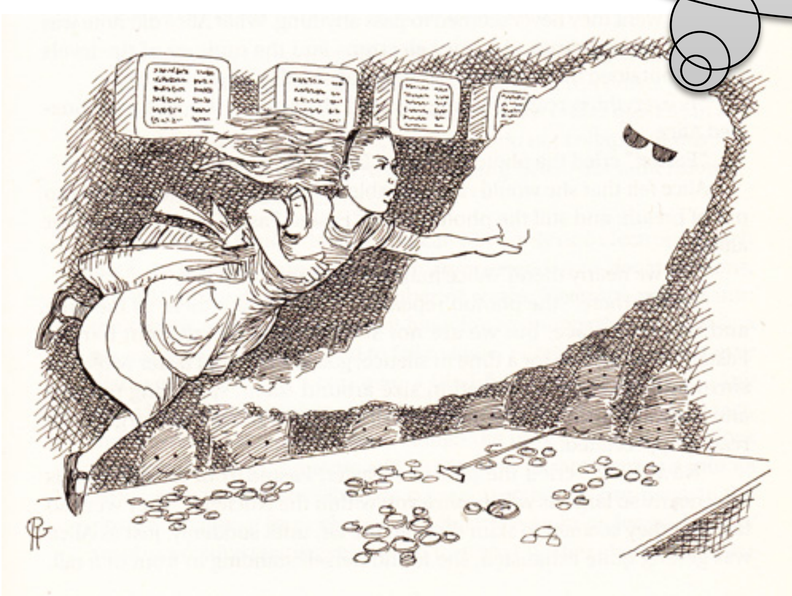
Good matching of the results



Conclusions

MIST entered in the quantum land, who knows what's there?

If you're interested in simulating quantum noise with MIST, contact me (vajente@caltech.edu) and download MIST: http://sourceforge.net/projects/optics-mist/ *
and subscribe to the **google group**: https://groups.google.com/forum/#!forum/mist-users-group



TO DO SOON:

- add automatic insertion of vacuum fields at all lossy ports (including clipping points)

* (almost) all the examples shown in this talk are included in the MIST package documentation